Report on Doctoral Thesis of Ivan Kolář: Spacetimes with symmetries in a general dimension

In this Thesis, the author studies a class of higher-dimensional spacetimes with enhanced symmetry described by the (generalized) Kerr–NUT–AdS family of metrics. The main results are gathered in 3 parts, two of which have already been published in Physical Review D and the last submitted for publication recently.

In the first part the author generalizes Carter’s separability ansatz, that has played an important role for the study of 4-dimensional black holes and their properties, to higher dimensions to re-obtain two (previously known) important families of metrics with separable Klein–Gordon equation. The warped products of these are also studied.

In the second part, the interpretation of parameters of Kerr–NUT–AdS spacetimes is further studied, the roots of metric functions are analyzed and the admissible ranges of coordinates determined; the main focus is on understanding the rotational and NUT charges—two important limiting cases of near horizon geometries and NUT-like limits are obtained.

In the third part, the axes and regularity of Kerr–NUT–AdS metrics are studied, drawing an analogy with the spinning cosmic string spacetimes. I consider this part the most physically interesting.

Overall, the Thesis is of high quality. It is well written and with minimum number of typos, it is presented in a unified notation, and contains a number of original technical results. The author has performed a lot of involved calculations that help to elucidate the complicated structure of the studied family of spacetimes and their subtle limiting sub-cases. At the same time I find the notation a bit heavy and the focus on generality is at several places at the expense of readability. The work would definitely benefit from adding simple examples and metrics written in standard coordinates the reader could relate to. In this respect, I really appreciated the discussion of the spinning string spacetimes in the third part.

In any case I am very happy to recommend this manuscript to be recognized as a Dissertation Thesis and the author to be awarded PhD degree in physics. I support the overall evaluation “excellent”.

I propose the following questions for the discussion:

1. In part I, a very special “simple separable metric ansatz” was studied and shown to (only) yield already known metrics. I wonder as to why a more general ansatz, described by the theory of Separability Structures reviewed in the Introduction—in particular by Eq. (54), has not been used and further studied. Can the author identify the matrices $M$ and $N$ in (54) for his ansatz in Part I, say in 4d, and discuss which possible further generalizations (if any) might be interesting to consider for future studies?

2. On page 19, the author mentions that the massless Klein–Gordon equation separates in the Plebanski–Demianki class of spacetimes. Is this really true, or one would have to consider a conformally invariant scalar field?
3. Page 38, uppermost paragraph: it is claimed there that the obtained spacetimes are new solutions in higher dimensions. I wonder if this is really true and whether these were not already found in arXiv:0805.0838 as the “generalized Kerr–NUT–AdS family”. Some comments on this issue are due.

4. Page 48, the statement that the “Kerr–NUT–AdS metrics remain the most general higher-dimensional solutions with a spherical horizon topology” is certainly not true. There are various analytic supergravity generalizations, “bumpy black holes” constructed numerically in arXiv:1410.4764, and so on. Perhaps some comments on this are due.

5. In part III, the axes of Kerr–NUT–AdS family are analyzed in detail. I wonder what the author has to say about the 4d Kerr–NUT–AdS spacetime identified for example in (overall) references [42] and [43]. Such solutions seem to possess, apart from the NUT-related Misner strings, also the unbalanced conical singularities—thence hinting on the presence of acceleration. I wonder if the same issue remains upon the new analysis of the solution (presented in this Thesis) and whether it is also present in higher dimensions. If so, the quest for finding C-metric in higher dimensions could actually already be resolved right here :) Can the 4d Kerr–NUT–AdS solution be written “only with Misner strings” but no acceleration?

6. In Carlip’s paper, gr-qc/9906126, it is claimed that the symmetry axes of the Taub–NUT solution are Killing horizons. Is this also true for the axis of the spinning string spacetime studied in this Thesis?

Finally, let me just mention 2 typos: i) Page 17, between (71) and (72): “Darboux coframe” and ii) Page 34, before (3.2) Missing gap before “The Christoffel” and double “ff” should be there.