Supervisor's report on the PhD thesis

Miroslav Olšák: Generalizing CSP-related results to infinite algebras

The thesis consists of 5 papers, denoted (A) – (E), and an introduction providing a wider background and discussion of the results. The results extend some parts of the universal algebraic theory, developed in connection with the Constraint Satisfaction Problem (CSP), from finite algebras to infinite ones.

The results are very strong, interesting, and often surprising. One of the highlights is a theorem from (A) (published in Bulletin of LMS) proving that a specific system of equations is, in some sense, a weakest nontrivial system for all idempotent algebras. The existence of such a weakest system is perhaps *the* most surprising recent result in universal algebra. It has quickly become widely known in the community and the involved operations are now commonly referred to as the Olšák terms. It is also interesting that the same system of equations has found significant application in computational complexity, in an improved hardness result for approximate graph coloring (Bulín, Krokhin, Opršal).

The novelty and variety of proof methods is another strength of the thesis. Although some parts of (A) use (but further significantly develop) the technique of absorbing sets, the methods in the remaining four papers (B) – (E) are strikingly novel. Paper [B] establishes a connection between semirings and certain subsets of free algebras, papers [C] and [D] employ complex primitive positive interpretations (instead of working on the algebraic side as is usually done in the literature), and paper [E] introduces a technique to deal with extremely long terms. The methods promise further applications in universal algebra and CSP. A first work (by Gillibert, Jonušas, Pinsker) directly inspired by [C],[D] has already been submitted.

Miroslav has achieved the results independently. My input was very small, all of the included papers are single-author. Apart from the results in the thesis, Miroslav has made significant contributions to other papers as well: 2 papers on the infinite-domain CSP (LICS'17 + Journal of Mathematical Logic together with B., Kompatscher, Pinsker, Van Pham, LICS'19 with Bodirsky, Mottet, Pinsker, Opršal), Promise CSPs (ICALP'19 with Ficak, Kozik, Stankiewicz), and AI (NIPS'18 with Kaliszyk, Urban, Michalewski).

In summary, the thesis is excellent and I strongly recommend Miroslav Olšák is awarded a PhD in Mathematics.

Prague, 16 May 2019

Libor Barto