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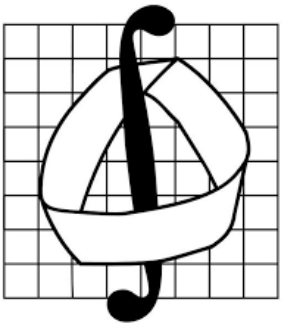
FACULTY OF MECHANICS AND MATHEMATICS



Dear Dean Kratochvíl,

It is my big pleasure to provide a report on the doctoral thesis of Mr. Miroslav Olšák. The thesis consists of 5 papers discussing Maltsev conditions and a generalization of loop lemmas.

Maltsev conditions are systems of equations in some signature, and an algebra satisfies a Maltsev condition if there exist terms satisfying all the equations. Many important properties of an algebra can be described via Maltsev conditions (congruence permutability, congruence distributivity, congruence modularity and so on). Additionally, Maltsev conditions proved their importance while studying the complexity of the Constraint Satisfaction Problem over a finite language of constraints. For instance, the Constraint Satisfaction Problem on a finite set is tractable (can be solved in polynomial time) if and only if the algebra of all polymorphisms of the constraint language has a WNU term (or, equivalently, a cyclic term, or a Siggers term, or a Taylor term), i.e. if the algebra satisfies a nontrivial Maltsev condition. A loop lemma is a claim saying that each digraph satisfying some structural assumptions (such as connectedness in some sense) and compatible with an algebra satisfying some properties has a loop. Such claims showed their effectiveness in studying the complexity of the Constraint Satisfaction Problem but they also provided new results to universal algebra.

Each paper included into this thesis contains solutions of highly nontrivial open questions formulated by well-known mathematicians. Especially, I would like to mention the first paper in which the author found the weakest nontrivial Maltsev condition for idempotent algebras. Precisely, he proved that every idempotent algebra that satisfies a non-trivial equational condition has a term satisfying two identities (now such a term is called Olšák term). This outstanding and surprising result shows that the author is capable of solving very challenging mathematical problems.

In the second paper the author gives a nice Maltsev condition characterizing SD($\wedge$) algebras. The remaining three papers are devoted to loop conditions and the generalization of loop lemmas for infinite domain, where loop conditions are Maltsev

conditions given by exactly one equation. The questions considered in this thesis are important for both universal algebra and computational complexity, and the results obtained by the author have already been used to study the complexity of the Constraint Satisfaction Problem on infinite domains. Additionally, I would like to point out that each paper not only contains a solution of an open question, it also provides a broad introduction showing a high mathematical background of the author. Probably, the only thing missing in the introduction is an elementary example showing that loop lemmas are strongly connected to the complexity of the Constraint Satisfaction Problem on finite/infinite domains, but I am not sure that such an example exists.

Also, I want to emphasize that all the papers are written in a very clear way and could be recommended to mathematicians who are not experts in universal algebra. I really enjoyed how the author reveals the connection between Maltsev conditions, positive primitive definitions, and loop lemmas for digraphs. His proofs contain nice explanations, diagrams, and motivations for next steps, which makes reading much easier. This also shows high mathematical level of the author.

Taking into account the significance of the mathematical results and the quality of the mathematical text, I strongly recommend Mr. Miroslav Olšák to be awarded a PhD in Mathematics.

Sincerely yours,

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