Inequalities for discrete and continuous supremum operators

Rastislav Olhava

In this thesis we study continuous and discrete supremum operators. In the first part we investigate general properties of Hardy-type operators involving suprema. The boundedness of supremum operators is used for characterization of interpolation spaces between two Marcinkiewicz spaces. In the second part we provide equivalent conditions for boundedness of supremum operators in the situation when the domain space in one of the classical Lorentz spaces $\Lambda^p_{w_1}$ or $\Gamma^p_{w_1}$ and the target space $\Lambda^q_{w_2}$ or $\Gamma^q_{w_2}$. In the case $p \leq q$ we use inserting technique obtaining continuous conditions. In the setting of coefficients $p > q$ we provide only partial results obtaining discrete conditions using discretization method. In the third part we deal with a three-weight inequality for an iterated discrete Hardy-type operator. We find its characterization which enables us to reduce the problematic case when the domain space is a weighted $\ell^p$ with $p \in (0,1)$ into the one with $p = 1$. This leads to a continuous analogue of investigated discrete inequality. The work consists of author’s published and unpublished results along with material appearing in the literature.