TECHNISCHE UNIVERSITÄT
CHEMNITZ

Technische Universität Chemnitz · 09107 Chemnitz

Building: Reichenhainer Straße 41
Room: 41/604
Phone: +49 371 531-22530
Fax: +49 371 531-836538
E-Mail: roland.herzog@mathematik.tu-chemnitz.de
Internet: http://www.tu-chemnitz.de/mathematik/part_dgl

Place, Date: Chemnitz, 10/6/2019

Report on the Dissertation

Towards Efficient Numerical Computation of Flows of Non-Newtonian Fluids

by Jan Blechta

Summary and Main Contributions

The thesis submitted by Jan Blechta considers a variety of aspects relevant for efficient numerical computations of fluid flow problems. In fact, the three chapters of the dissertation provide a systematic study of non-Newtonian constitutive relations and new existence proofs (Chapter I), new proofs for the localizability of the operator norm of bounded linear functionals on first-order Sobolev spaces with applications to a posteriori error estimation for nonlinear second-order partial differential equations (Chapter II), as well as a new analysis of so-called pressure convection-diffusion (PCD) preconditioners with practical boundary conditions for flows governed by the Navier-Stokes equations (Chapter III).

Before going into the details, it should be stated clearly that the dissertation covers a remarkable variety of topics in which the author has been able to make new contributions, connected by the common theme of incompressible fluids.

Detailed Report

Chapter I provides a systematic study of the constitutive laws for a class of incompressible fluids, and provides new proofs of existence of associated flow models in 3D domains, both in the stationary and instationary regimes. The material is based on a manuscript submitted to a peer-reviewed journal, written jointly by the author, J. Málek and J. Rajagopal (Texas A&M). The first part of this chapter is devoted to modeling aspects. Different classes of models based on continuous shear rate-driven or shear stress-driven constitutive relations are considered. Of particular interest are the models described in section 2.5, which possess two different behavioral regimes. These models become *activated* and change their behavior either when the stress, or the shear rate, respectively, exceed a certain value. They include so-called activated

Euler fluids, which are frictionless when the shear rate is low. Apparently this class of fluids has previously gone unnoticed. Boundary conditions which exhibit an activated behavior are also considered.

The second part of Chapter I comprises section 3, in which results for the new class of activated Euler fluids are derived, for which the admissible pairs of shear stress and shear rate belong to certain maximal monotone graphs. Despite the degeneracy of the differential operator in the balance of momentum, the authors can show the existence of a solution even for large data under appropriate, mild regularity assumptions. The proofs are innovative and require a careful choice of Sobolev exponents, Galerkin approximation, fixed point theorems, regularization of the constitutive relation and measure theoretic arguments.

Chapter II of the thesis establishes a new proof for the localizability of the operator norm of bounded linear functionals on first-order Sobolev spaces of type $W_0^{1,p}(\Omega)$, i.e., the $W^{-1,q}(\Omega)$-norm, where q and p are dual exponents. This chapter, as an article written jointly with J. Málek and M. Vohralík (Paris), has been published in the flagship IMA Journal on Numerical Analysis. The full scale $1 \leq p, q \leq \infty$ is considered. The main purpose of this study is to obtain a direct proof of the equivalence of global and local versions of the $W^{-1,q}(\Omega)$-norm for the residual of a general class of nonlinear second-order partial differential equations in divergence form. The localization happens over vertex-based patches. This theory allows the derivation of an a posteriori error estimation framework, utilizing its global reliability and local efficiency established elsewhere, which is analogous to the well-known results in the Hilbert space setting ($p = q = 2$). The result is first obtained in the presence of Galerkin orthogonality, see Theorem 3.7 in this chapter. An extension to the case of inexact solves, i.e., in the absence of Galerkin orthogonality, is provided in section 4. Numerical results for the p-Laplacian confirm the theoretical findings.

Finally, Chapter III considers the efficient solution of the linear systems arising as linearization of Oseen type of the stationary Navier-Stokes equations. The material in this chapter is previously unpublished work solely by the present author. The setting considered includes inflow, no-slip and do-nothing outflow boundary conditions, as relevant for practical applications. The author analyzes the two variants of the so-called pressure convection-diffusion (PCD) preconditioners. He is following the approach of operator preconditioning, i.e., provides an analysis in function spaces. The results comprise a number of well posedness, a priori and spectral results with carefully worked out Sobolev exponents and constants, under various (mild) assumptions on the "wind" function. Discrete realizations of the preconditioner are worked out in section 3, with an emphasis on the "correct incorporation of boundary conditions into PCD" (p.106) for various choices of the pressure space discretization.

Summary and Evaluation

The dissertation submitted by Jan Blechta contains a wealth of new results revolving around the common theme of incompressible fluids. I believe that the approach of a systematic classification of a broad class of fluids by means of their constitutive relation as well as the associated existence results for large data presented in Chapter I are genuinely novel and will serve as a major reference in this field in the future. This study has even led to the (mathematical) discovery of previously unnoticed class of actived Euler fluids, which are yet to discovered experimentally. Chapter II provides a new proof to a posteriori error estimation for general, nonlinear second-order problems. The impact of this part of the thesis reaches well beyond fluid flow problems. It is not surprising that these results have already been published in

TECHNISCHE UNIVERSITÄT
CHEMNITZ

one of the best numerical analysis journals. Finally, Chapter III makes important, rigorous and novel contributions to the analysis of the well-known pressure convection-diffusion (PCD) preconditioners for the stationary Navier-Stokes system, in the presence of inflow, no-slip and do-nothing outflow boundary conditions. A remarkable amount of technical estimates are required for this purpose. The author corrects mistakes and inaccuracies made elsewhere in the literature. I believe that the results of this chapter should be published in one or two scientific articles.

Altogether, Jan Blechta demonstrates in his thesis his ability for creative scientific work of the highest quality in a remarkable breadth of areas. The thesis is very well written, nicely illustrated and I found hardly any typos at all.

I strongly recommend acceptance of the thesis.

Roland Herzog