In the first part of this thesis we are concerned with the constitutive theory for incompressible fluids characterized by a continuous monotone relation between the velocity gradient and the Cauchy stress. We, in particular, investigate a class of activated fluids that behave as the Euler fluid prior activation, and as the Navier-Stokes or power-law fluid once the activation takes place. We develop a large-data existence analysis for both steady and unsteady three-dimensional flows of such fluids subject either to the no-slip boundary condition or to a range of slip-type boundary conditions, including free-slip, Navier's slip, and stick-slip.

In the second part we show that the $W^{-1,q}$ norm is localizable provided that the functional in question vanishes on locally supported functions which constitute a partition of unity. This represents a key tool for establishing local a posteriori efficiency for partial differential equations in divergence form with residuals in $W^{-1,q}$.

In the third part we provide a novel analysis for the pressure convection-diffusion (PCD) preconditioner. We first develop a theory for the preconditioner considered as an operator in infinite-dimensional spaces. We then provide a methodology for constructing discrete PCD operators for a broad class of pressure discretizations. The principal contribution of the work is that a clear and pronounced methodology for dealing with the artifical boundary conditions is given, including the inflow-outflow case, which has not been adequately addressed in the existing literature.