Posudek práce

předložené na Matematicko-fyzikální fakultě
Univerzity Karlovy

✠ posudek vedoucího ❑ posudek oponenta
✠ bakalářské práce ❑ diplomové práce

Autor: **Pavel Kůs**
Název práce: **Konformní symetrie a víry v grafenu**
Studijní program a obor: **Obecná fyzika**
Rok odevzdání: **2019**

Jméno a tituly vedoucího: **Doc. Alfredo Iorio**
Pracoviště: **Ústav částicové a jaderné fyziky**
Kontaktní e-mail: **alfredo.iorio@mff.cuni.cz**

Odborná úroveň práce:
✠ vynikající ❑ velmi dobrá ❑ průměrná ❑ podprůměrná ❑ nevyhovující

Věcné chyby:
❑ téměř žádné ✠ vzhledem k rozsahu přiměřený počet ❑ méně podstatné četné ❑ závažné

Výsledky:
❑ originální ✠ původní i převzaté ❑ netriviální kompilace ❑ citované z literatury ❑ opsané

Rozsah práce:
✠ veliký ❑ standardní ❑ dostatečný ❑ nedostatečný

Grafická, jazyková a formální úroveň:
❑ vynikající ✠ velmi dobrá ❑ průměrná ❑ podprůměrná ❑ nevyhovující

Tiskové chyby:
❑ téměř žádné ✠ vzhledem k rozsahu a tématu přiměřený počet ❑ četné

Celková úroveň práce:
✠ vynikající ❑ velmi dobrá ❑ průměrná ❑ podprůměrná ❑ nevyhovující

Slovní vyjádření, komentáře a připomínky vedoucího:

This thesis is dedicated to study aspects of the analog gravity stemming from the description of the low-energy spectrum of deformed graphene in terms of a massless Dirac field theory on curved backgrounds, in the spirit of [1]. There, special emphasis is put on emergent spacetimes that are of the form: $\mathbf{R} \times \mathbf{\Sigma}$, that is the product of flat time $\mathbf{R}$ and curved surfaces $\mathbf{\Sigma}$. Noticeable spacetimes of this kind were found to enjoy conformal flatness in three-dimensions, that amounts to $\mathbf{\Sigma}$s of constant curvature [1], and the associated nontrivial behaviors of the Dirac fields of graphene were later analyzed, see [2] for a review. As well known, for $\mathbf{\Sigma}$s of constant curvature, the conformal factor of the metric satisfies the Liouville equation, that enjoys symmetry under the Virasoro conformal algebra. Moving within that scenario, the focus of this thesis is on certain vortex solutions of the Liouville equation, characterized by a natural number N, and found in [3].

In this thesis, the Candidate first learned the necessary background on the advanced topics discussed earlier, as witnessed by the first three Chapters. There he introduces the massless Dirac field theory, well describing electronic properties of graphene in the low energy limit. Then discusses how to deal with spinors in a curved spacetime, and then illustrates the fact that the massless Dirac field theory is invariant under Weyl transformations, which has far-reaching consequences, some of which I illustrated above.

After this preparatory study, the Candidate faced the assignment I gave him, namely to solve a problem posed in [1], that is the identification of the kind of surfaces to which the vortex solutions of [3] correspond. This is an interesting problem in its own right, and may be relevant for further studies that would probe possible nontrivial behaviors of the Dirac field on the associated spacetimes of the kind $\mathbf{R} \times \mathbf{\Sigma_{vortices}}$. He did that by solving the difficult mathematical problem of identifying the appropriate change of coordinates, from the isothermal coordinates (natural for the Liouville equation), to the canonical coordinates for surfaces of revolution (necessary to plot and analyze surfaces). He found that, for a generic N the $\mathbf{\Sigma_{vortices}}$ are surfaces of positive constant Gaussian curvature, $K > 0$, of the Bulge type, and that only for $N = 1$ such surfaces are spheres. The correspondence he found is exact, and the explanation of the geometrical/physical meaning of the natural number N he provided is very clear. All of this is in the fourth Chapter of this thesis.

The Bulge type surfaces are "barrel shaped" surfaces with singular boundaries, that, together with the Spindle type of surfaces, are the (usually overlooked) alternatives to the sphere as representatives of a surface of constant $K > 0$. In fact, as long intrinsic properties are concerned, a coordinate redefinition maps both the Bulge and the Spindle shaped singular surfaces into the boundary-less sphere. Nonetheless, in a laboratory, where extrinsic properties matter too, these three types of surfaces are distinct. All these geometric issues are very well presented in a dedicated Chapter of the thesis, showing the Candidate's mastering of the subject. Noticeable is that, the singular boundaries of the "barrels" can be mapped into the corresponding plots of the conformal factors, essentially modifying the results of [3] in this respect (see the plots of [4]). Since here we have in mind the possible realization of these structures with real graphene membranes, to have a sphere, or a Bulge surface makes a crucial difference.

The thesis closes with a Section dedicated to a preliminary brief study of the 2+1-dimensional spacetimes, $\mathbf{R} \times \mathbf{\Sigma_{vortices}}$. There the Candidate shows the possible connection with the Bondi-Lemaitre-Tolman spacetime. Finally, two long and well structured Appendices are dedicated to deepen the discussion on conformal symmetry and on the physics of the three spacetimes discussed in Chapter 3, that are the Rindler, the de Sitter, and the Bañados-Teitelboim-Zanelli spacetimes.

The work just described is non-trivial, and requires both computational talent and imaginative thinking, besides the acquisition of knowledge on advanced topics. The Candidate performed all that with a high degree of independence, by developing his own computational tools, both analytic and computer-aid, and showing a sincere eagerness for learning. These facts show sharpness of mind, commitment and enthusiasm. These qualities led the Candidate to successfully complete the assigned task, and to actually produce a work that went beyond my expectations.

[1] A. Iorio, Weyl-gauge symmetry of graphene, Ann. Phys. **326** (2011) 1334.
[2] A. Iorio, Curved spacetimes and curved graphene: A status report of the Weyl symmetry approach, Int. J. Mod. Phys. D **5** (2015) 1530013.
[3] P. Horvathy, J.-C. Yera, Vortex solutions of the Liouville equation, Lett. Math. Phys. **46** (1998) 111.
[4] P. Horvathy, P. Zhang, Vortices in (abelian) Chern-Simons gauge theory, Phys. Rept **481** (2009) 83.

Případné otázky při obhajobě a náměty do diskuze:

Q1. First introduce all surfaces of constant positive Gaussian curvature, in general. Then, explain the effect of the singular boundaries of the Bulge type of surfaces you found on the corresponding conformal factors of the vortex solutions in point.

Q2. There are infinite surfaces of constant negative Gaussian curvature. Explain why they cannot be reduced to one, as for the sphere. You showed in the thesis that they cannot be associated to vortex-like solutions. Nonetheless, there are other important topological structures associated to surfaces of constant negative Gaussian curvature. Can you tell which ones? and why?

Práci

✠ doporučuji
❑ nedoporučuji
uznat jako bakalářskou.

Navrhuji hodnocení stupněm:
✠ výborně ❑ velmi dobře ❑ dobře ❑ neprospěl/a

Místo, datum a podpis vedoucího:

Praha, 4.6.2019

Alfredo Iorio