Abstract

This study provides an introductory insight into the complex field of graphene and its relativistic-like behaviour. The thesis is opened by an overview to this topic and draws special attention to interesting non-topological vortex solutions of the Liouville equation found by P. A. Horváthy and J.-C. Yéra, which emerge in a context of the Chern-Simons theory [1], [2] and have been put into context of graphene [3], [4]. We introduce the massless Dirac field theory, well describing electronic properties of graphene in the low energy limit, and point to the fact that the action of the massless Dirac field is invariant under Weyl transformations, which has far-reaching consequences. When the graphene membrane is suitably deformed, we assume that the correct description is that of a Dirac field on a curved spacetime. In particular, an important case is that of conformally flat 2+1-dimensional spacetimes. These are obtained when the spatial part of the metric describes a surface of constant intrinsic curvature [3]. In other words, the conformal factor of such spatial metrics has to satisfy the Liouville equation, an important equation of mathematical physics.

In this work, we have identified the kind of surfaces to which the Horváthy-Yéra conformal factors, above recalled, correspond, and have provided the geometrical explanation of the natural number N of such non-topological solutions. We have done that by identifying the appropriate change, from the isothermal coordinates to the canonical coordinates for surfaces of revolution. We found here that, for the generic N, such surfaces are surfaces of positive constant Gaussian curvature of the Bulge type (barrel shaped surfaces, that present singular boundaries), and only for $N = 1$ coincide with the sphere. Finally, we briefly comment on the corresponding 2+1-dimensional spacetimes, and show the possible connection with the Bondi-Lemaitre-Tolman spacetime.

Keywords: gravity analogues, Dirac massless field theory, conformal and Weyl symmetry, Liouville equation, non-topological vortex solutions, 2+1 - dimensional spacetimes, graphene membrane