Report on Bachelor Thesis
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Latinské obdĺžniky s troma riadkami a asociativita

The thesis is concerned with the number K_n of reduced $3 \times n$ latin rectangles on symbols $\{1, \dots, n\}$. There exists a complicated formula due to Riordan whose paper is difficult to read. The author carefully derives the formula (Veta 10) and calculates the values K_n for $n \leq 20$. The main difficulty lies in counting the number of derangements that also disagree at every point with a given permutation of a specified cycle type. In Section 3, $3 \times n$ latin rectangles are counted for $n \leq 10$ up to conjugacy in S_n using Burnside's Lemma and computer calculation.

The thesis is carefully written and correct. I checked many values obtained in the paper against the numbers available in the literature. There are some minor issues that I point out below.

Overall, I recommend that the thesis be accepted as a bachelor thesis and awarded grade 1.

Comments

1) It is clear from the conclusion that the thesis was supposed to contain an applications of $3 \times n$ latin rectangles to associativity, aiding in the construction of quasigroups with minimal number of associating triples. This is reflected in the title of the thesis that contains the word "associativity." Since this contribution did not materialize, the title is slightly misleading.

2) Notation could be improved throughout. For instance: a) the numbers d_k and w_k depend not only on k but also on n. This becomes important in several places, for instance in formula (2.3), where d_{k_i} is used not for a fixed n but rather for ℓ_i. b) The notation $U \cdot 0!$ that evaluates a polynomial U by substituting $i!$ for every x^i is odd but useful. However, the related notation $u = U.0!$ is used too liberally. For instance, in Lema 4 we read the formula $u_{(a)} = U_2^{a_2} \cdot U_3^{a_3} \cdots U_n^{a_n} \cdot 0!$, from which we are supposed to automatically deduce that the name of the polynomial $U_2^{a_2} \cdot U_3^{a_3} \cdots U_n^{a_n}$ is $U_{(a)}$. (The basic problem here is that the function $U \mapsto U.0!$ is not invertible.)

3) References to numbered equations should be rendered as $(x.y)$ in the text, not as $x.y$.

4) Lema 6 would be better stated as $U_i \cdot U_j = U_{i+j} + U_{|i-j|}$, which is what is actually proved and from which the current Lema 4 immediately follows. Also, the spade suit notation is distracting; one can display that equation instead and number it as usual. Finally, it should be said somewhere that $U_{-n} = U_n$, not just that $u_{-n} = u_n$.

5) In Section 3.2 the Orbit-Stabilizer theorem could be stated.

6) The bottleneck of the algorithm in 3.3 is not the generation of permutations that commute with a given permutation p. This is accomplished by `Stabilizer( SymmetricGroup( n ), p );` in GAP very quickly. Getting all permutations up to their cycle structure is also easy (for small values of n), cf.

```
List( ConjugacyClasses( SymmetricGroup( n ) ), C -> Representative );
```

The bottleneck consists of checking whether all suitable triples of permutations form a latin rectangle.

7) I urge some caution with the asymptotic behavior of K_n/J_n. Certain aspects of permutations need a large n to develop, such as the combinatorial explosion of distinct cycle structures.

Petr Vojtěchovský
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