The motion of a test particle in the Schwarzschild background models the merger of a compact object binary with extremely different masses known in the literature as Extreme Mass Ratio Inspiral. In the simplest geodesic approximation, this motion is integrable and there is no chaos. When one takes the spin of the smaller body into account, integrability is broken and prolonged resonances along with chaotic orbits appear. By employing the methods of Poincaré surface of section, rotation number and recurrence analysis we show for the first time that there is chaos for astrophysically relevant spin values. We propose a universal method of measuring widths of resonances in perturbations of geodesic motion in the Schwarzschild spacetime using action-angle-like variables. We apply this novel method to demonstrate that one of the most prominent resonances is driven by second order in spin terms by studying its growth, supporting the expectation that chaos will not play a dominant role in Extreme Mass Ratio Inspirals. Last but not least, we compute gravitational waveforms in the time-domain and establish that they carry information on the motion's dynamics. In particular, we show that the time series of the gravitational wave strain can be used to discern regular from chaotic motion of the source.