Posudek diplomové práce

Matematicko-fyzikální fakulta Univerzity Karlovy

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Název práce Geometry of Poisson-Lie T-duality
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Studijní program Mathematics **Studijní obor** Mathematical Structures

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Text posudku:

I refer to the detailed report in the attachment.

Práci doporučuji k obhajobě. I recommend the thesis for being defended. Furthermore I suggest the mark 1.

Práci nenavrhuji na zvláštní ocenění.

Datum 28 May 2019 **Podpis**

Report on "Geometry of Poisson-Lie T-duality", Master Thesis of Josef Svoboda

May 28, 2019

1 Summary of the content

The thesis presents an investigation of recent developments in the study of Poisson-Lie T-duality in the language of Courant algebroid geometry.

The author starts with a short recap of notions in classical differential geometry which are important for the thesis. Fibre bundles, principal bundles and their associated vector bundles as well as infinitesimal actions of Lie algebras on principal bundles are introduced. The introductory part continues with the definition of Lie algebroids with the Atiyah Lie algebroid as a particular example and concludes with the introduction of equivariant bundles, principal connections and the correspondence between splittings of the Atiyah sequence and principal connections.

In chapter 2, the author introduces the notion of Courant algebroid and explains their classification using the Severa class. After defining transitiveness, one of the main examples of Courant algebroids for the text is introduced: For a Lie algebra D and a Lie subgroup $G \subset D$ such that for the Lie algebra $\mathfrak{g} = \mathfrak{g}^{\perp}$ w.r.t. a non-degenerate G-invariant symmetric pairing on $\mathfrak{d}$, one has a Courant algebroid structure on $\mathfrak{d} \times D/G \to D/G$. Defining a generalized metric, it is already at this stage possible to state the idea of Poisson-Lie T-duality, relating two such Courant algebroids (constructed from D and two subgroups G_0, G_1), which the author states nicely in figure 2.2. Chapter 2 concludes with the introduction of pullbacks of Courant algebroids, the characterization of Dirac structures by 2-forms and definition and properties of generalized isometries.

Chapter 3 is the central part of the thesis. The goal is to define and investigate the notion of reduction for Courant algebroids. After introducing trivially extended actions and equivariant Courant algebroids, the relation between equivariant spittings and extended actions is investigated. It is proven that Courant algebroids over principal bundles admitting a trivially extended action using the connection on the bundle have vanishing first Pontryagin class. With these tools at hand, in chapter 3.3, two ways of reduction for Courant algebroids are discussed in detail. *Heterotic reduction* is explained as reduction of an equivariant Courant algebroid over a principal D-bundle P to a Courant algebroid over the quotient P/D. The space of sections of the latter is identified as D-invariant sections of the orthogonal complement of the image of the Lie algebra of D by the trivially extended action. *Isotropic reduction* is described as the reduction of a Courant algebroid over a principal D-bundle P to a Courant algebroid over P/G where $G \subset D$ is a Lie subgroup having $\mathfrak{g} = \mathfrak{g}^{\perp}$ for its Lie algebra $\mathfrak{g}$.

The results of chapter 3 are used in the final chapter 4 to give a sketch of Poisson-Lie T-duality. After a short review of T-duality, Poisson-Lie T-duality of surjective submersions is described and it is proven that they admit generalized isomorphic Courant algebroid structures. The essential idea of the proof is given. Finally, an example of Poisson-Lie T-duality using isotropic reductions is sketched in chapter 4.3.

2 Comments on structure, formatting and language

The following remarks concern passages where more information would have been nice:

- In the proof of proposition 1.3.9 the last line is not relevant for the proof. It is to prove that X^h is D-invariant.
- In the proof of p. 21, it should be shown that $H' = H + dB$, which is of course not difficult.
- In Lemma 2.2.8, the proof of 1 => 2 is not complete, surjectivity of ϕ and invariance of the metric is missing.
- In the statement of prop. 3.2.4, $R(\xi)$ is not stated correctly.
- In remark 3.2.2, the second dimension count is incomplete.
- In p. 39, the dimension counting should be for K_0, not for K.

Besides tivial typos there are the follwoing more serious ones:

- In the diagram (1.2), P should be Q and M should be N.
- The referencing to CA1 and CA2 (which appear only in chapter 2) should be replaced by proper referencing to definition 1.2.1.
- Line before (2.15), should be $\iota_Y d\iota_X B$.
- Lemma 2.2.8, first statment, V_i should be V_i^+.
- In the paragraph "Equivariant splitting", the argument of σ should be $[\xi^\#, X]$.
- In (3.8), $d^2\omega_\xi H$ is actually $d^2\omega_\xi$.
- After (3.11), ω_ξ is not $\mathfrak{d}$-valued.

3 Concluding comments

The thesis is well-written with detailed proofs of most of the statements. Reduction of Courant algebroids is explained in detail, including a sketch on their application to Poisson-Lie T-duality. I recommend the thesis for defense.