Report on the master thesis:

Archit Gupta: Second-order characteristics of point processes

The submitted theses studies the most important second-order characteristics of point processes, namely the K-function. Besides the classical Ripley's K-function, which describes the mean number of further points of the point process contained in a ball centered around a typical point of the point processes, another version, the multiparameter K-function is also considered. The latter uses a rectangle centered in a typical point of the point process instead of a ball. For the estimation of the K-function from the data pairs of observed points in the bounded observation window are used and thus edge-effects resulting from the unobserved points outside of the boundary of the observation window influence the quality of the estimator. Several edge-correction methods were suggested in the literature.

The goal of the theses is to investigate the statistical properties of the different versions of the estimator of the K-function from the literature. Namely bias and variance of the estimators are computed for the special case of the stationary Poisson point process (corresponding to the basic model of the complete spatial randomness). The derivation of the formulas is done in chapter 2 of the theses and it includes a lot of technical and tedious calculations. In chapter 3 a simulation study is provided, which verifies the accuracy of the variance approximation derived in chapter 2 and compares it with alternative variance approximations from the literature. Moreover the efficiency of the different estimators of the K-function is compared. The requirements from the assignment of the theses were fulfilled.

The theses is well organized and well written, the introduced notation keeps even the technical parts synoptic and readable. The theses is written in standard mathematical style, the results in chapter 2 are accompanied by proofs. The mathematical level of the thesis is very good. The derivations of the results from chapter 2 include many technical calculations, which are provided in detail in the thesis. The author obviously putted a lot of effort into this part. I also appreciate the summary part at the end of chapter 2, where all the theoretical achievements are listed. The simulation study in chapter 3 is well designed and conducted and the results are again clearly summarized in section 3.10.

Concerning the formal arrangement of the thesis, the only major criticism is that the typesetting of the mathematical formulas should be done with more care (overlapping letters, incorrect size of brackets etc.).

The topic of the thesis is surely not the most difficult but it is an interesting one and the closed form formulas for the variance of the considered estimators are not available in the literature. And as the simulation study confirms, they are better then the approximations of the variance standardly available. The authors own contributions are clearly stated in the conclusion chapter 4.

Thus in conclusion I can say, that the submitted thesis is a very good one and it surely should be accepted as a diploma thesis at Charles University, Faculty of Mathematics and Physics.

A list of minor remarks/questions folllows:

- p.9 after Definition 1.2.4 – there is said that the second-order factorial moment measure plays a similar role for spatial point processes like variance for real-valued random variables. Is it really so? What is the relation between $\alpha^{(2)}$ and $\mathrm{var}(\Phi(B)), B \in \mathcal{B}_o$?
- p.11, proof of Theorem 1.3.2 – why is it so obvious that $\alpha^{(2)}(\mathbb{R}^{dn} \setminus \mathbb{R}^{[dn]}) = 0$?
- p.15 – actually Fig. 1.3 is rather confusing, since at this place of the thesis the reader does not have a clue about the different estimators of the K-function shown in the figure.

- p.44, Monte Carlo verification – "after accounting for some random error"– what are the bounds for the "random error" in this situation? E.g. what is the precision of your Monte Carlo approximation of the constants in question?
- At many places in the thesis the terms estimate and estimator are interchanged.
- In the simulation study one fixed observation window and one fixed intensity of the Poisson process were chosen. Could you comment on the universality of your findings stated in the summary in section 3.10?

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