Abstract: This thesis is devoted to the rare decay  $\pi^0 \to e^+e^-$ . We firstly introduce all theoretical knowledge necessary for the description of the vertices used in this work. Subsequently, we introduce three two-loop chiral Feynman diagrams, which are of order  $O(\alpha^2 p^4)$ . These diagrams are firstly decomposed to Master integrals by Laporta algorithm. After that, we calculate each Master integral individually by the differential equation technique. We also focus on the renormalization of the process. Since we are working with the effective field theory, finite corrections  $\chi$ from counterterm diagrams naturally appear. With use of the calculations listed in this thesis, the determination of the parameter  $\chi_R$  could be further improved taking into account the whole two-loop correction.