Master Thesis

Semigroup-valued metric spaces

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Ramsey Theory originated in late twenties and beginning of thirties (Ramsey, van der Waerden). In the last quarter of 20th century it underwent a lively development in a structural form in works of Graham and Rothschild, Nešetřil and Rödl, Szemerédi, to name just a few of very many, and continues developing and producing very interesting results up to this day. The basic pattern of the problems can be illustrated in the categorical form: a category $\mathcal{C}$ has the *Ramsey property* (to be decided about) if for any two objects A, B and any natural number k there is an object C such that for every coloring c of the set of morphisms $A \to C$ there is a morphism $f : B \to C$ such that all the colors $c(f \cdot g)$ for all the morphisms $g : A \to B$ coincide. (The objects of the categories typically involved are sets with combined relational and partial algebraic structure, with natural homomorphisms.)

The thesis on hand is a substantial contribution (or, one can say, contains substantial contributions, because the results are numerous, interesting and useful) to the field. A very expedient general approach to be indicated shortly not only covers and generalizes many known hard results, but only brings new interesting very non-trivial facts.

The fundamental concept the author introduces and exploits is that of the $\mathfrak{M}$-metric space where $\mathfrak{M} = (M, +, \leq)$ is a partially ordered commutative semigroup with the specific property that $a \leq a + b$; such a space (X, d) is endowed by a metric $d : \{(x, y) \mid x, y \in X, x \neq y\} \to \mathfrak{M}$ with obvious properties (note that one works with a *semigroup*, not a monoid, hence there is no specific requirement on $d(x, x)$ – this has certain technical advantages). Slightly more generally, the author works with the $\mathfrak{M}$-edge-labelled graphs where only some pairs, edges, $\{x, y\}$ are initially given labels $d(x, y) \in \mathfrak{M}$; if such an object can be extended by labelling the remaining pairs to a $\mathfrak{M}$-metric space one speaks of a metric $\mathfrak{M}$-labelled graph.

As stated above, the Thesis contains many interesting, highly non-trivial and useful new results. Trying to reproduce them here would not be very profitable (and indeed, not possible: they are indeed many, and mostly rather technical). The author himself names as the main results two very general theorems about classes of $\mathfrak{M}$-metric spaces (X, d) with prohibited systems of d-labelled cycles, one of them concerning amalgamation and Ramsey property, the other about the extension property for partial homomorphisms. They have, of course, applications, and there are many facts that are not comprised.

It is an excellent Thesis, by far exceeding the requirements laid on such Theses. It should be noted that many of the results were already published, accepted or submitted in prestigeous journals (as far as I know, there are already 12 such papers). Furthermore, the defendant winned with his results the SVOČ competition.

Also, the Thesis is very well written; the results, although very demanding and often very technical, are presented in an as transparent form as the subject allows. I recommend the Thesis to be accepted and classified by the highest grade.

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