

The structural Ramsey theory is a field on the boundary of combinatorics and model theory with deep connections to topological dynamics. Most of the known Ramsey classes in finite binary symmetric relational language can be shown to be Ramsey by utilizing a variant of the shortest path completion (e.g. Sauer's S -metric spaces, Conant's generalised metric spaces, Braunfeld's Λ -ultrametric spaces or Cherlin's metrically homogeneous graphs). In this thesis we explore the limits of the shortest path completion. We offer a unifying framework — semigroup-valued metric spaces — for all the aforementioned Ramsey classes and study their Ramsey expansions and EPPA (the extension property for partial automorphisms). Our results can be seen as evidence for the importance of studying the completion problem for amalgamation classes and have some further applications (such as the stationary independence relation).

As a corollary of our general theorems, we reprove results of Hubička and Nešetřil on Sauer's S -metric spaces, results of Hubička, Nešetřil and the author on Conant's generalised metric spaces, Braunfeld's results on Λ -ultrametric spaces and the results of Aranda et al. on Cherlin's primitive 3-constrained metrically homogeneous graphs. We also solve several open problems such as EPPA for Λ -ultrametric spaces, S -metric spaces or Conant's generalised metric spaces.

Our framework seems to be universal enough that we conjecture that every primitive strong amalgamation class of complete edge-labelled graphs with finitely many labels is in fact a class of semigroup-valued metric spaces.