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Autoreferát disertační práce / Summary of the Doctoral thesis

Hydromechanické charakteristiky kaolinových suspensí
Hydromechanic charakteristics of clay suspensions

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Abstract

A mathematical model of two-phase systems, such as clay suspensions, consists of a set of partial differential equations which reflect both the general laws of mechanics and the relations connecting the involved characteristics of the particular system under consideration. The latter equations are known as constitutive relations. The aim of this study was to find the constitutive relations for kaolin suspensions that are necessary when solving forward problems of fine sludge thickening processes. The task was to design and carry out experimental research of the given suspension and to find a convenient method of utilizing the results for the sake of getting the sought relationships. It follows from the applied mathematical theory of two-phase systems that the sought relationships are hydraulic conductivity of the suspension as a function of the solid-phase concentration and the dependence of the solid-phase concentration on the solid-phase stress.

The first part of this study describes the experimental research. Since both the characteristics are difficult to measure, it was necessary to analyze the suspension's characteristics and their measurability. Subsequently, the process of the suspension preparation and the method of laboratory measurements were determined. The following sections present the way of utilizing the obtained data and getting the sought constitutive relations. The problem of the hydraulic conductivity determination is solved in two steps. In the first one, the general theory is utilized in order to get a correspondence between the measured height of the column of the suspension column and hydraulic conductivity. Making use of these results and applying an optimization method the sought functional dependence is then derived.

Solving the problem of finding the dependence of the solid-phase stress on the suspension concentration, the general theory makes it possible to find a relationship between the initial data, height of the suspension column and the total mass of solid phase, and the final solid-phase stress at a specially chosen point. Making use of the entire series of performed experiments, the sought constitutive equation is determined. In this case, two different solutions have been found; their comparison shows that, quantitatively, both solutions almost identically match within the entirety of their domain.

1 Introduction

Various industrial technologies involve processes connected with sedimentation and thickening of suspensions. Repositories of suspensions produced during the ore treatment and thickeners of communal sludge can serve as examples.

The reduction of the suspension volume minimizes, for example, the space of digestion tanks, improves the efficiency of centrifugation and makes it possible to get water available for other use. Concerning the industrial sludge, Concha (2014) points out that filtration, centrifuging and thickening are the key factors to the process of water recovery.

The gravity thickening is, because of its low energetic demands, one of the most efficient processes of lowering volume of suspensions and significantly increasing the concentration of their solid phase. The process can be divided into two distinguishable parts: simple sedimentation of the solid-phase particles and the subsequent compression due to the weight of the solid phase in the lower suspension layer where there are solid-phase particles in contact with one another.The position of the interface separating the parts results from the balance of mass exchange between them. The weight of the solid phase in the compression zone increases the liquid-phase pressure. Consequently, the liquid phase is pushed out of the compression zone which causes the required dewatering of the suspension.

The time dependence of the interface position reflects the mutual influence of the two layer–an serve as a source of comprehensive information of the suspension behavior.

The present research makes use of the possibility to register the position of the interface as a function of time. Applying further equations of a general theory of two-phase systems, (Mls, 1999), the obtained data can be utilized in accordance with the goal of the research.

The achieved results were published as the conference contribution Petrová & Mls (2011) and the papers Petrová & Mls (2013), Mls & Sedlácková (2017) and Mls & Sedlácková (2018).

2 Aim of the study

The most efficient methods of solving problems connected with the thickening of fine and compressible suspensions is to use mathematical models of their behavior. Solving forward problems of sedimentation and subsequent compression enables us to predict very efficiently the development of the stress and concentration of the studied suspension.

It can be shown that the general equations of two-phase systems do not make a complete set of equations. Every problem of prediction of suspension behaviour requires an additional set of equations. These equations define the necessary characteristics of the particular suspension and are known as constitutive relations. In view of this, it is obvious that any attempt to solve a forward problem must be preceded by a process that determines the respective constitutive relations.

The presented research was aimed at carrying out a set of experiments with a well-defined suspension of water and kaolin and developing a convenient method of getting the two constitutive relations that are necessary required when modelling the problems of thickening kaolin suspensions. In particular, the sought relations are

1. hydraulic conductivity of the suspension as a function of its solid-phase concentration and

2. the solid-phase concentration as a function of the solid-phase stress.

3 Material and methods

The experimental research was carried out with suspensions of water and kaolin. From the available materials, the Sedlec kaolin Ia was chosen as it is a well-defined material making it possible to repeat the experiments at any time later. It is also suitable because of its well-balanced granulometric curve, see Table 1. The applied water was deionized before mixed with the solid phase.

D(μm)	50	40	30	25	20	15	10	8
Mass %	99.0	99.4	100.0	100.2	98.7	97.5	94.1	91.7
D(μm)	7	6	4	3	2	1.5	1	
Mass %	89.6	86.4	77.7	70.9	60.9	54.4	45.9	

Table 1. Sedlec kaolin Ia – Granulometric distribution

The Sedlec kaolin Ia is not a raw material. In order to stimulate its coagulation, it is enriched with calcium and sodium cations. The coagulation obtained due to this enrichment is, however, too low to create a sufficiently sharp interface between the layer of suspension and the overlying layer of water. For the sake of getting a clearly visible interface to facilitate the laboratory measurements, calcium chloride was added. The chosen rate was 4 grams of calcium chloride per 1 kg of solid phase for every utilized suspension, i.e. regardless of its concentration.

The experiments were carried out in a set of vertical cylinders with impervious bottom. The inner diameter of the cylinders was 104 mm and their height was 2000 mm. Several experiments were repeated using cylinders of diameters 94 mm and 114 mm in order to check whether the results were affected by the rate of cross-section surface and perimeter.

The suspension was prepared in a separate vessel and then poured in the measuring cylinder. A special conic base under the cylinder and an adjustable holder in the upper part made it possible to set the cylinder in the vertical position. Each experiment starts from a homogeneous state of the suspension column determined by its initial concentration c_0 and its initial height L_0. The solid-phase particles of the suspension move downwards due to their density and build a zone of sedimentation. At the beginning, this zone covers all the suspension column.

The equations of mechanics of two-phase systems published by Mls (1999) are used in order to connect the measured data with other characteristics of the studied suspension. In the case of one (vertical) space dimension, the equations read

$$\frac{\partial n}{\partial t}(t,x) + \frac{\partial w}{\partial x}(t,x) = 0, \tag{3.1}$$

$$\frac{\partial n}{\partial t}(t,x) - \frac{\partial v}{\partial x}(t,x) = 0, \tag{3.2}$$

$$\frac{\partial w}{\partial t}(t,x) + gn(t,x) + \\ + \frac{n(t,x)}{\rho_w}\frac{\partial p}{\partial x}(t,x) + \frac{gn(t,x)}{K(t,x)}u(t,x) = 0, \tag{3.3}$$

and

$$\frac{\partial v}{\partial t}(t,x) + g\big(1 - n(t,x)\big) - \frac{1}{\rho_s}\frac{\partial \tau}{\partial x}(t,x) + \\ + \frac{1 - n(t,x)}{\rho_s}\frac{\partial p}{\partial x}(t,x) - \frac{g\rho_w n(t,x)}{\rho_s K(t,x)}u(t,x) = 0, \tag{3.4}$$

where t is time, x is space coordinate measured vertically upwards, w and v are the liquid-phase and the solid-phase volumetric flux densities, g is gravity acceleration, n is porosity, ρ_w is density of the liquid phase, ρ_s is density of the solid phase, p is the liquid-phase pressure, K is hydraulic conductivity of the suspension, τ is the solid-phase stress and u is the relative liquid-phase volumetric flux density, i.e. volumetric flux density of the liquid phase relative to the solid phase. The relative flux density can be expressed in the form

$$u = w - \frac{n}{1-n} v \,. \tag{3.5}$$

and the solid-phase concentration c as the solid-phase mass contained in unit volume of the two-phase system, i.e.

$$c = \rho_s (1 - n) \,. \tag{3.6}$$

A set of 50 measured experiments has been carried out in the laboratory of Department of hydrogeology of Charles university. The applied suspensions were mixtures of the above described kaolin and water in a precisely defined state of coagulation. Each test started from homogeneous state of the suspension column. Hence the initial condition of an experiment are

$$c(0, x) = c_0 \quad \text{and} \quad w(0, x) = 0 \quad \text{for } x \in (0, L), \tag{3.7}$$

where L is the initial height and c_0 is the initial concentration of the suspension column at the experiment. A visible interface develops in the suspension during the sedimentation process. The interface separates the zone of suspension from the overlying layer of water and moves downwards starting at time $t = 0$ from the level $x = L$. We denote $Y\,(t)$ its height above the bottom at time t. The thickening of the suspension begins at the impervious bottom where the particles of the solid phase are stopped. Consequently, another interface starts its motion from the bottom upwards, separating the layer of suspension of the initial concentration c_0 from the zone of thickening, $c(t, x) > c_0$, In view of this, the position $Z(t)$ of the lower interface at time $t,\ t > 0$, is defined by the equation

$$Z(t) = \sup\{x \geq 0; c(t,x) = c_0\}\,, \tag{3.8}$$

see Figure 1.

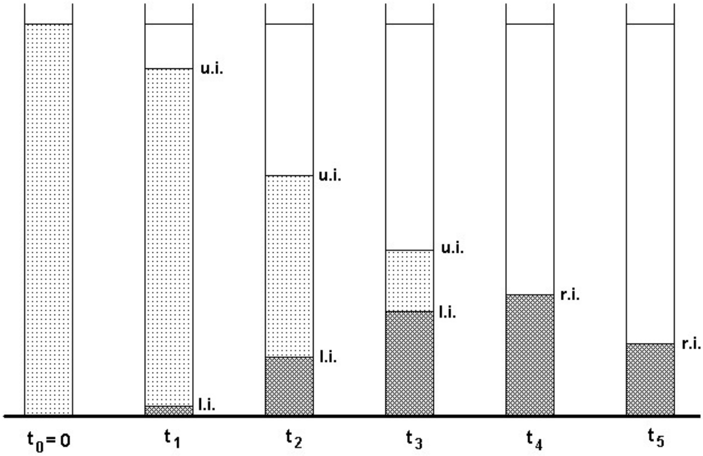

Figure 1: Positions of the upper and lower interfaces in time

In virtue of the known functions

$$c(t,x) = c_0 \qquad and \quad \tau(t,x) = 0 \tag{3.9}$$

in the domain

$$\Omega = \{(t,x) \in R^2; t > 0, x \in (Z(t), Y(t))\}, \tag{3.10}$$

Eqs. (3.3) and (3.4) give the following ordinary differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} + \frac{g\rho_w n_0}{(1-n_0)K(c_0)\big(\rho_w + n_0(\rho_s - \rho_w)\big)} w =$$

$$= \frac{g(\rho_s - \rho_w)n_0(1-n_0)}{\rho_w + n_0(\rho_s - \rho_w)}\,, \tag{3.11}$$

where n_0 is the initial value of porosity related to the initial concentration c_0 by equation (3.6).

With respect to the initial condition $w(0) = 0$, the solution to the problem is

$$w(t) = \rho(1-n_0)^2 K(c_0)\left(1 - exp\left(\frac{-gn_0t}{(1-n_0)(1+\rho n_0)K(c_0)}\right)\right), \tag{3.12}$$

Where

$$\rho = \frac{\rho_s - \rho_w}{\rho_w}\,. \tag{3.13}$$

Denoting by $V\,(t)$ the velocity of the solid-phase particles in domain Ω, it holds

$$V(t) = \frac{\mathrm{d}Y}{\mathrm{d}t}(t) \quad \text{and} \quad v(t) = (1-n_0)V(t). \tag{3.14}$$

The last equations and Eq. (3.12) enable us to formulate the following ordinary differential equation giving the height of the upper interface

$$\frac{dY}{dt} = -\rho(1-n_0)K(c_0)\left(1 - exp\left(\frac{-gn_0t}{(1-n_0)(1+\rho n_0)K(c_0)}\right)\right). \tag{3.15}$$

The solution to this equation that satisfies the initial condition $Y\,(0) = L$ is

$$Y(t) = L - \rho(1-n_0)K(c_0)t +$$

$$+\frac{(1-n_0)^2(1+\rho n_0)\rho K^2(c_0)}{g n_0}\left(1 - exp\frac{-g n_0 t}{(1-n_0)(1+\rho n_0)K(c_0)}\right). \tag{3.16}$$

In the last equation, only the constant $K(c_0)$ remains unknown. Since the function $Y\ (t)$ is the measured height of the upper interface, it was possible to compare the theoretical results with the measured values and to find the value $K(c_0)$ as the best fit, see Figure 2 where the squares are measured values and the curve is the theoretical height (3.16) height with the fitted value of $K(c_0)$.

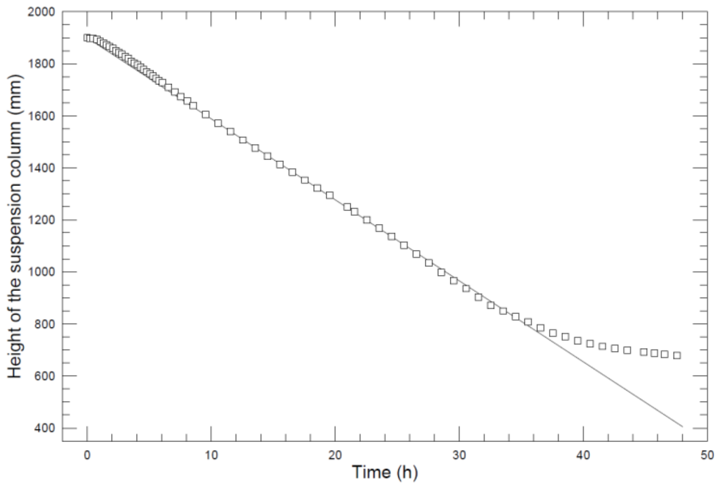

Figure 2: Test No. 6 – function $Y(t)$ and the measured values.

Having solved these optimization problems for the set of completed experiments, a new data set was obtained:

$$\{(c_i, K_i)\}_{i=1}^{45}, \tag{3.17}$$

where c_i stand for $(c_0)_i$ and $K_i = (K(c_0))_i$ are the fitted optimum values, $i = 1, \ldots, 45$.

Analyzing the data (3.17) it was realized that the hydraulic conductivity decreases with concentration in the whole region and that the nature of the decrease is of a power form for lower values of concentration and exponential for its higher values. Using the below defined kind of optimization and requiring moreover certain degree of smoothness, the unknown function $K(c)$ was determined. Particularly the sum

$$S = \sum_{i=1}^{N} \left[\ln(K_i) - \ln\big(K(c_i)\big)\right]^2 , \qquad (3.18)$$

was minimized, while two different expressions of the sought function $K(c)$ were used.

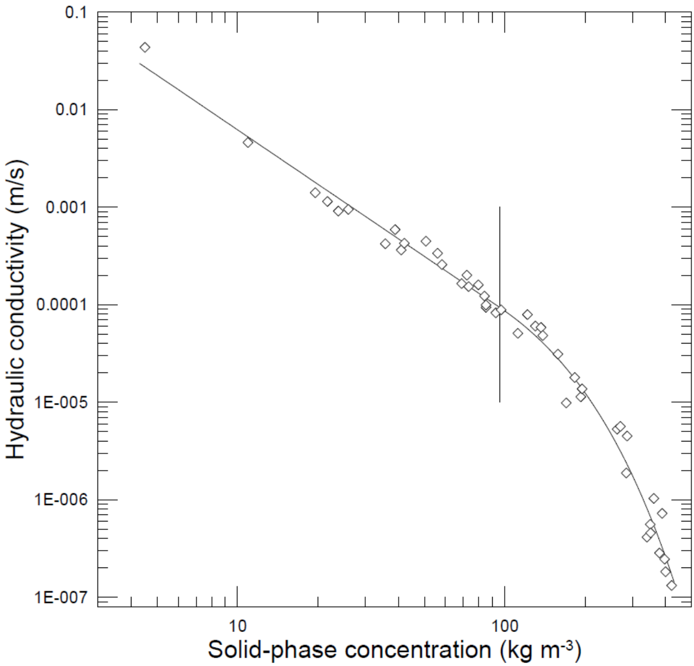

Figure 3: Function $K(c)$

Eventually, the following results were achieved:

$$D = [4.5023, 420.99], \tag{3.19}$$

$$\begin{aligned} K &= A_1 * c^{B_1} \quad \text{for } c \in [4.5023, 95.74375], \\ K &= A_2 * B_2^C \quad \text{for } c \in [95.74375, 420.99\,], \end{aligned} \tag{3.20}$$

Where

$$\begin{aligned} A_1 &= 0.4487224 \qquad B_1 = -1.857769 \\ A_2 &= 6.002750 \times 10^{-4} \qquad B_2 = 0.9807835\,. \end{aligned} \tag{3.21}$$

The obtained function $K(c)$ is depicted in the Figure 3, where the squares are the computed couples (3.17) and the curve is the function (3.20).

The other goal of this research was to determine the other constitutive equation, the solid-phase stress as a function of the solid-phase concentration, making use of the same set of measurements as in the previous case. We start considering the above introduced vertical cylinders and solving a steady state problem in the suspension column of height L. In this case

$$w(x) = v(x) = 0, \tag{3.22}$$

and equations (3.3) and (3.4) are

$$g + \frac{1}{\rho_w} \frac{\mathrm{d}p}{\mathrm{d}x}(x) = 0 \tag{3.23}$$

and

$$g\big(1 - n(x)\big) - \frac{1}{\rho_s} \frac{d\tau}{dx}(x) + \frac{1 - n(x)}{\rho_s} \frac{\mathrm{d}p}{\mathrm{d}x}(x) = 0\,. \tag{3.24}$$

Eqs. (3.24) and (3.24) give the following relation between the solid-phase concentration and derivative of the solid-phase stress.

$$\frac{\mathrm{d}\tau}{\mathrm{d}x}(x) = g\frac{(\rho_s - \rho_w)}{\rho_s}c(x)\,, \tag{3.25}$$

We introduced the following two assumptions that enabled us to make use of the known total mass of the solid phase in the column and its also known final height:

(*) The settling process has reached the quasi-stationary state and the height of the suspension column does not change.

(**) There exists a function $\xi \mapsto \Phi(\xi), \Phi: [0,\infty) \rightarrow [0,\infty)$ such that in any suspension column satisfying assumption (*), the specific mass of the solid phase contained in the top layer of depth λ is equal to $\Phi(\lambda)$.

In this way, function Φ makes a kind of material coordinate and can be related to the space coordinate x. Denote the utilized measured couples of limit height and total mass of the suspension column by

$$\{(L_i, M_i)\}_{i=1}^{N}\,, \tag{3.26}$$

where $N = 50$ in this case; these data are shown in Figure 4. In virtue of assumption (**), function Φ uniquely measures the solid-phase mass in a layer of the suspension

$$\Phi(\xi) = \int_0^{\xi} c(\zeta)\,\mathrm{d}\zeta\,, \tag{3.27}$$

where ξ is another space coordinate, this time measured positively downwards,

$$\xi = L - x,$$

and consequently, assumption (**) states

$$M_i = \Phi(L_i)\,, \qquad \text{for } i = 1, \ldots, N\,. \tag{3.28}$$

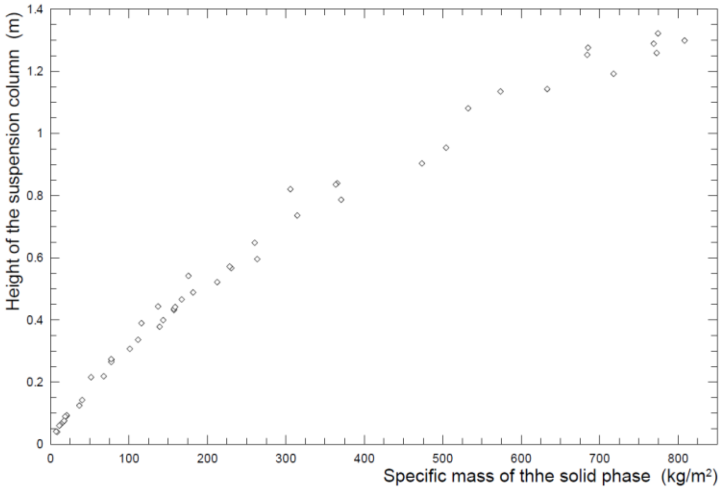

Figure 4: Final height and corresponding solid-phase mass

We choose the following two forms of function Φ

$$\Phi_p(\xi) = a\xi + b\xi^r \tag{3.29}$$

and

$$\Phi_e(\xi) = \alpha\xi + \beta(\exp(\gamma\xi) - 1)\,, \tag{3.30}$$

where $a, b, r,$ and α, β, γ are parameters that have to be determined in order to get the particular function. Minimizing the error in (3.28), it was found

$$a = 247 \pm 7, \qquad b = 256 \pm 4, \qquad r = 2.\pm 0{,}2 \tag{3.31}$$

and

$$\alpha = -1428 \pm 7,$$

$$\beta = (5.92 \pm 0{,}2) \times 10^3, \qquad \gamma = (282{,}4 \pm 0{,}9) \times 10^{-3}, \tag{3.32}$$

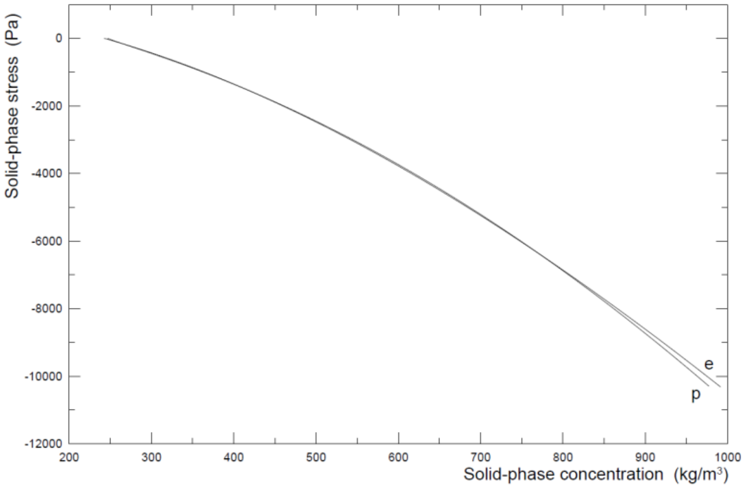

Figure 5: Functions τ_p and τ_e , distinguished by letters p and e

Finally, making use of Eqs. (3.25), (3.27), (3.29) and (3.30), the sought function was found in the following two forms

$$\tau_p(c) = -g\frac{\rho_s - \rho_w}{\rho_s}\big(bc + a(r-1)\big)\left(\frac{c-a}{rb}\right)^{1/(r-1)} \tag{3.33}$$

and

$$\tau_e(c) = -g\frac{\rho_s - \rho_w}{\rho_s}\left(\frac{c-\alpha}{\gamma} - \beta + \frac{\alpha}{\gamma}\ln\left(\frac{c-\alpha}{\gamma\beta}\right)\right), \tag{3.34}$$

that are depicted in Figure 5.

4 Results and discussion

In the study, I consider only monotonous processes; a process in a suspension column is monotonous if the inequality

$$\frac{\partial c}{\partial t}(t_1, m_1)\frac{\partial c}{\partial t}(t_2, m_2) \geq 0, \tag{4.1}$$

is satisfied for any two-time instants t_1 and t_2 and for any two values m_1 and m_2 of a material coordinate, where c is the suspension concentration, see also Petrová & Mls (2013). The restriction is introduced in order to avoid effects of hysteresis that can be expected when admitting reversion of the studied process. In view of this, all the resulting relationships can be applied only when processes with increasing concentration are studied, i.e. processes satisfying

$$\frac{\partial c}{\partial t}(t, m) \geq 0, \tag{4.2}$$

for any couple (t, m) of time and material coordinate.

Hydraulic conductivity of a suspension that changes its concentration is difficult to measure. It was shown that a different data, measured with sufficient accuracy, can be utilized in order to get the sought characteristic. The obtained constitutive relation (3.20) defined in the domain (3.19) shows a very good agreement with utilized data.

The resulting function $K(c)$ is smooth in its domain, $K \in C^1(D)$. The research brought a particularly interesting finding: the relationship changes its character in D; from the power function for lower values of concentration to the exponential function for higher concentrations. This phenomenon is visible in Figure 3.

Functions τ_e and τ_p are two solutions of the second problem. It is not possible to determine the function Φ uniquely; its form was chosen similarly as in the case of function K. This is acceptable in view of the well-known frequent ill-posedness of inverse problems. On the other hand, having obtained function Φ, the resulting function $\tau\ (c)$ is determined uniquely, it follows from Eqs. (3.25), (3.27) and the condition $\tau(L) = \Phi(0) = 0$.

5 Conclusions

Domains of the achieved relationships are defined. They start from very low concentrations and there is no need to enhance them in this direction. The upper bounds of the obtained domains are limited. It is a matter of the future research to enhance the domains in this direction. The height of the utilized columns and the necessity to start the experiments with the process of simple sedimentation limited the domains of the sought functions.

The important result, determination of the gel point, puts also certain limit to the domain of function K. As it is stated in the thesis, the obtained function can be accepted as a justified extrapolation of the function in the neighborhood of its domain's upper bound.

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Vybrané publikace
Selected publications

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