

CHARLES UNIVERSITY
FACULTY OF SOCIAL SCIENCES

Institute of Economic Studies



Cusp catastrophe theory:
Application to the housing market

Bachelor's thesis

Prague 2019

Author: Vojtěch Kořínek

Supervisor: PhDr. Jiří Kukačka Ph.D.

Year of defense: 2019

Bibliographic Note

KOŘÍNEK, Vojtěch. *Cusp catastrophe theory: Application to the housing market*. Prague 2019. 44 pp. Bachelor's thesis (Bc.) Charles University, Faculty of Social Sciences, Institute of Economic Studies. Thesis supervisor PhDr. Jiří Kukačka Ph.D.

Abstract

The bachelor's thesis applies the stochastic cusp catastrophe model to the housing market of the United States. Weekly data over the period from 2007 to 2017 are used. The current catastrophe theory literature related to the housing market is reviewed, the models found are assessed and expanded. Specifically, we have identified three deficiencies of the catastrophe models applied to housing market in the current literature and our contribution lies in the elimination of these deficiencies. In order to satisfy the constant volatility assumption of the model, the state variable is normalized by the estimated volatility derived from GARCH. Furthermore, multiple control variables are added to the model to represent the activity of fundamentalists and chartists. The results suggest that the cusp catastrophe model fits the data better than the linear and logistic models. The normalization of the state variable improves the model performance while the introduction of the additional control variables does not produce better results.

Keywords

Housing market, catastrophe theory, stochastic cusp catastrophe model, housing bubble, real estate, fundamental investors, speculation.

Abstrakt

Tato bakalářská práce aplikuje stochastický model založený na teorii katastrof na realitní trh USA. Použita jsou týdenní data v období let 2007 až 2017. Současná literatura spojená s aplikací teorie katastrof na realitní trh je zmíněna, použité modely jsou zhodnoceny a rozšířeny. V těchto modelech aplikovaných na realitní trh jsme identifikovali tři nedostatky a náš přínos spočívá v jejich eliminaci. Abychom splnili předpoklad konstantní volatility požadovaný modelem, stavová proměnná je normalizována pomocí odhadnuté volatility získané GARCHem. Dále, kontrolní proměnné, které jsou použity k odhadnutí aktivity fundamentalistů a chartistů, jsou přidány do modelu. Výsledky ukazují, že model založený na teorii katastrof popisuje data lépe než lineární a logistický model. Normalizace stavové proměnné zlepšuje výsledky modelu, zatímco přidání kontrolních proměnných v lepší výsledky neústí.

Klíčová slova

Realitní trh, teorie katastrof, stochastický model teorie katastrof, realitní bublina, reality, fundamentální investoři, spekulace.

Declaration of Authorship

The author hereby declares that he or she compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain any other academic title.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis in whole or in part and agrees with the thesis being used for study and scientific purposes.

Prague, May 9, 2019

Vojtech Korinek

Acknowledgments

I am grateful to my supervisor PhDr. Jiří Kukačka Ph.D. for his guidance, valuable advice, willingness and patience during writing the thesis. I would also like to express my gratitude to my parents for their support.

Typeset in FSV L^AT_EX template with great thanks to prof. Zuzana Havrankova and prof. Tomas Havranek of Institute of Economic Studies, Faculty of Social Sciences, Charles University.

Bachelor's Thesis Proposal

Author	Vojtěch Kořínek
Supervisor	PhDr. Jiří Kukačka Ph.D.
Proposed topic	Cusp catastrophe theory: Application to the housing market

Research question and motivation

The housing market has been at the forefront of discussion among economists in the past decades as it has been linked to major world events. The presence and burst of housing bubbles, most notably in the U.S. in late 2000s, but also in many European countries, significantly contributed to financial crises. Therefore, understanding the forces behind the housing market has become vital. Robert Shiller in his book *Irrational Exuberance* (2nd edition) claims that not only fundamental values, but also psychological factors play an important role in determining housing prices and developing bubbles. Thus, models assuming only fundamental investors and relying only on fundamental values may lead to erroneous conclusions as these models omit one of the important aspects.

As a result, models assuming heterogeneous expectations and interactions between heterogeneous agents have been developed (Kouwenberg and Zwinkels, 2015; Dieci and Westerhoff, 2015). These models have shown that the interaction between fundamental investors and speculators may produce results better describing the reality compared to models relying only on fundamental values. One such approach is catastrophe theory, whose first application to the financial markets was proposed by Zeeman in his paper *On the unstable behavior of stock exchanges* (1974). In this thesis, the catastrophe theory, specifically the cusp catastrophe model, will be applied to the housing market to assess its suitability to describe the market and its crashes.

Contribution

While the cusp catastrophe theory has been successfully applied to the stock market (Barunik and Vosvrda, 2009; Barunik and Kukacka, 2015), its application to the housing market has been limited. Diks and Wang (2016) used the theory to describe the housing market of 6 countries. However, as only the interest rate was used as the control variable, it gives the opportunity to explore this topic further. This thesis will attempt to use multiple control variables to represent the fundamental investors and the speculative money on the market and estimate their effects. The results of this thesis could be used for further policy making regarding the housing market and for better identification of conditions in the market leading to a possible crash.

Methodology

First, the housing market and the associated macroeconomic indicators will be described and qualitatively assessed whether their development and values correspond to the behavior of the market. Then, the cusp catastrophe theory will be applied and its fit evaluated and compared to other econometric models. The returns of the Dow Jones U.S. Real Estate Index will represent the state variable, while various indicators (volume of mortgage-backed securities held by banks, mortgage applications, 30-year mortgage rate etc.) will be tested as the control variables. Cobb's maximum likelihood method (Cobb, 1981a, 1981b) will be used to estimate the parameters of the model. As the volatility of the housing market returns cannot be considered constant over time, a GARCH model will be implemented to estimate the volatility. The returns normalized by the estimated volatility will then satisfy the constant volatility assumption necessary for the stochastic catastrophe model.

The data (macroeconomic indicators, housing market indicators, indices) will be collected using Thomson Reuters Eikon, The Federal Bank of St. Louis and, possibly, other sources.

Outline

1. Introduction
2. U.S. housing market before the 2008 crisis and today
3. Recent housing market research review and catastrophe theory literature review
4. Methodology
5. Model and results

6. Conclusion

7. References

Core bibliography

Barunik, Jozef and Kukacka, Jiri, (2015), *Realizing stock market crashes: Stochastic cusp catastrophe model of returns under the time-varying volatility*, Quantitative Finance, Vol. 15, No. 6, pp. 959-973.

Barunik, Jozef and Vosvrda, Miloslav, (2009), *Can a stochastic cusp catastrophe model explain stock market crashes?*, Journal of Economic Dynamics and Control, Vol. 33, No. 10, pp. 1824-1836.

Cobb, Loren, (1981a), *Estimation theory for the cusp catastrophe model*, MPRA Paper 37548, University Library of Munich, Germany, revised 05 Jun 2010.

Cobb, Loren, (1981b), *Parameter estimation for the cusp catastrophe model*, Behavioral Science, Vol. 26, No. 1, pp. 75-78.

Dieci, Roberto and Westerhoff, Frank, (2015), *Heterogeneous expectations, boom-bust housing cycles, and supply conditions: A nonlinear dynamics approach*, BERG Working Paper Series, No. 99, ISBN 978-3-943153-16-3, Bamberg University, Bamberg Economic Research Group (BERG), Bamberg

Diks, Cees and Wang, Juanxi, (2016), *Can a cusp catastrophe model explain housing market crashes?*, Journal of Economic Dynamics and Control, Vol. 69, pp. 68-88.

Kouwenberg, Roy and Zwinkels, Remco R. J., (2015), *Endogeneous Price Bubbles in a Multi-Agent System of the Housing Market*, PLoS ONE 10(6): e0129070, doi:10.1371/journal.pone.0129070.

Shiller, Robert J., (2005), *Irrational exuberance (2nd edition)*, Princeton, N.J., Princeton University Press

Shiller, Robert J., (2007), *Understanding Recent Trends in House Prices and Home Ownership*, No 1630, Cowles Foundation Discussion Papers, Cowles Foundation for Research in Economics, Yale University.

Zeeman, Erik C., (1974), *On the unstable behavior of stock exchanges*, Journal of Mathematical Economics, Vol. 1, No. 1, pp. 39-49.

Contents

Thesis Proposal	vii
List of Tables	xii
List of Figures	xiii
1 Introduction	1
2 U.S. Housing market before the 2007-2008 financial crisis and today	4
3 Recent housing market research review	8
3.1 Models of the housing market	11
4 Catastrophe theory literature review	13
4.1 Catastrophe theory	13
4.2 Recent applications in finance	16
5 Methodology	18
5.1 Basic framework	18
5.2 Model	20
5.3 Volatility	23
5.4 Estimation	24
5.5 Evaluation of fit	26
6 Data and results	28
6.1 State variable	28
6.2 Estimation of volatility	30
6.3 Fundamentalists and chartists	33
6.4 Comparison and final results	34

7 Conclusion	37
Bibliography	39

List of Tables

6.1	Summary of the model with one control variable (model 1) . . .	29
6.2	Summary of the model with standardized returns and one control variable (model 2)	32
6.3	Summary of the model with standardized returns and four con- trol variables (model 3)	34
6.4	Comparison of the estimated models	35

List of Figures

2.1	S&P/Case Shiller U.S. National Home Price Index	5
2.2	Dow Jones U.S. Real Estate Index	6
5.1	Cusp surface	20
6.1	Weekly returns of the Dow Jones U.S. Real Estate Index	30
6.2	Returns, estimated volatility and standardized returns of the Dow Jones U.S. Real Estate Index	31

Chapter 1

Introduction

As the last financial crisis, which occurred in late 2000s, was strongly related to the housing market and began by the burst of the housing bubble, the focus on the research and understanding of the real estate market has intensified. Even though the crisis hit at first only the United States, it soon spread around the world, affecting almost every developed economy. Not only was the crisis unprecedented in the geographical scope, but it was also the largest crisis since the Great Depression of 1930s. Housing bubbles, however, are not limited to the United States. They have occurred quite regularly around the world. Japan experienced an overall asset price bubble in late 1980s and early 1990s, after which the economy stagnated for many years. In 1990s, the residential housing market in Hong Kong showed signs of speculation and the residential housing prices skyrocketed. As the Asian financial crisis hit hard in 1997, the residential property prices reacted wildly and declined by about 40 % from its peak (Kalra et al., 2000). Europe has also been hit by housing bubbles. Countries, such as Spain or Ireland, experienced a rapid growth in housing prices before the last financial crisis and when the crisis eventually spilled over from the United States, a sharp decline in the housing prices ensued, affecting the economies substantially. More recently, we have also seen large increases in the house prices in many different countries and cities around the world. While we cannot be sure whether or not this growth has resulted in bubbles, the prices have often already surpassed the pre-crisis levels. Therefore, understanding the housing market better may not only help explaining the past, but it may be vital in predicting and reacting to present and future bubbles.

Moreover, unlike other financial markets, the housing market affects almost everyone. At some point in their lives, most of the people will find themselves in

a situation when they have to find a dwelling. The housing market is, therefore, different to other financial markets as almost everyone at certain point of time participates in it. Also, the purchase of the house is a once-in-a-lifetime event for many and such people do not often perceive it only as an investment, but rather as a place to live and to call home. On the other hand, it also shares many similarities with other financial markets and, as a result, many models introduced at first to the stock market are later applied also to the housing market. One of these models is also the catastrophe theory.

Catastrophe theory was first developed by Thom (1972) and later implemented within the field of social sciences in Zeeman (1974). One of the most important attribute of the theory is that, within its framework, it allows for and is able to describe systems where not only continuous but also discontinuous changes occur. As a result, it has found many applications in various fields, ranging from psychology through physics to economics. Zeeman (1974) merged the concepts of the catastrophe theory and the heterogeneous agent models and qualitatively analyzed the dynamics of a stock market. Through the interaction of two types of investors, fundamentalists and chartists, he showed that, under the right circumstances, the market may be internally driven to a period of strong overvaluation followed by a sudden crash. This successful application spurred interest in the theory and much research was done. Nevertheless, the popularity of the catastrophe theory in social sciences later decreased due to a strong criticism published in late 1970s (Sussman and Zahler, 1977, 1978a, 1978b). However, Rosser (2007) presented a case for the reintroduction of the theory in social sciences as he claimed that the critique had not been accurate and that some of the problems of the theory mentioned in the critique had since been solved. As a result, there has recently been new research done within the field of finance (Barunik and Vosvrda, 2009; Barunik and Kukacka, 2015; Diks and Wang, 2016).

The objective of this thesis is to apply the catastrophe theory, specifically the cusp catastrophe model, to the U.S. housing market in order to check whether the model fits the data well and, as a result, whether it can be used to assess the state of the housing market effectively. As the financial crisis occurs within the period of our interest, it is likely that sudden jumps of the market are present. Therefore, the use of the catastrophe theory appears to be justified in our case. Diks and Wang (2016) have already applied the theory to housing markets of several countries. However, as they omitted some important facts and relied on only one control variable, there is room for further exploration of

the topic. Based on Zeeman (1974), we estimate the activity of fundamentalists and chartists using several indicators as control variables. In order to evaluate the statistical fit of the model, we employ several measures and, subsequently, compare our model to two alternative models.

This thesis is divided into several chapters and the structure is as follows: Chapter 2 describes the state of the U.S. housing market in the lead up to, during and after the financial crisis, Chapter 3 presents some of the recent housing market research literature, Chapter 4 reviews the history and literature of the catastrophe theory, on which the model used in this thesis is based. Chapter 5 explains the methodology of this thesis while Chapter 6 presents the data and results. Finally, Chapter 7 summarizes our findings.

Chapter 2

U.S. Housing market before the 2007-2008 financial crisis and today

This thesis is focused on the housing market of the United States and, therefore, before empirically testing the chosen model, the state of the market and the evolution over the past decades should be described. This section qualitatively explores the market and presents an overview of major indicators. Furthermore, the relation of the housing market and the economy is assessed and the role which the burst of the housing bubble played during the financial crisis of late 2000s is described.

Even though there are many statistics, which can be used to depict the state of the housing market in the United States, the most important one is probably the price level. Some of the most widely applied relevant measures include the S&P/Case-Shiller Home Price Indices, which are based on the work of and named after American economists Karl Case and Robert Shiller. There are several versions of the indices based on the geographical scope, ranging from a national index through a several-city composite indices to indices for individual cities. The calculation is performed using repeat sales data of single family homes. As we are interested in the housing market of the entire country, Figure 2.1 shows the S&P/Case-Shiller U.S. National Home Price Index from January 1987 to February 2019. The data is not seasonally adjusted and the year 2000 serves as the base year.

It is apparent that, overall, there has been a rising trend. Over the period shown in the figure, the index increased from 65.753 in January 1987 to 205.041

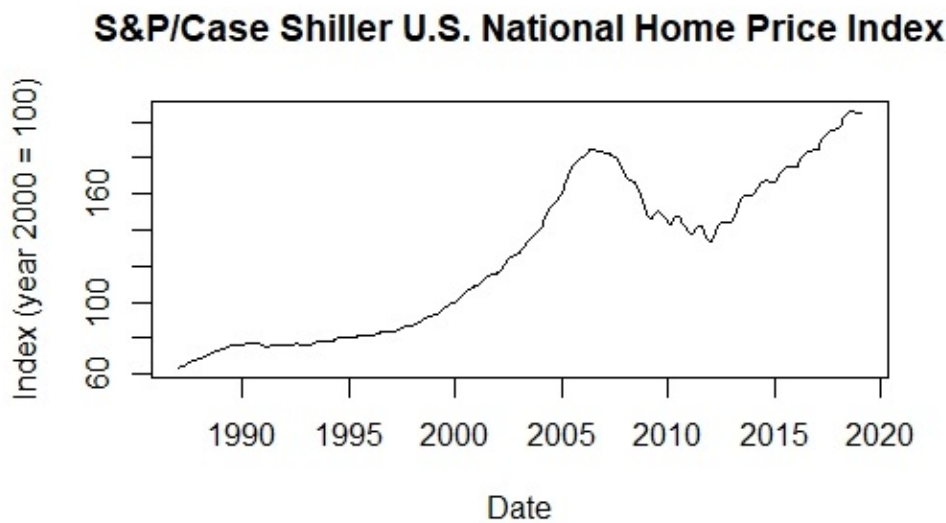


Figure 2.1: S&P/Case Shiller U.S. National Home Price Index

Source: Federal Reserve Bank of St.Louis

in February 2019. At the beginning, in late 1980s and early 1990s, the index was rising mildly. However, the growth began to accelerate in the second half of the 1990s and continued accelerating during the first half of 2000s, with the index reaching the value of 184.615 in July 2006. This apparent boom in house prices corresponds to the subprime mortgage bubble. This significant event is described in more detail later in this section. After the burst of the bubble, there was a steep decline and the index had a decreasing trend the majority of the time until 2012 only with several mild short-time fluctuations upwards. Since that, the market has experienced rapid growth again and has even surpassed the highs of 2006.

The S&P/Case Shiller Indices describe, however, only residential housing. In order to see the entire housing market, we need other measures. The Dow Jones U.S. Real Estate Index is designed to represent the Real Estate Super-sector as defined by the Industry Classification Benchmark (ICB). Figure 2.2 shows the performance of the index from February 2000 to February 2019.

We can see that the development is similar to the residential housing. There is a substantial growth until about the years 2006 and 2007. The peak occurred in February 2007 unlike the residential housing index, which peaked already in 2006. At first sight, there is much more short-term movement in the Dow Jones index compared to the S&P/Case Shiller index. However, this is explained by the fact that whereas S&P/Case Shiller index has only monthly data, daily data

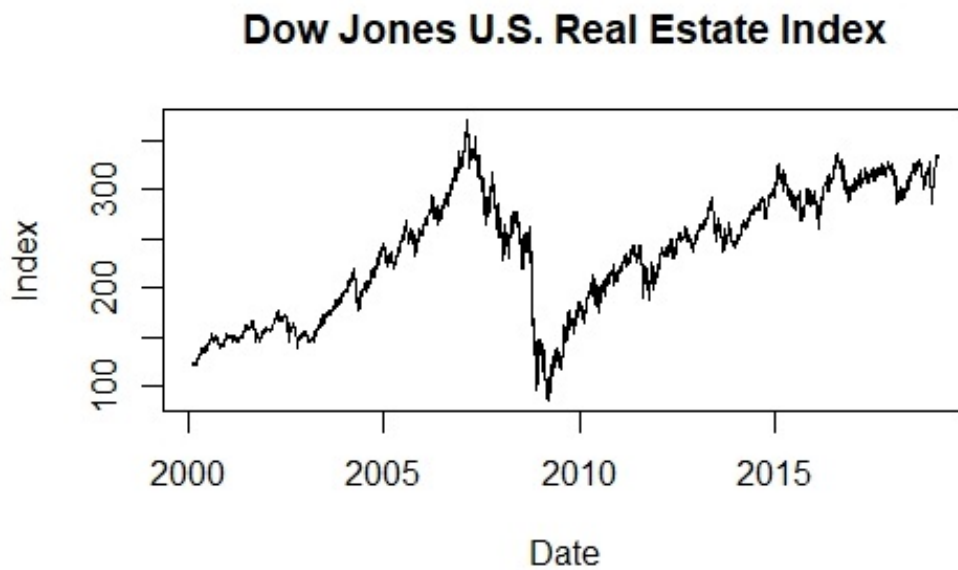


Figure 2.2: Dow Jones U.S. Real Estate Index

Source: Thomson Reuters

are available for the Dow Jones index. It is also apparent that the index fell to its minimum in 2009, after which it started increasing again and the overall trend has persisted to the day. That differs substantially from the development of the S&P/Case Shiller index, where the prices continued falling until 2012.

According to the National Bureau of Economic Research, three recessions have occurred since 1987. The first lasting from July 1990 to March 1991, the second from March 2001 to November 2001 and the last from December 2007 to June 2009. The first two are usually not associated with the housing market. This corresponds to the development of the S&P/Case Shiller index as the index does not appear to have been significantly affected by the two crises. However, that is not the case with the last crisis. There, the housing market played a substantial role. United States Financial Crisis Inquiry Commission (2011) claims that the burst of the housing bubble was the spark of the following financial crisis. After the recession of 2001, the interest rates were kept low, which together with easy lending standards and policies of the Clinton and Bush administrations, which encouraged homeownership, fueled the growth of the bubble. As more people began taking out mortgages, banks started packaging them into complex financial securities. These derivatives were then sold to investors. Due to the fact that this practice was found to be highly profitable, banks and other mortgage providers further relaxed their

lending standards as they wanted to have more mortgages to package, meaning even people with less than optimal credit score were able to obtain mortgage. These mortgages, called subprime, were extremely risky. Nevertheless, when the banks packaged thousands of them into a security, the rating agencies considered them investment grade. Furthermore, derivatives, such as credit default swaps, effectively allowed investors to bet on a performance of those mortgages without owning them, further fueling speculation. However, it all came down when people started defaulting on their mortgages and the investors realized that many of the derivatives are essentially worthless. By that point of time, large financial institutions had billions of dollars of these “toxic” assets on their balance sheets. As a result, many of them failed, most notably Bear Stearns and Lehman Brothers in 2008, and many, such as AIG, had to be bailed out by the U.S. government as they were deemed “too big to fail”. Following this, the confidence in the market plummeted causing institutions to stop lending each other. This led to a sharp fall in the stock market and the economy went into deep recession (United States Financial Crisis Inquiry Commission, 2011).

Chapter 3

Recent housing market research review

Housing market research has recently found itself at the forefront of discussion in economics due to its role in the financial crisis of late 2000s. As the development and subsequent burst of the real estate bubble was not successfully predicted and as it, therefore, led to the “Great Recession”, understanding the forces behind the housing market has become vital. Since then, many concepts have been developed and articles published. This section presents such literature as well as literature published before the crisis.

In finance, most of the traditional theory and models assume some form of the Efficient Market Hypothesis (EMH), which was developed by Fama (1970). The basic idea of EMH is that the market price reflects all available information and, therefore, it is impossible to buy undervalued or overvalued stocks. The only way to outperform the market is by chance, by taking on more risk or by having insider information. Even though the housing market has many similarities with other financial markets, such as stock market, it also differs substantially. For many individuals and households, buying a house or an apartment is once-in-a-lifetime event and they do not perceive it strictly as an investment with the goal of making money. It is rather a place, where they feel at home. Furthermore, people often spend a vast portion of their lives in one dwelling instead of moving regularly. That may, as a result, lead to lower liquidity on the housing market compared to the stock market.

Case and Shiller (1989) examined data on repeated sales prices of individual homes in several U.S. cities for the period of 1970-1986 and found that year-to-year changes in prices are autocorrelated. On the other hand, the prices did

not appear to reflect the information about the real interest rate. Moreover, Shiller (2007) argued that the boom in house prices occurring in 2000s is not possible to explain by changes in fundamentals. Rather, he suggests that a behavioral theory which associates the price increases with a feedback mechanism and social epidemic fits the data better. Shiller (2005) claims that the notion that investors tend to be euphoric and frenzied during price booms and pessimistic and panic-stricken during crashes is not credible. Therefore, the price movements cannot be considered to be caused entirely by investors who simply blindly follow the trend or the sentiment. However, he claims that there are behavioral biases, which he calls psychological anchors, present on the markets and that they do affect the price.

Salzman and Zwinkels (2013) list and describe many possible biases on the real estate markets. They claim that not only household buyers are not completely rational, but that also other parties participating on the market suffer from bias. Therefore, biases from perspectives of three different participants are considered—the perspective of institutional investors, the household perspective and the appraiser’s perspective. Now, several biases will be mentioned to illustrate some of the aspects of the behavior on the housing market. However, in-depth analysis of the biases is beyond the scope of this thesis and, therefore, please refer to the original article by Salzman and Zwinkels (2013) for more detail.

One of the most common and most natural bias is herd behavior. This bias describes the fact that people often tend to follow “the crowd” and fail to think independently due to social pressure. Another important behavioral aspect is overoptimism. Households tend to underestimate the risk of an increase of interest rates and believe that buying a house does not involve a lot of risk (Case et al., 2003; Farlow, 2004). Regarding corporate real estate, it has been shown that investors invest more regularly in projects for which information is easily available. This is so called availability bias. Hence, the imperfect availability of information often lead the investors to abandon a rational process (Adair et al., 2004 as cited in Salzman and Zwinkels, 2013). Even the appraisers suffer from psychological bias. These range from giving in to client’s pressure to the confirmation bias, where the appraisers tend to adjust their price estimates less when given evidence which goes against their existing view than when given evidence which supports it.

Therefore, the evidence seems to suggest that the housing market is inefficient. In classical financial theory, this inefficiency would be exploited by

arbitrageurs, who would be able to earn returns on their money without taking on risk. However, Shiller (2007) claims that high transactions costs associated with the housing market prevents investors from exploiting the inefficiencies as it makes the process very expensive. Furthermore, Hommes (2006) mentions that it may be very risky for arbitrageurs to correct the inefficiency as many traders may choose to follow the trend and in the short run further worsen the mispricing. Finally, Farlow (2004) presents a comprehensive overview of the various risks associated with arbitrage, such as fundamental risk or margin risk. As a result, models relying on the EMH and considering only agents with rational expectations may be inherently biased when applied to the real estate market.

Recently, many researchers have, thus, turned their attention to models allowing for various forms of expectations and for interaction between multiple types of agents, the so called heterogeneous agent models (HAMs). This approach is by no means new. As is often times the case, the idea of HAMs was first developed to describe the stock market and only subsequently has it been applied to the housing market. Zeeman (1974) already developed a model with two types of investors and it is this model, upon which the model used in this thesis is based. As this model will be relevant to the methodology of this thesis, we will provide its more detailed description in later sections. One of the important aspects of HAMs is that they allow for comparison between several trading strategies and behavioral biases within one model. Furthermore, the interaction between heterogeneous agents may drive the market boom endogenously without a change in external factors. Hommes (2006) presents in detail and lists the HAM literature from its early beginnings to more advanced stage.

In 1990s, empirical studies were written attempting to describe events seen in financial data. Lux (1995) formalizes herd behavior in speculative markets. He describes a process of “infection” among traders which leads the system to equilibria not corresponding to fundamental values. Brock and Hommes (1997, 1998) model a system, where agents are boundedly rational. The traders make a rational choice between several predictors which are based on past performance and, over time, they can adapt and change their choice—an evolutionary learning process. De Grauwe and Grimaldi (2006) apply similar model to the exchange rate, where agents, based on an ex-post evaluation of past profitability can choose between multiple simple forecasting rules and decide whether to switch between the rules over time or not. They found that in such environment, the exchange rate most of the time deviate from its fundamental value.

Anufriev and Hommes (2012) described the evolutionary selection process and the aggregate outcomes in asset pricing experiments. They considered four simple heuristics—adaptive heuristic, strong and weak trend-following heuristic and, finally, anchoring and adjustment heuristic. The results showed that the heuristic switching model can lead to three aggregate outcomes observed in laboratory market forecasting experiments—slow monotonic price convergence, oscillatory dampened price fluctuations and persistent price oscillations.

3.1 Models of the housing market

As the evidence suggests, models with heterogeneous agents can describe the real world and the dynamics on financial markets better than classical models relying on rational expectations. Therefore, it has been recently widely applied in the housing market research as well.

Somervoll et al. (2010) used HAM to illustrate the connection between adaptive expectations and housing market fluctuations. They identify three functions of a dwelling—a shelter, a vehicle for investment and a mortgage collateral. They found that, under no credit constraints, the market experiences mild fluctuations. That is not the case, however, when credit constraints are introduced. The market then presents itself with periods of mild oscillations as in the case of no constraints, but these can be followed by sudden crashes of the market.

Dieci and Westerhoff (2012, 2013, 2016) develop a framework which attempts to show the effect of speculation on house prices. While allowing for heterogeneous agents and their interaction, they also consider the roles of housing demand and supply. In Dieci and Westerhoff (2016), a standard stock-flow housing model is implemented which relates the house prices with the housing stock and the rent levels. They then look how the fundamentals, mainly the supply of housing, interact with behavioral factors represented by expectation-driven agents interaction. The model was found to lead to periods of booms and crashes with periods of overvaluation and overbuilding. Furthermore, they arrived at three conclusions: “an inelastic supply results in longer and more persistent bubbles, the level of overbuilding reached during the boom phase is, along with changes in market expectations, a crucial factor for the timing and size of subsequent crashes and the price-rent ratio may be used as a warning indicator to predict the collapse of housing market bubbles” (Dieci and Westerhoff, 2016).

Finally, Kouwenberg and Zwinkels (2015) tested a HAM model on real world data. Using a multi-agent system, where the agents are heterogeneous, adaptive and boundedly rational, they estimate the model with historical U.S. house prices. The results suggest that the model fit the data well, closely following the historical house prices. Furthermore, the interaction between agents lead to endogenously induced boom-bust cycles.

Chapter 4

Catastrophe theory literature review

In this section, the catastrophe theory is described and the associated literature presented. Furthermore, the various applications of the theory on real-world phenomena will be mentioned.

4.1 Catastrophe theory

Catastrophe theory is a subset of theory of nonlinear dynamical systems. It was developed by a French mathematician René Thom (1923-2002), who introduced the theory in his 1972 book (Thom, 1972). The book was translated into English by a British mathematician David. H. Fowler (1937-2004) in 1975 (Thom and Fowler, 1975). A major contribution of the theory is that it tries to explain and describe various systems of complex behavior where not only continuous changes may occur, but where also a sudden discontinuity may be present. Before that, much of the research in social and natural sciences was based on the concepts of calculus developed independently in the late 17th century by the English scientist Isaac Newton (1642-1727) and the German polymath Gottfried Wilhelm Leibniz (1646-1716). While being very useful in many sciences, it has limitations. As many real-world processes are abrupt and present with a discontinuity, such as stock market crash, fall of a bridge or sudden change of behavior in psychology, a new concept, which would describe such discontinuities better, was needed. The catastrophe theory filled the gap and provided a framework with which such systems may be described.

The systems within the framework of the catastrophe theory are, in general, defined by a potential function, whose critical points correspond to the equilibria of the system. For certain values of parameters and variables, there arise multiple equilibria, between which the system can move abruptly. It is this sudden jump between equilibria that is called a catastrophe.

Gilmore (1993) developed criteria to determine the presence of catastrophes:

- Bimodality: for certain values of parameters and variables, there are multiple equilibria (modes),
- Sudden jump: the transition between the equilibria may occur abruptly with only a small change in the parameters of the system,
- Inaccessibility: the outcomes located between the equilibria (modes) are highly unlikely,
- Hysteresis: the transition from one mode to the other is asymmetrical, i.e. the jump from equilibrium 1 to equilibrium 2 does not occur at the same point in the control parameter space as the jump from equilibrium 2 to equilibrium 1,
- Divergence: a small change in the control parameters usually leads to a small change in the initial and final values of the state variables, for the neighborhood of the critical points, however, a small change in the control parameter initial values may lead to a large change in the final values of the control variable.

In 1974, a British mathematician Erik Christopher Zeeman (1925-2016) bridged the gap between a theory and an application within the field of social sciences. In Zeeman (1974), he proposed a model based on the catastrophe theory, which could explain some of the unstable behavior on the stock exchanges. Within the seven hypotheses stated in the article, he claimed that the price movements of a certain stock index including a stock market crash are caused by interaction between two types of investors—fundamentalists and chartists. Whereas fundamentalists buy and sell stock using fundamental analysis such as evaluation of the firm financial statements, supply and demand and other important economic factors, chartists base their strategy on charts of the past behavior of the market and on the extrapolation of the information to the future. This idea that financial market crashes may be caused not by external factors, but by internal forces within the market, i.e. the interaction between

market participants, has been developed and applied also outside catastrophe theory. For examples of literature of such applications in the field of financial and housing markets, refer to Chapter 3.

Since then, the application of catastrophe theory has not been limited to finance but has evolved into a framework used in various fields. To name a few, van der Maas et al. (2003) applied the catastrophe models to the dynamics of attitude formation and change, Holt et al. (1978) used it for description of changes in international violence, such as wars, Chen et al. (2014) suggested the application of the cusp catastrophe model in medicine to model health outcomes, such as grip strength, while Chen and Chen (2017) used the theory to evaluate an HIV prevention intervention program. Catastrophe theory also provided an approach to explain the sudden change in traffic stream behavior (Navin, 1986) as well as phenomena in biology (van Harten, 2000), chemistry (Wales, 2001) and physics (Tamaki et al., 2003). In economics, one of the most popular early work was done by Varian (1979), in which he analyzed the business cycles.

In late 1970s, however, the catastrophe theory as a model came under pressure. Sussman and Zahler (1977, 1978a, 1978b) launched a critique on the theory, especially on its use in economics and social sciences, where the phenomena are not as exact as in physics or engineering. Their main objection to the theory is that it is largely qualitative and is not easily testable on real-world data. Furthermore, they present several criteria which determine whether or not a model is useful (Sussman and Zahler, 1978a). Even if a model satisfies some of the criteria, it may still be valuable as long as it does not satisfy all of them. However, the authors count the catastrophe theory among models which do satisfy all of the criteria which, therefore, renders it useless. Due to the criticism, the use of catastrophe theory in economics and finance in the following decades substantially decreased (Rosser, 2007).

However, Rosser (2007) reviewed the critique in detail and pointed out the inaccuracies in the claims presented by Sussman and Zahler. Although the catastrophe theory has deficiencies and has to be applied carefully, he rejects the notion of completely avoiding it. He says that some of the main claims in the critique are unfounded or have been since dealt with and makes a case for the return of application of the catastrophe theory within the field of economics.

4.2 Recent applications in finance

Due to the recent justification for the reintroduction of the catastrophe theory into finance and due to the financial crisis of late 2000s, where events showing discontinuities may have been present, researchers returned to the catastrophe model.

Barunik and Vosvrda (2009) applied the stochastic cusp catastrophe model to the U.S. stock market and focused specifically on periods of two crashes, the Black Monday of 1987 and the crisis following the attacks on the World Trade Center on September 11, 2001. Their idea was that while the crash of 2001 was caused by external forces, namely the terrorist attack, the 1987 crash resulted from internal forces within the market. Therefore, the hypothesis was that the catastrophe theory would describe the 1987 data well due to the presence of bifurcation caused by the internal forces. On the other hand, the catastrophe model should not describe the 2001 data better than the linear model as the crash was assumed to be caused by external factors. The S&P 500 index was used as the variable representing the stock market. Furthermore, the daily change of total trading volume, ratio of advancing stocks volume and declining stocks volume, OEX put/call ratio, Dow Jones Composite Bond Index and one-day lag of S&P 500 returns were added to the model as control variables. The findings of the article suggest to confirm the assumptions made by the researchers. In the case of the 1987 crash, the catastrophe model describe the data better than alternative models and there was a high probability that the data show bifurcations. Regarding the 2001 data, the catastrophe model did not outperform the alternative models.

As the model used by Barunik and Vosvrda (2009) assumed a constant volatility of returns, it was not possible to easily apply the theory over time spans longer than periods of few years as, over a longer period of time, the volatility cannot be assumed to be constant. In order to deal with this fact and, as a result, to be able to test the catastrophe model on longer periods of data, Barunik and Kukacka (2015) introduced the concept of realized volatility to the model to estimate the volatility with which the returns could be normalized. The normalized returns were then assumed to exhibit constant volatility. Having satisfied the assumption, the authors examined the daily S&P 500 returns from 1984 to 2010. The time frame, therefore, includes several crises and crashes, specifically the Black Monday of 1987, the burst of the Dot-Com bubble in 2000, the attacks on the World Trade Center, the bankruptcy of Lehman

Brothers of 2008 and the Flash Crash on March 6, 2010. In a similar way to Barunik and Vosvrda (2009), the ratio of advancing stocks volume and declining stocks volume and the OEX put/call ratio were used as control variables to depict the activity of fundamental investors and chartists, respectively. The results suggest that, compared to alternative models, the data over the long-term period are better described by the catastrophe model.

Following the success of application to the stock market, Diks and Wang (2016) turned to the housing market as it shows periods of booms and crashes much like the stock market. Using the stochastic cusp catastrophe model, they studied the housing market of six countries, the United States, the United Kingdom, the Netherlands, Japan, Sweden and Belgium. The data spanned more than 30 years, from 1970 to 2013. As the state variable, the relative deviation of housing price from the estimated fundamental price, which was based on expected future rental prices, was used while the long-term interest rate represented the only control variable. When keeping the control parameters constant, the housing markets of the U.S., the U.K. and the Netherlands were likely not to exhibit bifurcations while, on the other hand, in Sweden, Japan and Belgium the markets did have multiple equilibria. They then also studied the stability of the system by allowing the control variable to vary and found that there is evidence that the interest rate can induce the systems to move to the bifurcation region, but that the paths are different for each country.

More recently, Wesselbaum (2017) estimated several catastrophe models of the financial crisis. First, using quarterly data from 1991 to 2015, the bank failure model developed in Ho and Saunders (1980) was discussed. There, they approximated the bank failure probabilities, or the state variable, by the delinquency rate on all commercial loans. Furthermore, they used the loan loss reserves and the Chicago FED financial stress index as control variables. However, the model did not generate jumps, which they claim is due to the small sample size or due to misspecification. Next, they augment the model by also adding the effect of monetary policy represented by the Federal Funds rate. In this model, catastrophic events are observed and the model fits the data better than alternative models. Therefore, these results suggest that the monetary policy plays an important role in the build-up to and during the financial crisis.

Chapter 5

Methodology

This section presents the methodology. The model is based on catastrophe theory, specifically cusp catastrophe model. One of the main aspects of this theory is that it allows for sudden (discontinuous) changes in the state variable given only small smooth changes in the control variables. As there have been many crises in the housing market around the world as was mentioned in Chapter 1 and as the financial crisis in the United States in the late 2000s occurred within the period of our interest, it is appropriate to apply the catastrophe model here.

5.1 Basic framework

This thesis is based on Zeeman (1974), in which he applied the catastrophe theory to the stock market and introduced internal dynamics by allowing for two types of investors. Often, the catastrophe model is depicted using a 3-dimensional equilibrium surface, where the outcome is shown dependent on the values of the parameters α and β . Figure 5.1 shows the cusp catastrophe equilibrium surface.

Zeeman (1974) argued that in markets where fundamentalists and chartists are present, it is the chartists who drive the bifurcation, which may eventually lead to a crash. On the other hand, the fundamentalists are a stabilizing force in the market. Let's assume that there is no excess demand for an index. Then the market finds itself in equilibrium which is the origin in Figure 5.1. However, the equilibrium is unstable as even a small perturbation will cause chartists to enter the market and to follow the trend of the perturbation. When there is a positive change in price, the index will, therefore, begin to rise. In normal case,

as the returns continue rising, the fundamentalists will begin leaving the market as, using the market knowledge and fundamental analysis, they will perceive the index to be overvalued. There will, thus, be a downward pressure on the returns. At certain point, in Figure 5.1 at point *A*, the downward pressure will be stronger than the activity of the chartists and the returns will begin to fall. A recession will ensue and the chartists by following the trend will begin to leave the market as well. The returns will then slowly continue falling until certain point, in Figure 5.1 point *B*, when the fundamentalists will begin to buy again as they will perceive the index to be undervalued and the system will return to the origin.

The dynamics described above is only valid for systems with limited number of chartists. When we consider a system where the number of chartists is much higher, the path will be different. Again, when there is a small perturbation upwards, the rising index will attract chartists who will drive the returns higher. At point *C*, *C'* or *C''*, the downward pressure exerted by the fundamentalists who perceive the index to be overvalued will overpower the chartists and the returns will begin to decrease. However, this will cause a large number of chartists to leave the market at the same time. As a result, the system will find itself within the bifurcation region in Figure 5.1 and the index will fall abruptly. This event is called a crash, which in our case represents the catastrophe. Afterwards, there will be a period of instability when the index might rise and decline again substantially. In this period, the chartists will cease to be active in the market as there is no clear trend to follow. Once this has occurred (point *E*, *E'* or *E''*), the fundamentalists perceiving the index to be undervalued will begin to drive the returns back to the origin.

It is apparent that the value of the factor β determines whether the system is within the bifurcation region. On the other hand, the factor α determine whether the system finds itself on the lower-value region or on the higher-value region. Based on Zeeman (1974), the bifurcation factor is represented by the activity of the chartists and the asymmetry factor by the fundamentalists.

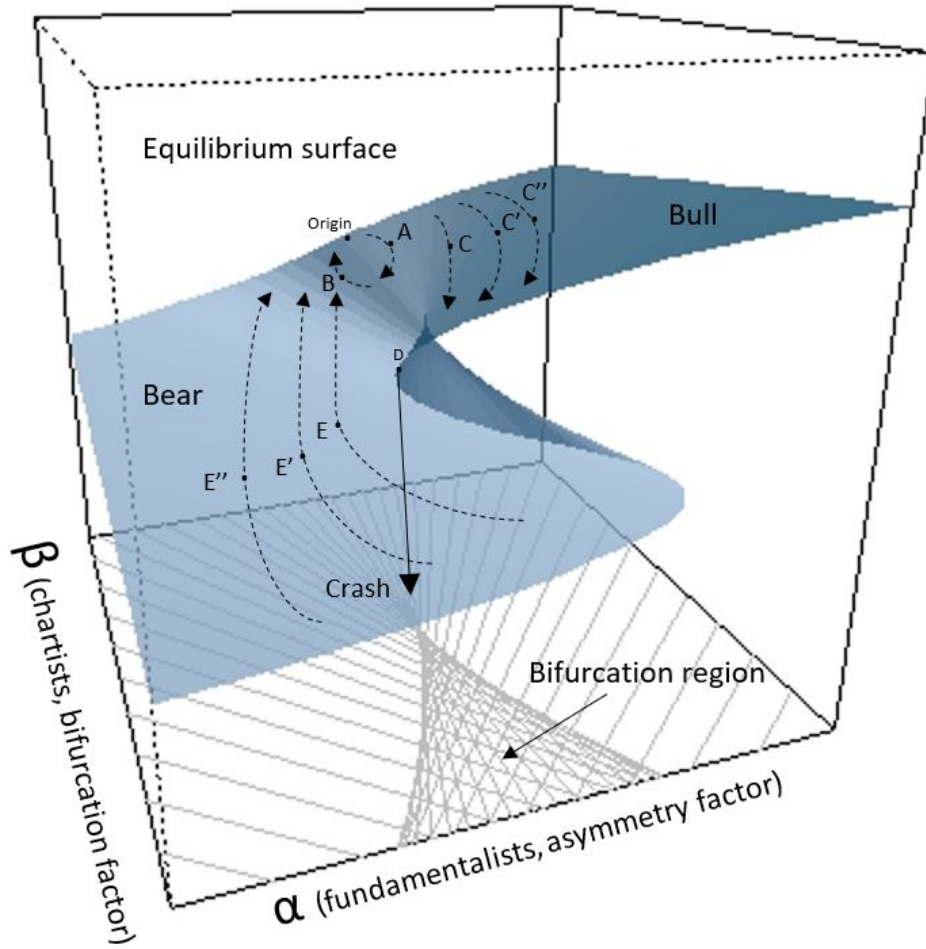


Figure 5.1: Cusp surface

Source: cusp R package

5.2 Model

Now, the model behind the main idea and framework introduced above will be described. In general, the model is defined by the equation

$$\frac{dy_t}{dt} = -\frac{\partial V(y_t; \theta)}{\partial y_t}, \quad (5.1)$$

where y_t is the canonical state variable describing the state of the system, $V(y_t; \theta)$ is a smooth potential function controlled by the control parameter (single or multiple) θ . It is apparent that the equilibria of the system occur when $\frac{dy_t}{dt} = 0$ and, therefore, when $\frac{\partial V(y_t; \theta)}{\partial y_t} = 0$. Due to the construction of the equation, the system revolves around an equilibrium as the system is driven

towards it when a perturbation occurs.

As one of the most widely applied types of the catastrophe theory in social sciences is the cusp version and as it also is the type suggested in Zeeman (1974), we focus on that as well. The model has, therefore, two control parameters, α and β , and the potential function is

$$V(y_t; \alpha; \beta) = \frac{1}{4}y_t^4 - \frac{1}{2}\beta y_t^2 - \alpha y_t, \quad (5.2)$$

where the parameter α is called the normal or asymmetry control factor while β the splitting or bifurcation control factor (Stewart and Peregoy, 1983). From there, it comes that the equilibria are the points for which

$$\frac{\partial V(y_t; \alpha; \beta)}{\partial y_t} = y_t^3 - \beta y_t - \alpha = 0. \quad (5.3)$$

Based on the value of Cardan's discriminant $\delta = 27\alpha^2 - 4\beta^3$, the number of roots of the equation can be derived. Cardan's discriminant arises when solving cubic equations using the Cardan's method. In general, the cubic equation has to be transformed to a reduced form of shape $x^3 + px + q = 0$. It is apparent that in our case the equation is already in this shape with $p = -\beta$ and $q = -\alpha$. The solution of the equation is then searched for in form: $x = v + w$. It has been shown that the solution can be written in the following form called the Cardano's formula:

$$x = v + w = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}. \quad (5.4)$$

Cardan's discriminant is then defined as $\delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$. In our case, it is, therefore, $\delta = \frac{\alpha^2}{4} - \frac{\beta^3}{27}$ or $\delta = 27\alpha^2 - 4\beta^3$. The equation has one real root for $\delta > 0$. When $\delta < 0$, the system finds itself in the bifurcation region and there are three distinct real roots, two local minima, which represent the stable equilibria, and a local maximum, which is an unstable equilibrium. Hence, there are two predicted values for the stable variable. This bifurcation is one of the most important aspects of the catastrophe theory as it allows for a sudden jump, which may represent a stock or a housing market crash. In the final case, when $\delta = 0$, the equation has a repeated root and all roots are real and

the predicted value is directly at a catastrophe point (Cobb, 1981).

The above approach represents the deterministic model. However, in reality, most of the systems cannot be regarded as fully deterministic, either due to a measurement error or to the nature of the system. In Zeeman (1974), it is proposed to introduce a stochastic process to the model in order to add in the effect of external factors. To move from the deterministic model to a stochastic one, we follow the approach suggested in Cobb and Watson (1980), where a white noise driving term is added. The model is then defined by the following equation

$$dy_t = -\frac{\partial V(y_t; \alpha; \beta)}{\partial y_t} dt + \sigma_{y_t} dW_t, \quad (5.5)$$

where the term W_t is a standard Wiener process. A standard Wiener process W_t for $t \geq 0$ is a stochastic process with the following properties: (1) $W_0 = 0$, (2) with probability 1, W_t is continuous in t , (3) the increment $W_{t+s} - W_t$ has the $Normal(0, s)$ distribution and (4) $W_{t+s} - W_t$ is independent of the past values W_u , $u < t$. The diffusion function σ_{y_t} determines the intensity of the Wiener process and can be understood as a randomness effect. As σ_{y_t} approaches 0, the system will be closer to the deterministic version. (Cobb, 1978).

In general, σ_{y_t} is a function of the state variable y_t . However, when it changes with y_t , the behavior of the system becomes complex and difficult to describe. On the other hand, it has been shown (Cobb, 1978, 1980; Cobb and Watson, 1980; Wagenmakers et al., 2005) that, when the diffusion function σ_{y_t} is constant, $\sigma_{y_t} = \sigma$, the probability density function describing the distribution of the system's states conditional on the control parameters α and β turns into the following shape:

$$f_{pdf}(y_t | \alpha, \beta) = k \exp \left(\frac{-\frac{1}{4}y^4 + \frac{1}{2}\beta y^2 + \alpha y}{\sigma} \right), \quad (5.6)$$

where k is a normalizing constant.

It is apparent that it has extrema at points where

$$-y_t^3 + \beta y_t + \alpha = 0. \quad (5.7)$$

Thus, they correspond to the equilibria predicted by the deterministic model. When the Cardan's discriminant $\delta = 27\alpha^2 - 4\beta^3$ is negative, there are two modes and an antimode and the system finds itself within the bifurcation region. The value of the control parameter β determines whether the probability density function in Equation 5.6 is unimodal or bimodal, whereas the parameter α determines the height of the modes.

5.3 Volatility

As we have seen above, the stochastic cusp catastrophe model works well when the assumption of constant diffusion parameter σ_{y_t} is satisfied. When modeling financial markets, the diffusion function is the volatility of the market returns. While the volatility of the returns of a housing market index is not directly observable and, thus, we cannot immediately determine σ_{y_t} , it would be highly unrealistic to assume that it is constant over a longer period of time. Barunik and Vosvrda (2009), who applied the cusp catastrophe to stock market, dealt with the problem by focusing on only short periods of time where it was reasonable to assume constant volatility, while Barunik and Kukacka (2015) implemented the concept of realized volatility in order to estimate the "true" volatility.

Even though the approach of Barunik and Kukacka (2015) proved to be successful, the lack of high-frequency data in our case prevents us from emulating it. Therefore, the volatility is, in our case, estimated by implementing a generalized autoregressive conditional heteroskedasticity (GARCH) model.

GARCH model was introduced in Bollerslev (1986) and is an extension of the ARCH model developed by Engle (1982). It has been widely applied in finance for estimating volatility, which is useful for measuring the risk associated with holding a security. According to the model, the estimate of the variance in the next period is the weighted average of the long-term variance, the variance in this period, and of the variance predicted for this period. In general, the GARCH(p, q) model, where p is the number of GARCH terms and q is the number of ARCH terms, for variance conditional on past information is as follows

$$h_t = \alpha + \sum_{i=1}^p \beta_i h_{t-i} + \sum_{j=1}^q \gamma_j \epsilon_{t-j}^2, \quad (5.8)$$

where $\alpha, \beta_1, \dots, \beta_p, \gamma_1, \dots, \gamma_q$ are coefficients, h_t is the conditional variance for period t and ϵ_t^2 is the actual observed variance for period t . The coefficients $\alpha, \beta_1, \dots, \beta_p, \gamma_1, \dots, \gamma_q$ must be positive.

After obtaining the estimates from GARCH, the state variable will be standardized and the system will be assumed to follow the following equation

$$dr_t = d\left(\frac{y_t}{\sqrt{h_t}}\right) = -\frac{\partial V(r_t; \alpha; \beta)}{\partial y_t} dt + dW_t. \quad (5.9)$$

Thus, the diffusion function is equal to 1 in every period and the model now satisfies the constant-volatility assumption of the theory.

5.4 Estimation

First, the GARCH model specified above is evaluated and the estimated volatility obtained. The returns are then standardized by the estimated volatility and this is our state variable.

As was stated in the definition of the model, however, the state variable is canonical, which is obtained by a smooth transformations of the original variables. We follow the approach used in Grasman et al. (2009), Barunik and Kukacka (2015) and Diks and Wang (2016), where the new variable is assumed to be a linear combination of the actual observed variables, that is

$$z_t = w_0 + w_1 y_{t,1} + w_2 y_{t,2} + \dots + w_p y_{t,p}, \quad (5.10)$$

where w_0, w_1, \dots, w_p are coefficients.

In our case, since we only have one dependent variable, we have

$$z_t = w_0 + w_1 y_t. \quad (5.11)$$

Regarding the control parameters α and β , we shall make the simplifying assumption similar to Cobb (1981) that they are linear functions of the independent variables, that is

$$\alpha = a_0 + a_1 x_1 + a_2 x_2 + \dots + a_k x_k \quad (5.12)$$

$$\beta = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_k x_k, \quad (5.13)$$

where x_1, x_2, \dots, x_k are the independent (exogenous) variables and $a_0, a_1, \dots, a_k, b_0, b_1, \dots, b_k$ are coefficients.

Hence, we will need to estimate the parameters $w_0, w_1, a_0, a_1, \dots, a_k, b_0, b_1, \dots, b_k$. To do this, we will follow the maximum likelihood approach of Cobb and Watson (1980) augmented in Grasman et al. (2009). Given the probability density function in Equation 5.6 and observed values of the dependent and independent variables, the likelihood function is

$$\mathcal{L} = \prod_i f_{pdf}(z_i | \alpha_i, \beta_i) = \prod_i \left[k_i \exp \left(-\frac{1}{4} z_i^4 + \frac{1}{2} \beta_i z_i^2 + \alpha_i z_i \right) \right]. \quad (5.14)$$

Applying simple arithmetic rules, we have

$$\mathcal{L} = \prod_i k_i \exp \left[\sum_i \left(-\frac{1}{4} z_i^4 + \frac{1}{2} \beta_i z_i^2 + \alpha_i z_i \right) \right]. \quad (5.15)$$

Now, we transform the above likelihood into negative log-likelihood

$$-\log \mathcal{L} = -\sum_i \log k_i - \sum_i \left(-\frac{1}{4} z_i^4 + \frac{1}{2} \beta_i z_i^2 + \alpha_i z_i \right). \quad (5.16)$$

Finally, the negative log-likelihood function $-\log \mathcal{L}$ is minimized with respect to the parameters $w_0, w_1, a_0, a_1, \dots, a_k, b_0, b_1, \dots, b_k$.

5.5 Evaluation of fit

In order to estimate how well the model fits the data, several measures have been developed. We will use the tools suggested in Grasman et al. (2009). The basic idea is to estimate alternative models and then compare the fit of the cusp catastrophe model relative to them.

First, a simple linear model is implemented. Cobb (1998) and Hartelmann (1997), as cited in Grasman et al. (2009), propose the following pseudo- R^2 measure

$$R^2 = 1 - \frac{\text{error variance}}{\text{Var}(y_t)}. \quad (5.17)$$

A problem arises with the definition of the error variance. As the cusp catastrophe model may predict multiple predicted values, a convention is needed to select which predicted values of the independent variable will be considered. First, systems obey the delay convention when they remain in the equilibrium in which they are until it disappears. On the other hand, the systems which seek a global minimum of potential follow the Maxwell's convention (Saunders, 1980). Thus, the mode of the density which is closest to the predicted value is used under the delay convention whereas under the Maxwell's convention the mode whose density is highest is selected. The error variance is then defined as the variance of the differences between the chosen predicted value and the observed value (Grasman et al., 2009). In our case, the delay convention is used. To further compare the two models, Cobb (1978) also suggests to use a likelihood-ratio χ^2 test.

On one hand, a linear regression is probably the most widely used model and, therefore, a comparison of the cusp catastrophe model to it is certainly useful. On the other hand, the cusp catastrophe model is usually used to describe events where sudden changes occur. By definition, the linear model will most likely be very poor in describing such events. Thus, Hartelman (1997), as cited in Grasman et al. (2009), suggests comparing our model to a non-linear least squares regression with the logistic curve

$$y_i = \frac{1}{1 + e^{-\alpha_i/\beta_i^2}} + \epsilon_i \quad , \quad i = 1, \dots, n, \quad (5.18)$$

where ϵ_i are zero-mean random disturbances.

This curve is more suitable to describing the events for which we attempt to use the cusp catastrophe model as it allows for exponential relationships and, thus, may model the sudden but continuous changes better than the linear model.

The comparison then can be made using the Akaike information criterion (AIC), described in Akaike (1974), and the Bayesian information criterion (BIC), developed in Schwartz (1978), where the smaller the criteria, the better the fit.

Chapter 6

Data and results

In this chapter, the model specification is presented. Furthermore, the data used are described, the results presented and the implications of the findings assessed. The cusp catastrophe theory has already been applied to the housing market, most notably in Diks and Wang (2016). However, their approach shows several limitations. Therefore, we expand on the approach presented in their paper by identifying three points upon which we elaborate.

6.1 State variable

As a measure assumed to be representative of the market, Diks and Wang (2016) used the relative deviation of the house price from the fundamental price, where the value of the fundamental price is based on the expected future rental prices as introduced in Bolt et al. (2014). However, the use of this state variable may be problematic as the estimation of the fundamental price is inherently arbitrary. There is no clear method to derive the value and, as a result, for various economists, a fundamental value may attain different meanings. Hence, we have decided to abandon the concept of fundamental value in our approach and to fit the model using only market data. As our state variable, we use the weekly log returns of the Dow Jones U.S. Real Estate Index, which represents a subsection of the Dow Jones U.S. Index and which is designed to depict the Real Estate Supersector as defined by the Industry Classification Benchmark (ICB). The time period covered runs from March 23, 2007 to May 12, 2017. Therefore, we have 530 observations. Even though there are data for the index available before 2007 as well as daily data, we do not use them as the data used for control variables specified later are limited to

this period and are published on weekly basis. The reference dates in our data are Fridays and the weekly log returns are calculated as the sum of the daily log return for the particular week. The data were collected using the Thomson Reuters Eikon database.

In order to be as close as possible to the approach of Diks and Wang (2016), we first estimate the cusp catastrophe model with the long-term interest rate as the only control variable used for determining both the asymmetry and bifurcation factor. Thus, we have

$$z_t = w_0 + w_1 y_t, \quad (6.1)$$

$$\alpha = a_0 + a_1 x_1, \quad (6.2)$$

$$\beta = b_0 + b_1 x_1, \quad (6.3)$$

where y_t are the weekly log returns and x_1 the long-term interest rate. The yield of the 10-year U.S. Government bonds was collected to represent the long-term interest rate. The estimation was performed using the cusp package in the software R. Table 6.1 shows the summary of the estimated cusp catastrophe model.

	Estimate	Std. Error	z-value	P(> z)
a_0	9.90	0.43	22.88	<2e-16 ***
a_1	-177.76	15.71	-11.32	<2e-16 ***
b_0	-11.29	0.78	-14.42	<2e-16 ***
b_1	228.16	0.04	5192.91	<2e-16 ***
w_0	0.82	0.09	9.56	<2e-16 ***
w_1	9.53	0.36	26.63	<2e-16 ***

	pseudo R^2	AIC	BIC
Linear	0.002	-1913.41	-1900.60
Logistic	0.006	-1911.42	-1890.05
Cusp	-0.020	1491.04	1516.68

Table 6.1: Summary of the model with one control variable (model 1)

We can see that the Akaike and the Bayesian information criteria have different signs for the linear and logistic models compared to the cusp model. That is due to the fact that the R software estimated the log-likelihoods positive for the two former models. This would suggest that the linear model fits the data best by far. This unexpected result may arise due to the fact that we try to

estimate six coefficients with only one independent variable and one dependent variable. Therefore, the fit might be complicated and artificial. In any case, the measures for the linear and logistic models should be used with caution. The coefficients for the state variable as well as for the control variables are all significant at the 99% confidence level. The coefficients a_1 and b_1 are large in absolute values indicating that the long-term interest rate drives both the bifurcation and the asymmetry.

However, as we have described in Chapter 5, Barunik and Kukacka (2015) showed that the stochastic cusp catastrophe model works well only if the constant volatility assumption is satisfied. The weekly log returns of our index cannot be assumed to be constant over time and, therefore, the estimates of the model shown in Table 6.1 may be inaccurate. Figure 6.1 depicts the returns of the Dow Jones U.S. Real Estate Index from March 23, 2007 to May 12, 2017. It is clear that the volatility of the returns really does vary over time.

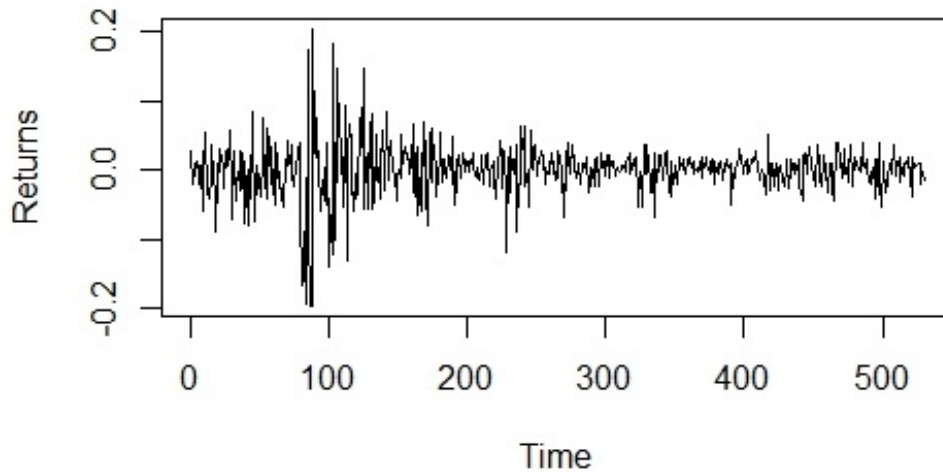


Figure 6.1: Weekly returns of the Dow Jones U.S. Real Estate Index

Source: Thomson Reuters

6.2 Estimation of volatility

Even though Diks and Wang (2016) cite the paper of Barunik and Kukacka (2015), they completely omit the constant volatility assumption from their model. However, the volatility of the relative deviation from the fundamental value, their state variable, cannot be assumed to be constant similarly to our case. Therefore, their estimation may lead to inaccuracies. We expand the

model by implementing the concept of constant volatility. As we describe in Chapter 5, we do not possess high-frequency data and, as a result, cannot use realized volatility, which was done in Barunik and Kukacka (2015), here. We rather estimate a standard GARCH model on the weekly log returns of the index from March 23, 2007 to May 12, 2017.

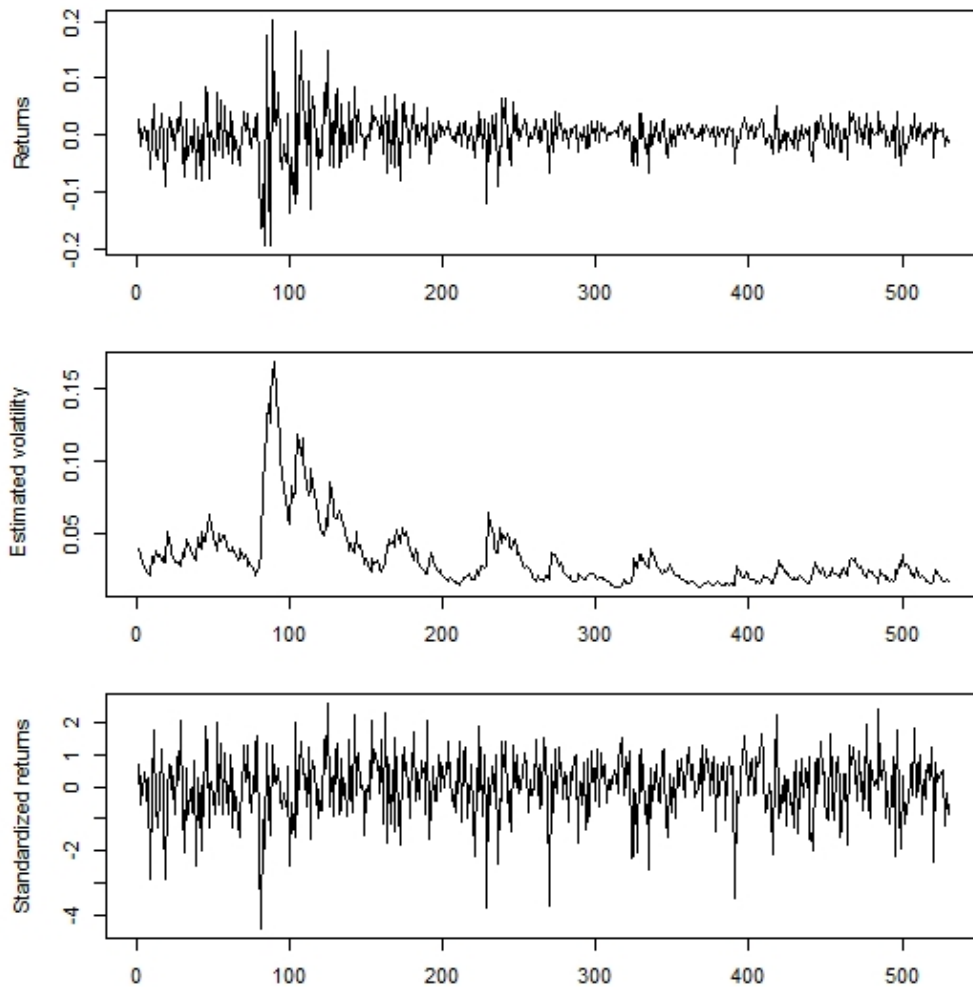


Figure 6.2: Returns, estimated volatility and standardized returns of the Dow Jones U.S. Real Estate Index

Source: Thomson Reuters, own calculation

First, using the Augmented Dickey-Fuller test we checked for stationarity. Next, we employed the Akaike information criterion to choose the most suitable ARIMA model. The model was selected and then, based again on AIC, the “best” GARCH model estimated. After estimating the GARCH model, we were able to determine the estimated weekly volatility and use it to normalize the

returns. Figure 6.2 shows the returns, the estimated volatility from GARCH and the normalized returns.

With the normalized returns, we then estimated the cusp catastrophe model using the long-term interest rate as the only control variable as in the case of the previous model with non-normalized returns. Table 6.2 shows the estimates. We can see that the estimated coefficients changed substantially compared to the previous model. w_1 became much smaller indicating that a more subtle transformation of the state variable is needed. Also, with the introduction of the estimated volatility, the information criteria for the cusp and alternative models are now much more comparable. Furthermore, the cusp model fits the data best based on the information criteria. This evidence points to the importance of the constant volatility assumption. Although the pseudo R-squared does not confirm that the cusp model fits the data better than the alternative models, it is a relatively weaker measure compared to the criteria based on log-likelihoods and, therefore, should be used only as a supplement. The intercept coefficients for the control variable as well as both coefficients for the state variable are significant at 99% confidence level. The coefficients a_1 and b_1 are large in absolute values and significant at 99% confidence level and 95% confidence level, respectively. This suggests that the long-term interest rate drives both sides of the market similarly to the previous model.

	Estimate	Std. Error	z-value	P(> z)
a_0	7.63	0.93	8.18	2.94e-16 ***
a_1	-95.68	34.17	-2.80	0.005 ***
b_0	-3.84	0.92	-4.19	2.82e-5 ***
b_1	71.53	33.42	2.14	0.032 **
w_0	1.26	0.02	69.77	<2e-16 ***
w_1	0.40	0.01	31.34	<2e-16 ***

	pseudo R^2	AIC	BIC
Linear	0.002	1507.24	1520.06
Logistic	0.008	1508.08	1529.45
Cusp	0.002	1484.74	1510.38

Table 6.2: Summary of the model with standardized returns and one control variable (model 2)

6.3 Fundamentalists and chartists

Finally, Diks and Wang (2016) consider only one control variable, the long-term interest rate. However, that is not consistent with the framework presented in Zeeman (1974) where the interaction between two types of investors, the fundamentalists and the chartists, drives the dynamics of the market. These two types of investors are assumed to follow different strategies. It may therefore appear unreasonable to estimate both the asymmetry and bifurcation factors using only one control variables as it should not be representative of the activity of both types of investors. While we by no means reject the view that the long-term interest rate plays an important role in determining the dynamics of the housing market (in fact, we include it in our model), we also add other control variables in order to depict and estimate the different activity of the two types of investors.

The fundamentalists base their strategy on fundamental analysis. Therefore, we estimate that their activity will be correlated with indicators such as the mortgage rate or the number of mortgage applications. Specifically, the weekly log change of the MBA refinance index and the 30-year fixed mortgage rate, both published weekly by the Mortgage Bankers Association, are added to the model. As we assume that the fundamentalists should drive the asymmetry, we use these two control variables to estimate only the asymmetry factor.

On the other hand, chartists do not follow such indicators. As the speculators in the build-up to the financial crisis used various derivatives whose value was based on the underlying mortgages, such as mortgage-backed securities, collateralized debt obligations or credit default swaps, we estimate their activity by the weekly log change in the volume-weighted-average-price (VWAP) of the iShares MBS ETF which seeks to track the investment results of an index composed of investment-grade mortgage-backed securities. The data were collected from the Thomson Reuters Eikon database. As the chartists are assumed to drive the bifurcation, we add this control variable only to the estimation of the bifurcation factor

Furthermore, we keep the long-term interest rate represented by the yield of 10-year U.S. Government bonds in the model. Overall, we therefore have

$$\alpha = a_0 + a_1x_1 + a_2x_2 + a_3x_3, \quad (6.4)$$

$$\beta = b_0 + b_1x_1 + b_2x_4, \quad (6.5)$$

where x_1 is the long-term interest rate, x_2 the weekly log change of MBA refinancing index, x_3 the 30-year fixed mortgage rate and x_4 the weekly log change of VWAP of the iShares MBS ETF. The normalized returns of the Dow Jones U.S. Real Estate Index are used as the state variable similarly to our second estimated model. Table 6.3 summarizes the estimates for this model.

	Estimate	Std. Error	z-value	P(> z)
a_0	8.48	0.55	15.53	<2e-16 ***
a_1	5.83	10.29	0.57	0.571
a_2	-0.37	0.84	-0.44	0.662
a_3	-91.29	20.00	-4.56	5.03e-6 ***
b_0	-3.01	0.09	-34.13	<2e-16 ***
b_1	56.39	2.27	24.87	<2e-16 ***
b_2	6.25	38.40	0.16	0.871
w_0	1.24	0.03	41.60	<2e-16 ***
w_1	0.42	0.01	30.69	<2e-16 ***

	pseudo R^2	AIC	BIC
Linear	0.017	1505.28	1530.91
Logistic	0.049	1491.70	1525.89
Cusp	0.015	1483.91	1522.36

Table 6.3: Summary of the model with standardized returns and four control variables (model 3)

The results show that, compared to the alternative models, the cusp model clearly performs better when assessed using the information criteria. While several parameters are significant at 99% confidence level, we can see that the coefficients a_1 , a_2 and b_2 are not significant at any reasonable level. This suggests that the bifurcation factor is affected mainly by the long-term interest rate and not by the weekly log change of VWAP of the iShares MBS ETF. Regarding the fundamentalist side of the market, we see that only the parameter a_3 is significant suggesting that the asymmetry is driven mainly by the 30-year fixed mortgage rate.

6.4 Comparison and final results

Overall, we have estimated three versions of the cusp model. Table 6.4 provides the summary of the estimates of coefficients and BIC for each of the three cusp models. We have shown that the first model does not satisfy the constant

	Model 1	Model 2	Model 3
	Estimate	Estimate	Estimate
a_0	9.90 ***	7.63 ***	8.48 ***
a_1	-177.76 ***	-95.68 ***	5.83
a_2	—	—	-0.37
a_3	—	—	-91.29 ***
b_0	-11.29 ***	-3.84 ***	-3.01 ***
b_1	228.16 ***	71.53 **	56.39 ***
b_2	—	—	6.25
w_0	0.82 ***	1.26 ***	1.24 ***
w_1	9.53 ***	0.40 ***	0.42 ***
	BIC	BIC	BIC
Linear	-1900.60	1520.06	1530.91
Logistic	-1890.05	1529.45	1525.89
Cusp	1516.68	1510.38	1522.36

Table 6.4: Comparison of the estimated models

volatility assumption of the stochastic cusp catastrophe model and, therefore, its inclusion here may seem questionable. However, we present it here as its approach is closest to the approach used in Diks and Wang (2016). Hence, it may be useful for comparison. In the cases with normalized returns (Models 2 and 3), the cusp model fitted the data better than the linear and logistic models. It is apparent that with the introduction of estimated volatility and the normalization of returns (Model 2), the cusp model performs better than without normalization (Model 1). Also, the coefficient w_1 is much lower when using the normalized returns indicating a more subtle transformation of the state variable. Therefore, normalizing the returns was beneficial not only from the theoretical point of view, but also empirically.

Lastly, we added multiple control variables to represent the activity of the two sides of the market (Model 3). However, based on BIC, the cusp model fits the data worse than both the Model 1 and Model 2. It is also interesting to note that the coefficients for x_2 and x_4 are not significant. The coefficient for the 30-year fixed mortgage rate is significant, but that may be due to the fact that it may be highly correlated with the long-term interest rate as a_1 is no longer significant and decreased substantially in absolute value. Thus, the inclusion of the control variables meant to represent the activity of fundamentalists and chartists did not improve the model. The evidence points to the conclusion that the largest driver in our model may be the long-term interest rate.

The empirical results suggest that the use of the catastrophe theory was justified as the estimates indicate a better fit than the alternative models. However, the difference is not large and, therefore, caution should be exercised when implementing the catastrophe model to the housing market before further research is carried out as there is a question whether the slightly better results are worth of estimating the rather more complicated model.

Chapter 7

Conclusion

In this thesis, we estimate the cusp catastrophe model using weekly data on the U.S. housing market from 2007 to 2017. Even though the catastrophe theory has already been applied to the real estate market, there is plenty of scientific potential for further exploration of the topic. In the previous applications, the fundamental price played a role in determining the state variable. That is, however, problematic as the correct derivation of fundamental value is inherently unknown. Furthermore, two more deficiencies of the catastrophe models applied to housing market were identified. First, it has been shown that the stochastic cusp catastrophe model works well only when the assumption of constant volatility is satisfied. In the current literature, however, the catastrophe models applied to housing market do not deal with this fact. Moreover, the models found in literature estimate the activity of fundamentalists and chartists with only one control variable. That is, however, unreasonable as these two types of investors are assumed to follow different strategies. Our contribution lies in the elimination of these deficiencies.

Overall, we estimate three versions of the cusp model. First, we eliminate the concept of fundamental value from the derivation of the state variable and use the returns of a real estate index instead, for which data can be easily obtained and no derivation is needed. For the second version of the model, we normalize the returns by the estimated volatility derived by a standard GARCH model. This is an important step as the returns of the index cannot be assumed to be constant. Therefore, the normalization is needed. Finally, we try to introduce the concept of fundamentalists and chartists to the model as it has been shown that the interaction between them may drive the dynamics of stock as well as housing market. The incorporation was done by the addition

of multiple control variables, which were used as an estimate of the activity of the two types of investors.

After estimating the results, the cusp catastrophe model fits the data better than the linear and logistic models. The introduction of the normalized state variable improved the results. However, the addition of control variables used to depict the activity of fundamentalists and chartists did not improve the model performance and the long-term interest rate appeared to be the largest determinant of the dynamics of the system in our models. While the results suggest a better fit of the cusp model compared to the alternative models, the difference is not large. In the future, it would be useful to further explore the possible variables, which could specifically explain the roles of fundamentalists and chartists. Further research could therefore be carried out in this direction.

Bibliography

ADAIR, A.S., BERRY, J.N. and MCGREAL, W.S. (1994). "Investment Decision Making: A Behavioural Perspective". *Journal of Property Finance*, Vol. 5, No. 4, pp. 32-42.

AKAIKE, H. (1974). "A new look at the statistical model identification". *IEEE Transactions on Automatic Control*, Vol. 19, No.6, pp. 716-723.

ANUFRIEV, M. and HOMMES, C. (2012). "Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments". *American Economic Journal: Microeconomics*, Vol. 4, No. 4, pp. 35-64.

BARUNIK, J. and KUKACKA, J. (2015). "Realizing stock market crashes: stochastic cusp catastrophe model of returns under time-varying volatility". *Quantitative Finance*, Vol. 15, No. 6, pp. 959-973.

BARUNIK, J. and VOSVRDA, M. (2009). "Can a stochastic cusp catastrophe model explain stock market crashes?". *Journal of Economic Dynamics and Control*, Vol. 33, No. 10, pp. 1824-1836.

BOLLERSLEV, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity". *Journal of Econometrics*, Vol. 31, No. 3, pp. 307-377.

BOLT, W., DEMERTZIS, M., DIKS, C., HOMMES, C. and VAN DER LEIJ, M. (2014). "Identifying booms and busts in house prices under heterogeneous expectations". DNB Working Papers 450, Netherlands Central Bank, Research Department.

BROCK, W. and HOMMES, C. (1997). "A rational route to randomness". *Econometrica*, Vol. 65, pp. 1059-1095.

BROCK, W. and HOMMES, C. (1998). "Heterogeneous beliefs and routes to chaos in a simple asset pricing model". *Journal of Economic Dynamics Control*, Vol. 22, pp. 1235-1274.

CASE, K.E., QUIGLEY, J.M. and SHILLER, R.J. (2003). "Home-buyers, Housing and the Macroeconomy". Paper presented at the Conference on Asset Prices and Monetary Policy.

CASE, K.E. and SHILLER, R.J. (1989). "The Efficiency of the Market for Single-Family Homes". *The American Economic Review*, Vol. 79, No. 1, pp. 125-137.

CHEN, D.G. and CHEN, X. (2017). "Cusp Catastrophe Regression and Its Application in Public Health and Behavioral Research". *Int J Environ Res Public Health*, Vol. 14, No. 10.

CHEN, D.G., LIN, Feng, CHEN, X., TANG, W. and KITZMAN, H. (2014). "Cusp Catastrophe Model A Nonlinear Model for Health Outcomes in Nursing Research". *Nursing research*, Vol. 63, pp. 211-220.

COBB, L. (1978). "Stochastic Catastrophe Models and Multimodal Distributions". *Behavioral Science*, Vol. 23, pp. 360-374.

COBB, L. (1980). "Estimation Theory for the Cusp Catastrophe Model". *1980 Proceedings of the American Statistical Association*, Section on Survey Research Methods, pp. 772-776.

COBB, L. (1981). "Parameter Estimation for the Cusp Catastrophe Model". *Behavioral Science*, Vol. 26, No. 1, pp. 75-78.

COBB, L. (1998). "An Introduction to Cusp Surface Analysis". Technical report, Aetheling Consultants. Louisville, CO, USA.
url: <http://www.aetheling.com/models/cusp/Intro.htm>.

COBB, L. and WATSON, W.B. (1980). "Statistical catastrophe theory: An overview". *Mathematical Modeling*, Vol. 1, No. 4, pp. 311-317.

DE GRAUWE, P. and GRIMALDI, M. (2006). "Exchange rate puzzles: a tale of switching attractors". *European Economic Review*, Vol. 50, pp. 1-33.

DIECI, R. and WESTERHOFF, F. (2012). "A simple model of a speculative housing market". *Journal of Evolutionary Economics*, Vol. 22, No. 2, pp. 303-329.

- DIECI, R. and WESTERHOFF, F. (2013). "Modeling House Price Dynamics with Heterogeneous Speculators". In: BISCHI G., CHIARELLA C., SUSHKO I. (eds) *Global Analysis of Dynamic Models in Economics and Finance*. Springer, Berlin, Heidelberg.
- DIECI, R. and WESTERHOFF, F. (2016). "Heterogeneous expectations, boom-bust housing cycles, and supply conditions: a nonlinear dynamics approach". *Journal of Economic Dynamics and Control*, Vol. 71, pp. 21-44.
- DIKS, C, and WANG, J. (2016). "Can a stochastic cusp catastrophe model explain housing market crashes?". *Journal of Economic Dynamics and Control*, Vol. 69, pp. 68-88.
- ENGLE, R.F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation". *Econometrica*, Vol. 50, No. 4, pp. 987-1007.
- FAMA, E.F. (1970). "Efficient Capital Markets: A Review of Theory and Empirical Work". *The Journal of Finance*, Vol. 25, No.2, pp. 383-417.
- FARLOW, A. (2004). "The UK Housing Market: Bubbles and Buyers". Paper presented at the Credit Suisse First Boston Housing Market Conference.
- GILMORE, R. (1993). "Catastrophe theory for scientists and engineers". New York, NY. Dover Publications.
- GRASMAN, R.P.P.P, VAN DER MAAS, H. and WAGENMAKERS, E.J. (2009). "Fitting the Cusp Catastrophe in R: A cusp Package Primer". *Journal of Statistical Software*, Vol. 32, No. 8.
- HARTELMAN, P.A.I. (1997). "Stochastic Catastrophe Theory". Ph.D. thesis. University of Amsterdam. Amsterdam, the Netherlands.
- HO, T. and SAUNDERS, A. (1980). "A catastrophe model of bank failure". *The Journal of Finance*, Vol. 35, pp. 1189-1207.
- HOLT, R.T., JOB, B.L. and MARKUS, L. (1978). "Catastrophe Theory and the Study of War". *Journal of Conflict Resolution*, Vol. 22, No. 2, pp. 171-208.

- HOMMES, C. (2006). "Heterogeneous Agent Models in Economics and Finance". *Handbook of Computational Economics*, Vol. 2, pp. 1109-1186.
- KALRA, S., MIHALJEK, D. and DUENWALD, C. (2000). "Property Prices and Speculative Bubbles: Evidence from Hong Kong SAR". Working Paper of the International Monetary Fund.
- KOUWENBERG, R. and ZWINKELS, R.C.J. (2015): "Endogenous price bubbles in a multi-agent system of the housing market". *PLoS ONE*, Vol. 10, No. 6, p.e0129070.
- LUX, T. (1995). "Herd behavior, bubbles and crashes". *Economic Journal*, Vol. 105, pp. 881-896.
- NAVIN, F.P.D. (1986). "Traffic congestion catastrophes". *Transportation Planning and Technology*. Vol. 11, No. 1, pp. 19-25.
- ROSSER, B. (2007). "The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater?". *Journal of Economic Dynamics and Control*, Vol. 31, pp. 3255-3280.
- SALZMAN, D.A. and ZWINKELS, R.C.J. (2013). "Behavioural Real Estate". No 13-088/IV/DSF58. *Tinbergen Institute Discussion Papers*. Tinbergen Institute.
- SAUNDERS, P. (1980). "An Introduction to Catastrophe Theory". Cambridge. Cambridge University Press.
- SCHWARZ, G. (1978). "Estimating the Dimension of a Model". *Annals of Statistics*, Vol. 6, pp. 461-464.
- SHILLER, R.J. (2005). "Irrational Exuberance: (2nd Edition)". Princeton University Press.
- SHILLER, R.J. (2007): "Understanding recent trends in house prices and home ownership". Working Paper 13553. National Bureau of Economic Research.
- SOMMERVOLL, D.E., BORGERSEN, T.A. and WENNEMO, T. (2010). "Endogenous housing market cycles". *Journal of Banking and Finance*, Vol. 34, No. 3, pp. 557- 567.

STEWART, I. and PEREGOY, P.L. (1983). "Catastrophe theory modeling in psychology". *Psychological Bulletin*, Vol. 94, pp. 336-362.

SUSSMAN, H.J. and ZAHLER, R. (1977). "Claims and accomplishments of applied catastrophe theory". *Nature*, Vol. 269, pp. 759-763.

SUSSMAN, H.J. and ZAHLER, R. (1978a). "Catastrophe theory as applied to the social and biological sciences". *Synthese*, Vol. 37, pp. 117-216.

SUSSMAN, H.J. and ZAHLER, R. (1978b). "A critique of applied catastrophe theory in applied behavioral sciences". *Behavioral Science*, Vol. 23, pp. 383-389.

TAMAKI, T., TORII, T. and MAEDA, K. (2003). "Stability analysis of black holes via a catastrophe theory and black hole thermodynamics in generalized theories of gravity". *Physical Review D*, Vol. 68, No. 2, 024028.

THOM, R. (1972). "Stabilité Structurelle et Morphogénèse: Essai d'une Théorie Générale des Modeles". Benjamin. New York.

THOM, R. and FOWLER, D.H. (1975). "Structural Stability and Morphogenesis: An Outline of a General Theory of Models". W. A. Benjamin, Massachusetts.

UNITED STATES FINANCIAL CRISIS INQUIRY COMMISSION. (2011). "The Financial Crisis Inquiry Report: Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States". Washington DC.

VAN DER MAAS, H., KOLSTEIN, R. and VAN DER PLIGT, J. (2003). "Sudden transitions in attitudes". *Sociological Methods and Research*, Vol. 32, pp. 125-152.

VAN HARTEN, D. (2000). "Variable nodding in *Cyprideis torosa* (Ostracoda, Crustacea): An overview, experimental results and a model from catastrophe theory". *Hydrobiologica*, Vol. 419, pp. 131-139.

VARIAN, H.R. (1979). "Catastrophe theory and the business cycle". *Economic Inquiry*, Vol. 17, pp. 14-28.

WAGENMAKERS, E.J., VAN DER MAAS, H. and MOLENAAR, P. (2005). "Fitting the cusp catastrophe model". *Encyclopedia of Statistics in Behavioral Science*, Vol. 1, pp. 234-239.

WALES, D.J. (2001). "A microscopic basis for the global appearance of energy landscapes". *Science*, Vol. 293, pp. 2067-2070.

WESSELBAUM, D. (2017). "Catastrophe theory and the financial crisis". *Scottish Journal of Political Economy*, Vol. 64, No. 4, pp. 376-391.

ZEEMAN, E.C. (1974). "On the unstable behaviour of stock exchanges". *Journal of Mathematical Economics*, Vol. 1, No. 1, pp. 39-49.