**CHARLES UNIVERSITY**

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# Numerical fiscal rules and fiscal institutions in an economy with a dynamic common pool problem

Bachelor's thesis

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Study program: Ekonomická teorie

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Prague, May 9, 2019

Lukas Janasek

Abstract

The focus of this bachelor's thesis is fiscal policy of a fragmented government that represents symmetric socio-economic groups. For the analysis of fiscal policy, I develop the model of a dynamic common pool. In the model, the fiscal choices of interest are 1) a level of tax revenue, 2) a level of public productive spending and 3) an intertemporal choice of a level of group's consumption spending. For each of the fiscal decisions, I describe a distortion associated with the fiscal choice stemming from the decentralized decision making. Next, I examine the impact of a deficit ceiling and fiscal institutions that centralize separate fiscal choices of groups on the three distortions. Due to the symmetry of groups, the analysis abstracts from the efficiency-equity trade-off. Among the key results is that three fiscal frameworks can attain the socially desirable fiscal policy: 1) centralization of the productive spending and the tax revenue combined with a deficit ceiling, 2) centralization of the consumption spending, 3) centralization of a budget size and the productive spending.

**JEL Classification** H30, H50, H60, H62, E61, E62
**Keywords** Fiscal policy, Common pool problem, Fiscal institutions, Numerical fiscal rules

**Title** Numerical fiscal rules and fiscal institutions in an economy with a dynamic common pool problem

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# Abstrakt

Tato bakalářská práce se zabývá fiskální politikou vlády tvořenou symetrickými socio-ekonomickými skupinami. Pro popis fiskální politiky je vytvořen model obsahující rozhodování ohledně 1) výše daňových příjmů vlády, 2) veřejných investic a 3) časového rozložení skupinových spotřebních výdajů. Pro každé z těchto tří rozhodnutí je popsáno zkreslení pramenící z decentralizovaného rozhodování vlády. V práci je dále zkoumáno, jaký vliv má na fiskální politku a tři dané distorze dluhový strop a fiskální instituce, které centralizují jednotlivá rozhodnutí. Z důvodu symetrie skupin analýza neobsahuje kompromis mezi efektivitou a rovností. Jedním z hlavních výsledků této práce je, že sociálního optima lze dosáhnout 1) pomocí centralizace veřejných investic a daňových příjmů kombinovaných s optimálním deficitním stropem, 2) pomocí centralizace rozhodnutí ohledně skupinových zájmových výdajů, 3) pomocí centralizace výše vládního rozpočtu kombinovaného s centralizací veřejných investic.



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| **Klasifikace JEL** | H30, H50, H60, H62, E61, E62 |
| **Klíčová slova** | Fiskální politika, Common pool problem, Fiskální instituce, Numerická fiskální pravidla |


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| **Proposed topic** | Numerical fiscal rules and fiscal institutions in an economy with a dynamic common pool problem |

**Research question and motivation** There is a consensus among economists that the government should tighten public spending in good times and use accumulated reserves in bad times. However, counter-cyclical fiscal policy is rarely observable in practice. Also, there is a recent trend to constrain fiscal policy-making through numerical fiscal rules or fiscal institutions. But is it possible to mitigate the debt bias and avoid cyclical fiscal policy in the presence of fragmented decision-making (a common pool problem)? Which fiscal rules have to be used to do so? Is it better to use fiscal numerical rules or financial institutions to mitigate the debt bias and make fiscal policy more counter-cyclical?

**Methodology** In seminal political economy models of fiscal policy (e.g., Persson and Tabellini, 2002), counter-cyclicality is attributed to a dynamic common pool problem. In the thesis, I will analyze two remedies to the deficit bias and the counter-cyclicality: fiscal numerical rules and fiscal institutions, assuming that the deficits are the result of a dynamic common pool problem.

**Expected Contribution** A game-theoretic analysis of fiscal numerical rules and financial institutions using the canonical model of a dynamic common pool in Persson and Tabellini (2002) and a model of fiscal rules as in Krogstrup and Wyplosz (2010) and the closely related literature.

## Outline

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Author                                        Supervisor

# Chapter 1

# Introduction

"In a perfect world of fully informed policymakers solely motivated by social welfare maximization, complete discretion enables them to optimally respond to changing circumstances at any time" (Beetsma *et al.* 2018, p. 3). However, the rise in public debt in OECD countries since the 1970s has shown us that it has not always to be the case for fiscal policy. It started a debate leading to theoretical modelling and empirical research on constraining fiscal policy through *numerical fiscal rules* and *fiscal institutions* (e.g. Poterba & von Hagen 1999). In Europe, the last reminder of importance of fiscal discipline was the European sovereign debt crisis and the topic of fiscal discipline received increased attention again in recent years. As a response to the observed imperfections, there has been a growing trend to constrain fiscal policy-making through numerical fiscal rules and fiscal institutions. According to the IMF, there are 39 countries in the world which implemented independent fiscal institution. Two two-thirds of them were established after 2007 and 10 of them were established in Europe after 2013 (Beetsma *et al.* 2018).

Broadly speaking, the aim of fiscal rules and fiscal institutions is to ensure health and sustainability of public finances and to mitigate a so-called *deficit bias*. The deficit bias tries to explain why elected governments may fail to deliver socially desirable fiscal policy and run excessive deficits. It may come from many sources such as common pool problems, informational problems, impatience, electoral competition, time inconsistency, exploitation of future generations and others (see Calmfors & Wren-Lewis 2011). In this thesis, I will focus solely on a fragmented fiscal decision making also referred to a *common pool problem*. Generally, the common pool problem can be described as interest groups competing for financial favors from the government disregarding the favor connected cost for others. As a consequence, each group funds eventually the favors of other groups resulting in a sub-optimal policy (Wyplosz 2012). In that case, a co-ordination failure emerges in a form of excessive deficits and overspending.

In this thesis, the analysis of fiscal rules and institution is based on a *model of a dynamic common pool.* The model is a generalization of two models of fiscal common pools, namely PT model and KW model. The PT model was used by Persson & Tabellini (2000) and describes a fragmented fiscal decision making, where the fiscal choice of interest is a level of taxes and group spending across two periods. The KW model was used by Krogstrup & Wyplosz (2009) and describes a fragmented fiscal decision making where the fiscal choice of interest is a level of public productive spending and group spending across two periods. In both models, a deficit occurs as a result of fragmented decision making. Here, the generalized model of a dynamic common pool involves *three fiscal decisions of interest*:

1. **Decision on a level of tax revenue**
2. **Decision on a level of productive spending (public investments)**
3. **Intertemporal choice of group consumption spending across two periods**

In the model, we observe *three distortions* connected to the fiscal choices stemming from the common pool problem:

1. **Excessive taxation**
2. **Suboptimal productive spending**
3. **Intertemporal distortion (excessive spending in early period)**

The objective of this thesis is to describe how the outcome of fiscal policy and the aforementioned distortions are influenced by an *institution*, that mediates a centralization of decisions on fiscal variables (tax revenue, productive spending, group consumption spending and budget size), and a deficit ceiling *rule*. (By the centralization we mean that the groups decide on a common level of fiscal variables for each group.) For better understanding, the distortions are firstly analyzed without the presence of other fiscal choices.

Next, in the general model, the distortions are characterized by two approaches: 1) We determine the socially optimal fiscal policy as a policy that maximizes social utility given the constraints of a representative group. Then we calculate the Nash equilibrium outcome of fiscal policy that maximizes utility of a representative group given the constraints of a representative group in different fiscal frameworks. Next, we compare the outcomes. 2) We determine the socially optimal costs of fiscal variables in resource constraints of the representative group. Next, we compare the relative cost differences of fiscal variables in resource constraint of a representative group to its socially optimal counterpart. This approach can be perceived as a parallel to a consumer choice problem, where the fiscal variables represent goods with a certain marginal rate of transformation. This alternative perspective allows us to identify frictions in fiscal policy-making and to decompose the model to separate optimization problems in both periods. From the optimization problems, we can observe the origin of each inefficiency, that occurs in the fiscal policy-making, and the dynamic aspect of our model.

The structure of my bachelor thesis is as follows. In Chapter 2, I review possible remedies to the common pool discussed in the literature. In Chapter 3, I develop a model of a dynamic common pool based on the PT model and the KW model, setting the structure of fiscal policy decision making and preferences over fiscal policy. Next, I develop simple models demonstrating each of the three distortions. In Chapter 4, I conduct an analysis of the full model. Firstly, I determine the socially optimal fiscal policy using the two approaches. Secondly, I determine the outcome of fiscal policy and describe possible distortions that occur in the fiscal framework using the consumer choice parallel. In Chapter 5, I briefly discuss some topics related to the results of the analysis.

# Chapter 2

# Literature review of remedies to a fiscal common pool

Generally, the literature identifies three main remedies to the common pool problem in fiscal governance. These are 1) binding numerical rules, 2) delegation to an agent and 3) a better budgetary process. The broad function of the remedies is to offset the incentives generated by the common pool problem (Wyplosz 2012), eliminate connected externalities among groups or to directly suppress the negative outcome of fiscal policy. Simple numerical rules are the *first remedy.* They are usually in a form of upper limits on the budget balance, debt, spending or lower limits on tax revenues or a combination of those (Wyplosz 2011). They ensure fiscal discipline directly without any changes to incentives of interest groups. The main concern of fiscal rules is their compliance, which is dependent on the design of rules. For instance on simplicity, flexibility and enforceability of the rule (Debrun *et al.* 2018). However, if the fiscal indiscipline stems purely from the common pool problem then there is less worry about the compliance with rules. The government has no incentives to breach the rules since the fiscal indiscipline is due to a lack of coordination, which the rules partially provide. (Portes & Wren-Lewis 2014)

The *second remedy* is to delegate a part of fiscal decision to an independent "agent who is not exposed to pressure by interest groups" (Wyplosz 2012, p. 32). This can be either a finance minister or an independent fiscal council. Fiscal policy councils are a particular form of fiscal institutions. Their members are non-elected and independent on the government and their position can be parallel to central bank monetary policy committees. (Krogstrup & Wyplosz 2009) In the literature, we can find two remedies to the common pool problem that the fiscal council can provide. The first remedy is setting an optimal deficit balance and the second is centralization of fiscal decisions. The role of the fiscal council might also be to decide on other fiscal variables to achieve social optimum and not only on a deficit level. However, variables such as size and structure of public spending and taxation involve a redistribution of wealth and income and cannot be determined by a non-elected official (Wyplosz 2005). In contrast, C. Wyplosz argues that budget deficit itself has only limited effect on intra-temporal income and wealth redistribution and claims that it redistributes only across generations, which are not a part of the democratic control (Wyplosz 2005). C. Wyplosz concludes that the deficit ceiling is the only variable that can be set by a non-elected authority (fiscal council) to avoid a clash in principles of democracy.

The *third remedy* to the common pool problem is to enhance budgetary process in order to eliminate externalities among groups, which can be seen as centralization of budgetary process. The centralization of budgetary process can be achieved either by a fiscal institution, whose role might be to ensure the coordination of groups in order to internalize externalities among themselves, or some form of contracts between interest group (see Calmfors & Wren-Lewis 2011; Poterba & von Hagen 1999). The contracts are usually in form of pre-established budgetary targets and rules (Hallerberg *et al.* 2004) such as ex-ante unbreakable ceilings for each ministry budget or ex-ante total spending or budget balance (Wyplosz 2011).

The literature also suggests that the remedies to common pool problem should be combined. For instance, Krogstrup & Wyplosz (2009) have shown that an optimal ex-ante set deficit reduces productive spending to a suboptimal level and thus cannot achieve socially desirable fiscal outcome alone. The social optimum is achieved when the ex-ante set deficit ceiling is introduced together with a contract between groups on the productive spending.

# Chapter 3

# Model of a dynamic common pool

The aim of this section is to develop a model of a fiscal decision making where the source of debt bias is a decentralized (fragmented) fiscal decision making. The model is constructed from components of two models in the literature: PT model and KW model, and the resulting model of a dynamic common pool can be perceived as a generalized version of these two models. In Section 3.2, it is demonstrated on simplified models, that represents building blocks of fiscal decisions in the model of a dynamic common pool (and naturally in the models used in the literature), what inefficiencies may generally occur when the decentralized fiscal decision making takes place.

## 3.1 Key elements

For the model of a dynamic common pool assume a two-period economy consisting only of identical individuals. The individuals are divided into $n$ groups of equal size and each group $j$, where $j = 1, ..., n$, is represented in the government with the same proportion. The government is responsible for fiscal policy. The interpretation of the groups can be as in the PT model, where the groups are political parties (Democrats and Republicans), or as in the KW model, where the groups are perceived generally as interest groups represented by its decision maker, who can be for instance a spending minister.

### 3.1.1 Fiscal decision making

The government conducts fiscal policy in each period. It involves decision about a tax rate $\tau$, a structure and a level of non-productive consumption spending $c$ and productive spending $X$. The interpretation of the tax rate $\tau$ is straightforward as an income tax and it determines a tax revenue $T$. The relation between the tax rate $\tau$ and the tax revenue $T$ is described below in Subsection 3.1.2. The aggregate consumption spending $c$ is a sum of consumption spending of each group: $c = \sum_{i=1}^{n} c_i$. The level of $c_j$ is determined solely by a group $j$ (or a decision maker representing the group $j$) and it affects utility only of individuals belonging to the group $j$. Hence, we can interpret $c_j$ as group or collective public spending. In the KW model, they are interpreted as "pork-barrel" style spending. In the PT model, they are called simply as "favored good" of the groups. In this thesis, we will call them "consumption spending". The productive spending occurs only in the KW model. Their interpretation is either as investments in public infrastructure, human capital investments or costly reforms that raise future taxable income. Generally, they represent any form of public investments. Analogically to the consumption spending, the productive spending $X$ is given as $X = \sum_{i=1}^{n} X_i$, where $X_j$ is a contribution of group $j$ to the common productive spending $X$. This is a small change compared to the KW model, where the productive spending is set centrally, but this approach is useful since it simplifies solving and understanding the model without additional assumptions on the timing of fiscal decisions. In the general model, the productive spending does not affect utility functions of interest groups directly but only throughout an increase in disposable revenue of the government in the second period. The return to productive spending $X$ is given similarly as in the KW model by an increasing and concave function $\theta(X)$. The marginal product of productive spending expenditures is decreasing.

The timing of the fiscal decision making is as it follows: In the first period, the interest group $j$ decides simultaneously with other groups on size of its budget $g_j^1$, which is decomposed to the productive spending $X_j$ and the consumption spending $c_j^1$. Hence, we have that $g_j^1 = c_j^1 + X_j$. For simplicity, the initial tax revenue $T^1$ is not a subject of the fiscal policy and is fixed. (Adding the decision about the tax revenue in the first period would complicate a solution of the model without adding value to the analysis.) For instance, we can assume that the tax system is rigid and changing the tax rate is time demanding. Next, assume the government can borrow any amount by issuing government bonds and finance its deficit $B$, which is residually by the budget constraint of groups. Similarly as in both models, it is assumed that the interest rate on the government bond is normalized to zero.

In the second period, the returns to productive spending $X = \sum_{i=1}^{n} X_i$ are $\theta(X)$ and they all become a part of the public budget. The interest group $j$

decides on size of its budget $g_j^2$ without possibility to invest (since we assume only a two-period economy), hence $g_j^2 = c_j^2$. The tax revenue $T^2$ is residually set to fit the intertemporal budget constraint such that the deficit $B$ from the first period is fully repaid. In other words, we eliminate the possibility of the government to default.

In the KW model, it is assumed that the government is hit by exogenous revenue cycle $\gamma$. The revenue cycle is anticipated by all groups and does not affect intertemporal budget constraint.

In the first period, the government faces a constraint:

$$\sum_{i=1}^{n} g_i^1 = T^1 + B + \gamma$$

and in the second period the constraint is given as

$$\sum_{i=1}^{n} g_i^2 = T^2 + \theta(X) - B - \gamma$$

For the intertemporal budget constraint of government we have:

$$\sum_{i=1}^{n} g_i^1 + \sum_{i=1}^{n} g_i^2 = T^1 + T^2 + \theta(X)$$

### 3.1.2 Individual preferences over fiscal policy

The preferences of individuals about fiscal policy can be separated to an effect of the fiscal policy on *private consumption* and a direct effect on *utility from the consumption spending* of each group. Firstly, we can determine the effect of fiscal policy on private consumption and then set preferences about the consumption spending.

In both periods, a representative consumer decides on a private consumption $m$ and a labor supply $l$. Her utility function is given as

$$u = m + V(x)$$

where $x$ denotes leisure and it holds that $x = 1 - l$. There is a tax $\tau$ levied by the government on output of labor $l$. The level of output is for simplicity $l$. Thus, for the consumption $m$ we have that $m = (1 - \tau)l$. The representative consumer maximizes her utility $u$, where $u = (1 - \tau)l + V(1 - l)$. The outcome of the optimization is an optimal labor supply function of tax rate $L(\tau)$ and the private utility $u$ is then a function of the tax rate $\tau$ similarly as in the PT model. We can then set an indirect utility from the tax rate $\tau$ throughout the collected tax revenue $T$ given as

$$W(T) = W(\tau L(\tau)) \equiv u(L(\tau))$$

. The function $W$ is decreasing and strictly concave in the tax revenue $T$ and represents an increasing tax distortion on economic output in the model. It influences the private utility $u$ and, as a result, **the function $W$ captures utility from private economic outcomes.**

This approach towards the effect of the fiscal policy, namely of the tax revenue $T$, is taken from the PT model with small adjustments. In the PT model, the government collects tax revenue $\tau$ only in the second period and the consumer is allowed to work only in the second period whereas the consumption choice is made intertemporally. As a consequence, the tax revenue is present only in the second period. This puts additional asymmetry on government's revenue across periods and the preferences of individuals about the tax revenue in the first period are not defined.

Similarly as in the PT model, the **direct effect of fiscal policy** on the utility of group $j$ **is captured by** an increasing and concave **function** $h(c_j^t)$, for $t = 1, 2$. We assume that the group $j$ internalizes utility only from its own consumption spending $c_j^t$ and disregards the consumption spending of other groups.

An individual's utility in each period is given as a simple sum of the private utility and the utility from consumption spending:

$$\omega_j^t = h(c_j^t) + W(T^t)$$

Individuals do not discount future and their intertemporal utility is given as a simple sum of utilities in each period:

$$U_j = \omega_j^1 + \omega_j^2$$

### 3.1.3 A brief reference to PT model and KW model

The model of a dynamic common pool is constructed such that the PT model is a special case of the model of a dynamic common pool where the number of groups is generalized from 2 to $n$ and where the level of productive spending is fixed. There is only a minor difference in the consumer choice optimization.

In the KW model, the form of utility of individuals from the fiscal policy differs. It is given as

$$U_j = log(y_j^1 + y) + log(y_j^2 + y)$$

where $y_j^t$ represents net transfers of the group $j$ defined as a difference between the consumption spending $c_j^t$ and the tax expenditures $\frac{T}{n}$ incurred by the group $j$. The variable $y$ represents a minimal acceptable net transfer for the group $j$. We can rewrite the utility function as

$$U_j = log(c_j^1 - \frac{T}{n}^1 + y) + log(c_j^2 - \frac{T}{n} + y)$$

The issue (and on the other hand benefit) of this form of utility function is that it eliminates the possibility to examine the increasing distortion of taxation since the utility function is dependent only on the net transfers of individuals, which is for out purpose undesirable. However, $log$ is a concave function. Consequently, the KW model and the general model coincide when the tax revenue is fixed (it is not a tool of fiscal policy and the level of tax revenue is exogenously pre-determined) in both periods. Then we have that $h(c_j^t) = log(c_j^t + constant)$.

## 3.2 3 sources of suboptimal policy

As a consequence of the fragmented fiscal decision making, three suboptimal allocations of resources occur in the model of a dynamic common pool. These suboptimal allocations are:

1. **Excessive taxation:** too high consumption spending in the second resulting in too high tax distortion

2. **Suboptimal productive spending:** too low productive spending not maximizing its potential return

3. **Intertemporal distortion:** excessive spending in the first period resulting in an uneven distribution of the consumption spending across periods

which are discussed in Subsection 3.2.1, Subsection 3.2.2 and Subsection 3.2.3 respectively.

Before solving the general model of a dynamic common it is useful to construct simplified models capturing such inefficiencies in isolation from the other inefficiencies. In the simplified models, the form of distortion may differ compared to the distortion in the overall model of a dynamic common pool, but they capture the substantial fiscal decisions that the interest groups make.

### 3.2.1 Excessive taxation

The first inefficiency occurs when the property rights to the tax revenue are not well defined and the interest groups have incentives to overspend. To illustrate, assume a one-period economy without any productive spending in which each interest group decides about its (group) consumption spending $c_j$. The amount of tax revenue $T$ (where $T = L(\tau)\tau$) is then residually determined to fit the budget constraint:

$$\sum_{i=1}^{n} c_i = T$$

The interest group $j$ maximizes its utility given as

$$W(T) + h(c_j)$$

with respect to the consumption spending $c_j$ and subject to the budget constraint. From the first-order condition (hereafter FOC w.r.t. $c_j$) we obtain that:

$$\frac{\partial W}{\partial T}(T^*) + \frac{\partial h}{\partial c_j}(c_j^*) = 0 \text{ for } j = 1, ..., n$$

This equation refers to a Nash equilibrium outcome.

In contrast, to determine the socially optimal level of consumption spending we maximize social utility given as

$$nW(T) + \sum_{i=1}^{n} h(c_i)$$

and from FOC w.r.t. $c_j$ we obtain:

$$n\frac{\partial W}{\partial T}(T^{SO}) + \frac{\partial h}{\partial c_j}(c_j^{SO}) = 0 \; hbox for \; j = 1, ..., n$$

From the concavity of functions $W$ and $h$ and a comparison of the first-order conditions we have that the Nash outcome involves overspending: $c_j^* > c_j^{SO}$, and consequently, too high tax revenue: $T^* > T^{SO}$. This is because each interest group internalizes only $\frac{1}{n}$ of costs of the taxation from an increase in its consumption spending $c_j$.

### Consumer choice parallel

An alternative approach to solving the model and explaining the inefficiency is to look at the consumption spending $c_j$ and the tax revenue $T$ as two goods. The model can be then perceived as parallel to a consumer choice problem, where the costs (or prices) of consumption spending and tax revenue are given from the resource constraint that each group faces. The tax revenue $T$ represents, in this context, a harmful good with a negative price. It is because the higher taxation reduces private utility as is described in the Section 3.1 and increases disposable resources for the groups. The inefficient decentralized outcome is then a result of the different relative cost of the consumption $c_j$ and the tax revenue $T$ that the interest group $j$ faces compared to its socially optimal counterpart. Under the decentralized outcome the relative cost of consumption spending $c_j$ to the tax revenue $T$ is $MRT^*_{c_j,T} = (-1)$ (where $MRT$ denotes a marginal rate of transformation) since the resource constraint is given as

$$c_j - T \leq c_{-j}$$

In the optimum we have that $MRT^*_{c_j,T} = MRS^*_{c_j,T}$ (where MRS denotes a marginal rate of substitution).

In contrast, to determine the socially optimal level of consumption spending we need to take into account the negative externality of consumption spending imposed on the other groups throughout the higher tax revenue $T$. The resource constraint eliminating such negative externality is

$$nc_j - T \leq 0 \; hboxfor \; j = 1, ..., n$$

where $nc_j$, $nc_j = c$, is the aggregate consumption spending and we have that $MRT^{SO}_{c_j,T} = (-n)$. Since it holds that $MRS^*_{c_j,T} = MRS^{SO}_{c_j,T}$, the overspending under the decentralized fiscal policy stems solely from a relative cost (price) distortion that the interest group $j$ faces. This form of fiscal decision and the connected distortion is also present in the PT model.

#### Brief summary

We observe a problem of unrecognized *negative externality.* The externality generates too *high consumption spending* resulting in too *high taxation.*

### 3.2.2 Suboptimal productive spending

The second inefficiency occurs when the interest groups decide about a level of productive spending they will provide to the common productive spending from their budget. The issue is that the contribution of productive spending reduces an amount of their consumption spending without compensating for the resources provided to the other groups.

To illustrate, assume a one-period model, where the collected tax revenue $T$ is exogenously given and the function $W(T)$ is constant. Each interest group receives revenue $\frac{T}{n}$ and chooses how much of the productive spending $X_j$ to provide to the common productive spending $X$. The collected returns to productive spending $\theta(X)$ are then realized (became a part of the public budget) and equally split between the interest groups as increased resources for the consumption spending. We have that the total spending of the group $j$ are given as $g_j = \frac{T+\theta(X)}{n}$. For the interest group $j$ the consumption spending is given as

$$c_j = g_j - X_j = \frac{T + \theta(X)}{n} - X_j$$

From the FOC w.r.t. $X_j$, we can determine the Nash equilibrium outcome of the level of productive spending:

$$\theta'(X^*) = n$$

The socially optimal level of productive spending, however, maximizes the overall consumption and it holds that:

$$\sum_{i=1}^{n} c_i = T - \sum_{i=1}^{n} X_i + \theta(X)$$

From FOC w.r.t. $X_j$ we obtain:

$$\theta'(X^{SO}) = 1$$

By comparing both first-order conditions, we have that:

$$X^* < X^{SO}$$

The interest groups have incentives to contribute less then is socially desirable since they realize a full cost of an increase in their contribution $X_j$ but only $\frac{1}{n}$ of their benefit.

#### Alternative approach

As in the previous model, we can look at the model as a consumer choice problem with two goods $c_j$ and $X_j$. However, in this case, the reason for the suboptimal productive spending outcome is not a different marginal rate of transformation between the productive spending $X_j$ and the consumption spending $c_j$ (which is irrelevant for the outcome) but a distorted return to productive spending $X_j$. Under the decentralized fiscal policy the interest group $j$ faces a revenue maximization problem:

$$max\ R_j \quad \text{where} \quad R_j = \theta(X_j + X_{-j}) - nX_j$$

For the revenue maximization problem capturing the positive externality of productive spending on the overall revenue we have that $X = nX_j$ and thus we solve:

$$max\ R \quad \text{where} \quad R = \theta(nX_j) - nX_j$$

Notice that we attain the same outcome when we let the interest groups set their consumption spending first. The remaining resources are then used as the common productive spending $X$ as it is done in the KW model. The interest groups then face the same optimization problem and these two approaches are interchangeable.

#### Brief summary

We observe a standard problem of an *unrecognized positive externality.* The externality generates too *low productive spending.*

### 3.2.3 Intertemporal distortion

The third inefficiency occurs when several interest groups decide about their consumption spending sequentially in multiple periods. In the following simplified model, assume a two-period economy without a possibility to increase the revenue throughout the productive spending. In each period there is an exogenously given level of tax revenue $T$ and each interest group decides on its consumption spending $c_j^1$ in the first period and consequently about its consumption spending $c_j^2$ in the second period. Since the government can freely borrow any amount $B$ (which is fully repaid in the second period), the intertemporal budget constraint is given as

$$\sum_{i=1}^{n} c_i^1 + \sum_{i=1}^{n} c_i^2 = 2T$$

To determine the Nash equilibrium for the consumption spending $c_j^{2*}$ and $c_j^{1*}$ we need to solve the model backward. In the second period, the consumption spending is given residually as

$$c_j^{2*} = \frac{2T - \sum_{i=1}^{n} c_i^1}{n}$$

In the first period, each interest group takes into account the equal division of resources in the second period and maximizes its utility:

$$h(c_j^1) + h\left(\frac{2T - \sum_{i=1}^{n} c_i^1}{n}\right)$$

From FOC w.r.t. $c_j^1$ we obtain

$$\frac{\partial h}{\partial c_j^1}(c_j^{1*}) + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2*}) = 0 \; hboxfor \; j = 1, ..., n$$

From the concavity of function $h$ we have that $c_j^{1*} > c_j^{2*}$.

For the socially optimal outcome it holds that

$$\frac{\partial h}{\partial c_j^1}(c_j^{1,SO}) + \frac{\partial h}{\partial c_j^2}(c_j^{2,SO}) = 0 \; hboxfor \; j = 1, ..., n$$

and thus $c_j^{1,SO} = c_j^{2,SO}$. In the Nash outcome compared to the social optimum, we observe an overspending in the first period, clearly $c_j^{1,*} > c_j^{1,SO}$, resulting in borrowing the positive amount $B^*$, where $B^* > B^{SO} = 0$. Each interest group has incentives to spend more in the first period since increasing the consumption spending $c_j^1$ by one unit causes a reduction in $c_j^2$ only by $\frac{1}{n}$ of the unit.

### Consumer choice parallel

Similarly as in the two previous sections, the model can be interpreted as a parallel to a consumer choice problem with two goods $c_j^1$ and $c_j^2$. As in Subsection 3.2.1, the reason for the inefficient outcome of the fragmented fiscal policy is the different relative cost of consumption spending in the first period compared to the consumption spending in the second period that the interest group $j$ faces in contrast to the socially optimal relative cost. In the second period, each interest group realizes that the resources are equally split and perceives its revenue constraint due to the sequential decision making as

$$c_j^1 + nc_j^2 \leq 2T - c_{-j}^1$$

where we have that $MRT^*_{c_j^2, c_j^1} = n$. In the optimum we have that

$$MRT^*_{c_j^2, c_j^1} = MRS^*_{c_j^2, c_j^1}$$

In contrast, for the socially optimal resource constraint compensating for the distorted costs we have that $c^1 = nc_j^1$ and $c^2 = nc_j^2$ and thus:

$$nc_i^1 + nc_i^2 < T$$

The socially optimal marginal rate of transformation between consumption spending is 1. Since the marginal rate of substitution is unchanged, $MRS^*_{c_j^2, c_j^1} = MRS^{SO}_{c_j^2, c_j^1}$, the reason for the excessive spending in the first period is only the different marginal rate of transformation.

### Elasticity of substitution

The degree of uneven distribution of the consumption spending and the size of deficit $B^*$ depends on elasticity of substitution between $c_j^1$ and $c_j^2$. To illustrate, we can assume a constant elasticity utility function of consumption spending:

$$u(c_j^1, c_j^2) = ((c_j^1)^\rho + (c_j^2)^\rho)^{\frac{1}{\rho}}$$

for $\rho \leq 1$. Proceeding the same optimization process of the interest group $j$ we obtain the following result:

$$c_j^1 = \frac{2T}{n} \frac{1}{1 + n^{\frac{1}{\rho - 1}}}$$

For perfect substitutes we have that $\rho = 1$. Hence,

$$c_j^1 = \frac{2T}{n}$$

and the whole tax revenue from both periods is spent in the first period. For perfect complements we have that $\rho$ approaches $-\infty$. Therefore, we have that

$$c_j^1 = \frac{T}{n}$$

The consumption spending is equalized between both periods and the intertemporal distortion is completely eliminated. In our model of a dynamic common pool, it is reasonable to assume that the function $h$ satisfies that $\rho \in (-\infty, 1)$, i.e. it is strictly concave. The same analysis can be performed in Subsection 3.2.1, however, for the model of a dynamic common pool, it is not necessary.

#### Brief summary

We observe *uneven distribution of consumption spending across periods*. The government spend the resources *soon*.

# Chapter 4

# Analysis

## 4.1 Social optimum

To have a normative benchmark for our analysis it is useful to determine the socially optimal fiscal policy. As it has been shown in Section 3.2, the socially optimal fiscal policy can be described by two possible approaches. The first approach is to maximize social utility subject to the interest group $j$'s resource constraints. The second approach is to look at the model as a parallel to a consumer choice problem, where the fiscal variables are perceived as a good. The we can determine socially optimal costs of fiscal variables in the resource constraints of the representative group $j$ and eliminate all relative cost (price) distortions and externalities stemming from the decentralization and maximize interest group $j$'s utility. Both procedures result in the same set of conditions and the same outcome of fiscal policy. It is then sufficient to solve the model by social utility maximization and, for additional understanding, determine the socially optimal form of resource constraint of group $j$ with a corresponding marginal rate of transformation and substitution of the interested variables.

### Social utility maximization

Following a long tradition in welfare economics, assume Utilitarian form of social utility. Hence, the overall social utility from both periods is given as a simple sum:

$$U = \sum_{i=1}^{n} U_i = nW(T^1) + nW(T^2) + \sum_{i=1}^{n} h(c_i^1) + \sum_{i=1}^{n} h(c_i^2)$$
$$= nW(T^1) + nW(T^2) + \sum_{i=1}^{n} h(g_i^1 - X_i) + \sum_{i=1}^{n} h(c_i^2)$$

where $nW(T^1)$ is constant and is not a subject of optimization.

In the second period, the tax revenue $T^2$ is residually given from the intertemporal budget constraint as

$$T^2 = \sum_{i=1}^{n} g_i^1 + \sum_{i=1}^{n} g_i^2 - T^1 - \theta(X)$$

From maximization of the social utility $U$ with respect to $g_j^1$, $g_j^2$ and $X_j$ we obtain the following conditions: FOC w.r.t. $g_j^1$:

$$n\frac{\partial W}{\partial T^2}(T^{2,SO}) + \frac{\partial h}{\partial c_j^1}(c_j^{1\,SO}) = 0 \text{ for } j = 1, ..., n \tag{4.1}$$

FOC w.r.t. $g_j^2$:

$$n\frac{\partial W}{\partial T^2}(T^{2,SO}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,SO}) = 0 \text{ for } j = 1, ..., n \tag{4.2}$$

FOC w.r.t. $X_j$:

$$n\frac{\partial W}{\partial T^2}(T^{2,SO})(-\theta'(X^{SO})) - \frac{\partial h}{\partial c_j^1}(c_j^{1\,SO}) = 0 \text{ for } j = 1, ..., n \tag{4.3}$$

From the equations (4.1) and (4.3) we have a condition for a level of productive spending:

$$\theta'(X^{SO}) = 1 \tag{4.4}$$

where the superscript $SO$ denotes a socially optimal level of a variable. From the equation (4.4) we can see that the productive spending is at the highest potential return. We have:

$$X^{SO} \in \arg\max \; \Big\{\theta(X) - X\Big\}$$

From the equations (4.1) and (4.2) we have that:

$$\frac{\partial h}{\partial c_j^2}(c_j^{2\,SO}) = \frac{\partial h}{\partial c_j^1}(c_j^{1\,SO}) \tag{4.5}$$

and from strict concavity of the function $h$ (monotonocity of $h'$) we can derive that $c_j^{1\,SO} = c_j^{2\,SO}$. Then it holds that:

$$T^1 + B^{SO} - X^{SO} + \gamma = T^{2\,SO)} - B^{SO} + \theta(X^{SO}) - \gamma$$

Hence,

$$B^{SO} = \frac{T^{2\,SO} - T^1}{2} + \frac{\theta(X^{SO})}{2} + \frac{X^{SO}}{2} - \gamma \tag{4.6}$$

Since both periods are the same, the optimal deficit smooths the consumption spending across the periods. It compensates differences in revenues caused by the different tax revenues $T^1$ and $T^2$ (respectively the tax rates $\tau^1$ and $\tau^2$) captured by the first term of equation (4.6), transfers the returns from productive spending to the first period and the cost of productive spending to the second period (captured by the second and the third term of equation (4.6)) and smooths the revenue cycle, directly captured by $\gamma$.

### Consumer choice parallel

Now we can look at the model as a parallel to consumer choice problem, where the fiscal variables $c_j^1$, $c_j^2$, $T^2$, $X_j$, $B$ represent a good for the group $j$. The consumer choice problem can be solved overall for the intertemporal budget constraint as it has been done in Subsection 3.2.3, but, in the upcoming sections, the decision making is decomposed to separate *optimization problems* in each period to examine the dynamic common pool effect and the distortions that in the model occur. To have a better comparison we can decompose the socially optimal conditions in each period as well. For the socially optimal fiscal choices associated with the full centralization (and that also eliminates all the externalities) we have that $X = nX_j$, $c^1 = nc_j^1$ and $c^2 = nc_j^2$ and the intertemporal budget constraint is given as

$$nc_j^1 + nc_j^2 + nX_j - T^2 \leq T^1 + \theta(nX_j) \tag{4.7}$$

*The first optimization problem* is in the second period: the interest group $j$ decides on the size of its consumption spending $c_j^2$ and the tax revenue $T^2$. The resource constraint it faces is given as

$$nc_j^2 - T^2 \leq \theta(nX_j) - B - \gamma \tag{4.8}$$

where we can denote the personal and aggregate resource constraint in the second period as

$$R^{2\,SO} = R_j^{2\,SO} = \theta(nX_j) - B - \gamma$$

In the optimum, it holds that:

$$MRS^{SO}_{c_j^2, T^2} = MRT^{SO}_{c_j^2, T^2}$$

where

$$MRS^{SO}_{c_j^2, T^2} = \frac{\frac{\partial \omega_j^2}{\partial c_j^2}}{\frac{\partial \omega_j^2}{\partial T^2}} = \frac{\frac{\partial h}{\partial c_j^2}}{\frac{\partial W}{\partial T^2}}$$

For the marginal rate of transformation we have that:

$$MRT^{SO}_{c_j^2, T^2} = (-n)$$

Then it holds that:

$$\frac{\frac{\partial h}{\partial c_j^2}}{\frac{\partial W}{\partial T^2}} = (-n) \tag{4.9}$$

The equilibrium level of $c_j^{2\,SO}$ and $T^{2\,SO}$ is then dependent solely on a level of revenue $R_j^{2\,SO}$, which affects utility $\omega_j^2$ indirectly. The effect of $R_j^{2\,SO}$ on $\omega_j^2$ can be perceived as a marginal utility of income given as

$$MUI^2 = \frac{\partial \omega_j^2}{\partial R_j^{2\,SO}}$$

*The second optimization problem* is in the first period: the interest group $j$ decides on variables $c_j^1$, $X_j$ and $B$, where the resource constraint is given as

$$nc_j^1 - B + nX_j \leq T^1 + \gamma \tag{4.10}$$

For the marginal rates of transformation we have that:

$$\begin{aligned} MRT^{SO}_{X_j, B} &= (-n) \\ MRT^{SO}_{c_j^1, B} &= (-n) \\ MRT^{SO}_{c_j^1, X_j} &= 1 \end{aligned} \tag{4.11}$$

and for the marginal rates of substitutions we have that:

$$MRS^{SO}_{X_j,B} = \frac{\frac{\partial U_j}{\partial X_j}}{\frac{\partial U_j}{\partial B}} = \frac{\frac{\partial \omega^2_j}{\partial R^{2\,SO}_j}\frac{\partial R^{2\,SO}_j}{\partial X_j}}{\frac{\partial \omega^2_j}{\partial R^{2\,SO}_j}\frac{\partial R^{2\,SO}_j}{\partial B}} = \frac{\frac{\partial R^{2\,SO}_j}{\partial X_j}}{\frac{\partial R^{2\,SO}_j}{\partial B}} = MRTS^{SO}_{X_j,B}(R^{2\,SO}_j) \tag{4.12}$$

where the expression $MRTS^{SO}_{X_j,B}(R^{2\,SO}_j)$ denotes a marginal rate of technical substitution of the resource constraint $R^{2\,SO}_j$ in the second period. We obtain the condition (4.12) since the variables $X_j$ and $B$ do not affect directly any utility function and they serve only as a transmitter of revenue from the first period to the second period. The interest group $j$ then searches an optimal structure of unconsumed revenue in the first period stored in the variables $B$ and $X_j$, which yields the highest possible return. The optimal structure is then given by the condition $MRTS^{SO}_{X_j,B}(R^{2\,SO}_j) = MRT^{SO}_{X_j,B}$. Further we have:

$$MRS^{SO}_{c^1_j,B} = \frac{\frac{\partial U_j}{\partial c^1_j}}{\frac{\partial U_j}{\partial B}} = \frac{\frac{\partial h}{\partial c^1_j}}{\frac{\partial \omega^2_j}{\partial R^{2\,SO}_j}\frac{\partial R^{2\,SO}_j}{\partial B}} \tag{4.13}$$

Analogically we obtain $MRS^{SO}_{c^1_j,X_j}$. From the general condition for a non-corner solution $MRS = MRT$ for all pairs of variables we arrive at the socially optimal outcome of fiscal policy as it has been described above.

## 4.2 Decentralized outcome

However, the actual outcome of fiscal policy is different. It depends on whether the decision on separate elements of fiscal policy is centralized or not. Now assume that the interest groups cannot coordinate and the decision about the size of budget, the productive spending and the tax revenue is fully decentralized.

### 4.2.1 Unconstrained outcome

To determine the actual outcome of fiscal policy, which refers to a Nash equilibrium of the decision process, we need to solve the model *backwards*. The timing of the decision making is described in Subsection 3.1.1 and the government is not constrained by any deficit requirements (hence "Unconstrained outcome").

In the second period, the tax revenue $T^2$ is residually given as

$$T^{2\,DO} = \sum_{i=1}^{n} g_i^1 + \sum_{i=1}^{n} g_i^2 - T^1 - \theta(X) \tag{4.14}$$

where the superscript $DO$ refers to Nash "Decentralized outcome". In the second period, the interest group $j$ decides on its budget size $g_j^2$ (where $g_j^2 = c_j^2$) and maximizes its utility:

$$W(T^{2\,DO}) + h(c_j^2)$$

From FOC w.r.t. $g_j^2$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2,\,DO}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO}) = 0 \text{ for } j = 1, ..., n \tag{4.15}$$

where the solution to the system of equations, which represent the best response functions of the groups, is an equilibrium function $g_j^{2\,DO} = g_j^2(\sum_{i=1}^{n} g_i^1, X)$ and it holds that $g_j^{2\,DO} = g_i^{2\,DO}$ since the interest groups are symmetric (they have symmetric preferences and their size is equal). In comparison with the condition for social optimum (4.2), we observe the overspending of each group as it has been described in Subsection 3.2.1. From the implicit function theorem applied on the equation (4.15) (or by decomposing an effect of the budget size in the first period and the productive spending on the resource constraint $R_j^{2\,DO}$ from the equation (4.22)), we can derive that

$$\frac{\partial g_j^{2\,DO}}{\partial X_j} = \frac{\partial g_j^{2\,DO}}{\partial g_j^1}\theta'(X) \tag{4.16}$$

In the first period, the interest group $j$ takes into account the outcome of the second period, decides simultaneously on its budget size $g_j^1$ and the productive spending $X_j$ and maximizes the expression:

$$W(T^{2\,DO}) + h(g_j^1 - X_j) + h(g_j^{2\,DO})$$

From FOC w.r.t. $g_j^1$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2,\,DO})\Big[1 + \sum_{i=1}^{n} \frac{\partial g_i^{2\,DO}}{\partial g_j^1}\Big] + \frac{\partial h}{\partial c_j^1}(c_j^{1\,DO}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO})\frac{\partial g_j^{2\,DO}}{\partial g_j^1} = 0$$
$$\text{for } j = 1, ..., n \tag{4.17}$$

From the equation (4.17) together with the equations (4.15) and (4.16) we can derive a relation between the consumption spending $c_j^{1\,DO}$ and the tax revenue $T^{2,\,DO}$ captured by the equation (4.18) and a relation between the consumption spending in both periods $c_j^{1\,DO}$ and $c_j^{2\,DO}$ captured by the equation (4.19). We have:

$$\frac{\partial W}{\partial T^2}(T^{2\,DO})\Big[1+\frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}\Big]+\frac{\partial h}{\partial c_j^1}(c_j^{1\,DO})=0 \text{ for } j=1,...,n \tag{4.18}$$

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,DO})=\frac{\partial h}{\partial c_j^2}(c_j^{2,\,DO})\Big[1+\frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}\Big] \text{ for } j=1,...,n \tag{4.19}$$

where we write that $g_-^2 j=\sum_{i=1}^n g_i^2-g_j^2$ to simplify the notation. The expression $\frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}$ is negative since higher budget size of the group $j$ in the first period limits the resources in the second period throughout the higher deficit $B$ or the lower productive spending $X_j$. Consequently, it limits the budget of other interest groups in the second period as well. As a result, the overspending in the first period is even stronger then in the second period and from the equation (4.19) we can derive that $c_j^{1\,DO}>c_j^{2\,DO}$.

It can be also shown that the expression $\frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}$ is higher than $-\frac{n-1}{n}$. From the budget constrain (4.14) we can derive that:

$$-\frac{\partial\sum_{i=1}^n g_i^{2\,DO}}{\partial g_j^1}+\frac{\partial T^{2\,DO}}{\partial g_j^1}=1$$

and it also holds that the term

$$\frac{\partial T^{2\,DO}}{\partial g_j^1}$$

is negative. From the symmetry of groups we can write that it holds in the equilibrium that:

$$\sum_{i=1}^n g_i^{2\,DO}=ng_j^{2\,DO}$$

Finally, we derive that:

$$\frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}>-\frac{n-1}{n} \tag{4.20}$$

The distribution of consumption spending across the periods is then less unequal as shown in the simplified model in Subsection 3.2.3. A detailed source of the inefficient outcome is described below using the consumer choice perspective towards the model.

From FOC w.r.t. $X_j$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2,\,DO})\Big[\sum_{i=1}^{n}\frac{\partial g_i^{2\,DO}}{\partial X_j}-\theta'(X^{DO})\Big]-\frac{\partial h}{\partial c_j^1}(c_j^{1\,DO}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO})\frac{\partial g_j^{2\,DO}}{\partial X_j}=0 \quad \text{for } j=1,...,n \qquad (4.21)$$

From the equation (4.21) together with the equations (4.17), (4.15) and (4.16) we eventually obtain that:

$$\theta'(X^{DO})=1$$

Even under the decentralized decision the amount of productive spending is at its maximal return and it holds that $X^{DO}=X^{SO}$. To understand why, we can look again at the model from the consumer choice perspective separated in each period.

### Consumer choice parallel

*The first optimization problem* is in the second period: the interest group $j$ decides on consuming goods $c_j^2$ and $T^2$ under the resource constraint given as

$$c_j^2-T^2\leq\theta(X)-B-\gamma-c_-^2j=R_j^{2\,DO} \qquad (4.22)$$

in contrast to the budget constraint (4.8) where the personal resource constraint is not dependent on the consumption spending of other groups. We can also denote the aggregate resource constraint as

$$R^{2\,DO}=\theta(X)-B-\gamma \qquad (4.23)$$

In the optimum, we have that:

$$MRS^{DO}_{c_j^2,T^2}=MRT^{DO}_{c_j^2,T^2}$$

where

$$MRT^{DO}_{c_j^2,T^2}=(-1)$$

The marginal rate of substitution is the same as when setting the socially desirable conditions since the utility function remains unchanged. Hence, we have that:

$$MRS^{DO}_{c_j^2, T^2} = MRS^{SO}_{c_j^2, T^2}$$

The too high level of consumption spending $c_j^{2\,DO}$ is given by the different marginal rate of transformation between $c_j^2$ and $T^2$ as described in the simplified model in Subsection 3.2.1. Notice that the equilibrium level of $c_j^{2\,DO}$ and $T^{2\,DO}$ is dependent solely on the level of aggregate revenue $R^{2\,DO}$. We have that:

$$g_j^{2\,DO} = g_j^2(B,\, X) = g_j^2(R^{2\,DO})$$

and the second period can be perceived as a subgame in the overall decision process. An outcome of the subgame is a function of aggregate revenue $R^{2\,DO}$ regardless of its composition of fiscal variables chosen in the first period. Denote

$$MUI^{2\,DO} = \frac{\partial \omega_j^2}{\partial R_j^{2\,DO}}$$

as a marginal utility of income in the second period.

*The second optimization problem* is in the first period: the interest group $j$ decides on the variables $c_j^1$, $X_j$ and $B$ given the resource constraint:

$$c_j^1 - B + X_j \leq T^1 + \gamma - g_{-j}^1 \tag{4.24}$$

We proceed the same optimization process as shown in Section 4.1 and for marginal rates of transformation we have that:

$$\begin{aligned} MRT^{DO}_{X_j, B} &= (-1) \\ MRT^{DO}_{c_j^1, B} &= (-1) \\ MRT^{DO}_{c_j^1, X_j} &= 1 \end{aligned} \tag{4.25}$$

For the marginal rates of substitutions we have that:

$$MRS^{DO}_{X_j, B} = \frac{\frac{\partial U_j}{\partial X_j}}{\frac{\partial U_j}{\partial B}} = \frac{MUI^{2\,DO}\frac{\partial R_j^{2\,DO}}{\partial X_j}}{MUI^{2\,DO}\frac{\partial R_j^{2\,DO}}{\partial B}} = \frac{\frac{\partial R_j^{2\,DO}}{\partial X_j}}{\frac{\partial R_j^{2\,DO}}{\partial B}} = MRTS^{DO}_{X_j, B}(R_j^{2\,DO})$$

and

$$MRS^{DO}_{c_j^1, B} = \frac{\frac{\partial U_j}{\partial c_j^1}}{\frac{\partial U_j}{\partial B}} = \frac{\frac{\partial \omega_j^1}{\partial c_j^1}}{MUI^{2\,DO}\frac{\partial R_j^{2\,DO}}{\partial B}}$$

In comparison with the socially optimal conditions, there are two differences. The first difference is a change in the marginal rate of transformation between productive spending $X_j$ and deficit $B$ and between consumption spending $c_j^1$ and deficit $B$. This is because of the decentralized decision of these variables. In the intertemporal budget constraint constructed from the constraints (4.22) and (4.24) given as

$$c_j^1 + c_j^2 + X_j + T^2 \leq T^1 + \gamma - g_{-j}^1 - c_{-j}^2 \tag{4.26}$$

the deficit $B$ cancels out and we can see the eventual impact of the decentralized fiscal policy on the relative costs between the consumption spending $c_j^1$ and $c_j^2$ and between the consumption spending $c_j^1$ and the tax revenue $T^2$.

The second difference is that the personal resource constraint in the second period $R_j^{2\,DO}$ is not only dependent on the returns to productive spending $\theta(X)$ and the deficit $B$ but also on the consumption spending of other groups $c_{-j}^2$. The consumption spending of other groups $g_{-j}^{2\,DO} = c_{-j}^{2\,DO}$ is a function of their corresponding resource constraint $R_i^{2\,DO}$. In the equilibrium of the second period, the consumption spending of each group is a function of the aggregate resource constraint $R^{2\,DO}$. Then we have that the effect of the deficit $B$ and the productive spending $X_j$ on the personal resource constraint $R_j^{2\,DO}$ is different then on its counterpart $R_j^{2\,SO}$ because of the additional effect on the consumption spending $g_{-j}^{2\,DO}$. By increasing the deficit or the productive spending the group $j$ decreases the consumption spending of other groups in the second period and thus increases its available resources in the second period. Explicitly, the effects are given as

$$\frac{\partial R_j^{2\,DO}}{\partial B} = -\frac{\partial g_{-}j^{2\,DO}}{\partial B} - 1$$

whereas the effect of deficit $B$ on $R_j^{2\,SO}$ is simply

$$\frac{\partial R_j^{2\,SO}}{\partial B} = (-1)$$

For the productive spending we have that:

$$\frac{\partial R_j^{2\,DO}}{\partial X_j} = \frac{\partial R_j^{2\,DO}}{\partial \theta}\theta'(X) = (1 - \frac{\partial g_{-}j^{2\,DO}}{\partial \theta})\theta'(X)$$

and similarly the effect of $X_j$ on $R_j^{2\,SO}$ is

$$\frac{\partial R_j^{2\,DO}}{\partial X_j} = n\theta'(X)$$

The changed effect of $\theta$ (or $X_j$) and $B$ on the resource constraint can be perceived as a source of a dynamic common pool problem. The fiscal choice in the first period has its strategic effect in the second period: to constrain other groups in order to increase personal resources in the future.

Now we can look at the explanation for the optimal level of productive spending $X^{DO} = X^{SO}$. Recall that the unconsumed revenue in the first period is divided into the productive spending and the deficit in a structure that maximizes the personal revenue in the second period. It is captured by the condition:

$$MRS^{DO}_{X_j,B} = MRTS^{DO}_{X_j,B}(R_j^{2\,DO}) = MRT^{DO}_{X_j,B}$$

and we have that:

$$\frac{\frac{\partial R_j^{2\,DO}}{\partial X_j}}{\frac{\partial R_j^{2\,DO}}{\partial B}} - = \frac{\frac{\partial R_j^{2\,DO}}{\partial \theta}\theta'(X)}{\frac{\partial R_j^{2\,DO}}{\partial B}} = -1 \tag{4.27}$$

Notice that it holds that:

$$\frac{\partial R^{2\,DO}}{\partial B} = -\frac{\partial R^{2\,DO}}{\partial \theta}$$

Then we can write that:

$$\frac{\partial g_j^{2\,DO}}{\partial \theta} = \frac{\partial g_j^{2\,DO}}{\partial R^{2\,DO}}\frac{\partial R^{2\,DO}}{\partial \theta} = -\frac{\partial g_j^{2\,DO}}{\partial R^{2\,DO}}\frac{\partial R^{2\,DO}}{\partial B} = -\frac{\partial g_j^{2\,DO}}{\partial B} \tag{4.28}$$

and we can then derive that:

$$\frac{\partial R_j^{2\,DO}}{\partial \theta} = \frac{\partial R^{2\,DO}}{\partial \theta} - \frac{\partial g_{-j}^{2\,DO}}{\partial \theta} = -\frac{\partial R^{2\,DO}}{\partial B} + \frac{\partial g_{-j}^{2\,DO}}{\partial B} = -\frac{\partial R_j^{2\,DO}}{\partial B} \tag{4.29}$$

As a consequence of the equation (4.29), we have in the condition (4.27) that the effect of deficit and return to productive spending on the personal resource constraint in the second period simply cancel out. Consequently, it holds that

$$\theta'(X^{DO}) = 1$$

**Brief summary**

The outcome of fully decentralized fiscal policy is significantly different from the socially optimal fiscal policy. 1) We observe overspending in the second period relative to the tax revenue captured by the equation (4.15) and even stronger overspending relative to the tax revenue in the first period captured by the equation (4.18). It is a consequence of the dynamic aspect of the model. 2) The productive spending is at the socially optimal level. 3) We observe an uneven distribution of consumption spending captured by the equation (4.19). The consumption spending in the first period exceeds consumption spending in the second period.

### 4.2.2 Optimal deficit ceiling

Now assume that there is a deficit ceiling $\hat{B}$ limiting the deficit B which is set *before* the fiscal decision making (we can imagine that the deficit ceiling is set in "zero period"). The deficit ceiling rule is known to all groups and is fully complied with. (We disregard completely the possibility of not complying with the fiscal framework.) In the following analysis, it is shown how the interest groups respond to the exogenously given level of deficit ceiling. Next, we derive what deficit ceiling $\hat{B}^{DO}$ should be set to achieve the highest possible social utility. Notice that it is not clear whether the optimal deficit ceiling $\hat{B}^{DO}$ coincides with $B^{SO}$. In the analysis, we will focus on a deficit ceiling that is lower then the Nash equilibrium deficit $B^{DO}$ and, therefore, is binding. To determine the optimal level of deficit ceiling $\hat{B}^{DO}$ before the fiscal policy is set we need to determine the outcome of fiscal policy as a function of the deficit ceiling.

**Fiscal policy given a deficit ceiling**

In the second period, interest group $j$ faces the same optimization problem as in the unconstrained situation. Hence, we have that:

$$T^{2\,DO\,ODC} = \sum_{i=1}^{n} g_i^2 + \hat{B} - \theta(X) + \gamma \tag{4.30}$$

From FOC w.r.t. $g_j^2$ we obtain:

$$\frac{\partial W}{\partial T^2}(T^{2,\,DO\,ODC}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO\,ODC}) = 0 \text{ for } j = 1, ..., n \tag{4.31}$$

and the solution of the system of equations (4.31), which refer to the best response functions of groups, is a function

$$g_j^{2\,DO\,ODC} = g_j^2(\hat{B}, X)$$

of the productive spending $X$, where $X = X_j + X_{-j}$, and the deficit ceiling $\hat{B}$. In contrast, $g_j^{2\,DO}$ is a function of the budget size in the first period instead of the deficit ceiling. From the implicit function theorem applied on the equation (4.31) (or by decomposing an effect of the deficit and the productive spending on the resource constraint $R_j^{2\,DO\,ODC}$ from the equation (4.39)) we have that:

$$\frac{\partial g_j^{2\,DO\,ODC}}{\partial X} = -\frac{\partial g_j^{2\,DO\,ODC}}{\partial \hat{B}}\theta'(X) \tag{4.32}$$

In the first period, the budget size of group $j$ is given by the fixed deficit ceiling and the resources are equally distributed between the interest groups:

$$g_j^{1\,DO\,ODC} = \frac{\hat{B} + T^1 + \gamma}{n} \tag{4.33}$$

The interest group $j$ decides on size of productive spending $X_j$ and maximizes:

$$W(T^{2,\,DO\,ODC}) + h(g_j^{1\,DO\,ODC} - X_j) + h(g_j^{2\,DO\,ODC})$$

From FOC w.r.t. $X_j$ we have:

$$\frac{\partial W}{\partial T^2}(T^{2,\,DO\,ODC})\Big[\sum_{i=1}^{n}\frac{\partial g_i^{2\,DO\,ODC}}{\partial X_j} - \theta'(X^{DO\,ODC})\Big] - \frac{\partial h}{\partial c_j^1}(c_j^{1\,DO\,ODC}) + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO\,ODC})\frac{\partial g_j^{2\,DO\,ODC}}{\partial X_j} = 0$$
$$\text{for } j = 1, ..., n \tag{4.34}$$

From the equations (4.34), (4.32) and (4.31) we can express a condition for the productive spending:

$$\theta'(X^{DO\,ODC}) = \frac{\frac{\partial h}{\partial c_j^1}(c_j^{1\,DO\,ODC})}{\frac{\partial h}{\partial c_j^2}(c_j^{2\,DO\,ODC})} \frac{1}{1 + \frac{\partial g_{-j}^{2,\,DO\,ODC}}{\partial \hat{B}}} \qquad \text{for } j = 1, ..., n \quad (4.35)$$

The solution of the system of equations (4.34), which refer to the best response functions of each group, is an equilibrium function

$$X_j^{DO\,ODC} = X_j(\hat{B})$$

of the variable $\hat{B}$. From the symmetry of the interest groups we have that:

$$X_j^{DO\,ODC} = X_i^{DO\,ODC}$$

#### Optimal deficit ceiling choice

Knowing the actual outcome of fiscal policy given the binding deficit ceiling we can calculate what the social utility maximizing level of deficit ceiling $\hat{B}^{DO}$ is. Notice that the groups are symmetric and therefore they prefer the optimal level of the deficit ceiling. The deficit ceiling can be set by the groups or an independent authority or a numerical fiscal rule without a change to the result. We maximize:

$$W(T^{2,\,DO\,ODC}) + h(g_j^{1\,DO\,ODC} - X_j^{1\,DO\,ODC}) + h(g_j^{2\,DO\,ODC})$$

and from FOC w.r.t. $\hat{B}$ we have:

$$\begin{aligned} &\frac{\partial W}{\partial T^2}(T^{2,\,DO\,ODC})\Big[\sum_{i=1}^{n}\Big(\frac{\partial g_i^{2\,DO\,ODC}}{\partial X}\frac{\partial X^{DO\,ODC}}{\partial \hat{B}} + \frac{\partial g_i^{2\,DO\,ODC}}{\partial \hat{B}}\Big) + 1 \\ &\quad - \theta'(X^{DO\,ODC})\frac{\partial X^{DO\,ODC}}{\partial \hat{B}}\Big] + \frac{\partial h}{\partial c_j^1}(c_j^{1\,DO\,ODC})\Big[\frac{1}{n} - \frac{\partial X_j^{DO\,ODC}}{\partial \hat{B}}\Big] \\ &\quad + \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO\,ODC})\Big[\frac{\partial g_j^2}{\partial X}\frac{\partial X^{DO\,ODC}}{\partial \hat{B}} + \frac{\partial g_j^{DO\,ODC}}{\partial \hat{B}}\Big] = 0 \end{aligned} \qquad \text{for } j = 1, ..., n \quad (4.36)$$

From the equation (4.36) together with the equations (4.34), (4.32) and (4.31) we eventually obtain a relation between the consumption in the first and the second period when the optimal deficit ceiling $\hat{B}^{DO}$ is set:

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,DO\,ODC})\Big[\frac{1}{n}+\frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}(\hat{B}^{DO})\Big]= \frac{\partial h}{\partial c_j^2}(c_j^{2\,DO\,ODC})\Big[1+\frac{\partial g_{-j}^{2\,DO\,ODC}}{\partial \hat{B}}(\hat{B}^{DO})\Big] \quad \text{for } j=1,...,n \quad (4.37)$$

Compared to the condition (4.19) there is a new term

$$\frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}$$

in the equation (4.37). (Also we have that $\frac{\partial g_{-j}^{2\,DO\,ODC}}{\partial \hat{B}} = \frac{\partial g_{-j}^{2\,DO}}{\partial g_j^1}$). The term is positive since a higher deficit ceiling $\hat{B}$ increases overall resources in the first period and consequently increases the productive spending. It can be shown that the term $\frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}$ is lower than $\frac{n-1}{n}$. From the resource constraint (4.33) it is given that:

$$\frac{1}{n}=\frac{\partial g_i^{DO\,ODC}}{\partial \hat{B}}=\frac{\partial X_i^{DO\,ODC}}{\partial \hat{B}}+\frac{\partial c_i^{DO\,ODC}}{\partial \hat{B}}$$

and the term

$$\frac{\partial c_i^{DO\,ODC}}{\partial \hat{B}}$$

is positive. Then we have that:

$$\frac{\partial X_i^{DO\,ODC}}{\partial \hat{B}}<\frac{1}{n}$$

and from the identity of the functions $X_i^{DO\,ODC}$ we have:

$$\frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}<\frac{n-1}{n}$$

The deficit ceiling thus mitigates the overspending in the first period since for the whole term we have:

$$\frac{1}{n}+\frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}(\hat{B}^{DO})<1$$

From the equations (4.35) and (4.37) we can derive a level of productive spending when the optimal deficit ceiling $\hat{B}^{DO}$ is set:

$$\theta'(X^{DO\,ODC}(\hat{B}^{DO})) = \frac{1}{\frac{1}{n} + \frac{\partial X_{-j}^{DO\,ODC}}{\partial \hat{B}}(\hat{B}^{DO})} \tag{4.38}$$

The whole term is bigger than one and thus the productive spending $X^{DO\,ODC}$ is lower then $X^{SO}$ and it is not at its highest potential return. To understand what is the role of deficit ceiling in the model we can solve the model again from the consumer choice perspective:

#### Consumer choice parallel

*The first optimization problem* in the second period is the same as in the unconstrained fiscal policy with the identical resource constraint (4.22):

$$c_j^2 - T^2 \leq \theta(X) - \hat{B} - \gamma - c_-^2 j = R_j^{2\,DO} = R_j^{2\,DO\,ODC} \tag{4.39}$$

In the first period, *the second optimization problem*, however, changes. The interest group $j$ decides on the goods $c_j^1$ and $X_j$ and the resources are equally split due to the binding deficit ceiling as

$$g_j^1 = \frac{\hat{B} + \gamma + T^1}{n}$$

The interest groups $j$ then faces a different resource constraint compared to (4.24), which also includes the deficit ceiling $\hat{B}$:

$$nc_j^1 + nX_j \leq \hat{B} + \gamma + T^1 = R_j^{1\,DO\,ODC} \tag{4.40}$$

Here we can see the direct impact of deficit ceiling on the costs of consumption $c_j^1$ and productive spending $X_j$ relative to other variables. Nonetheless, the marginal rate of transformation between $X_j$ and $c_j^1$

$$MRT_{c_j^1, X_j}^{DO\,ODC} = 1$$

remains unchanged. For the marginal rate of substitution we have:

$$MRS_{c_j^1, X_j}^{DO\,ODC} = \frac{\frac{\partial U_j}{\partial c_j^1}}{\frac{\partial U_j}{\partial X_j}} = \frac{\frac{\partial \omega_j^1}{\partial c_j^1}}{\frac{\partial \omega_j^2}{\partial R_j^{2\,DO\,ODC}} \frac{\partial R_j^{2\,DO\,ODC}}{\partial X_j}} = 1 \tag{4.41}$$

It has the same form as under the decentralized fiscal policy without the binding deficit rule.

It was said that the role of optimal deficit ceiling $\hat{B}^{DO}$ is to maximize the social utility. From the perspective of consumer choice problem, we can look at the deficit ceiling $\hat{B}^{DO}$ as two separate goods: deficit ceiling in the first period and the repaid deficit in the second period. Now, we can solve *the third optimization problem* in the "zero period". Under this perspective, the optimal deficit ceiling in optimum satisfies:

$$MRS_{1,2} = MRT_{1,2}$$

where the marginal rate of transformation

$$MRT_{1,2} = (-1)$$

represents a relative cost of the deficit ceiling $\hat{B}$ across the periods. In case that we would assume an interest rate $r$ (not normalized to zero) imposed on the deficit $B$ in the second period, the relative cost would be $\frac{-1}{1+r}$. Then we have:

$$MRS_{1,2} = \frac{\frac{\partial \omega_j^1}{\partial \hat{B}}}{\frac{\partial \omega_j^2}{\partial \hat{B}}} = \frac{\frac{\partial \omega_j^1}{\partial R^{1\,DO\,ODC}} \frac{\partial R^{1\,DO\,ODC}}{\partial \hat{B}}}{\frac{\partial \omega_j^2}{R_j^{2\,DO\,ODC}} \frac{\partial R_j^{2\,DO\,ODC}}{\partial \hat{B}}} = (-1) \tag{4.42}$$

where the expression

$$\frac{\partial \omega_j^1}{\partial R^{1\,DO\,ODC}} = \frac{\partial \omega_j^1}{\partial c_j^1} \frac{\partial c_j^1}{\partial R^{1\,DO\,ODC}}$$

is a marginal utility of income in the first period. The effect of deficit ceiling $\hat{B}$ on the resource constraint $R^{1\,DO\,ODC}$ is straightforward and equals to 1. From the conditions (4.42) and (4.41) we obtain a following relationship:

$$\frac{\frac{\partial R_j^{2\,DO\,ODC}}{\partial X_j}}{\frac{\partial R_j^{2\,DO\,ODC}}{\partial \hat{B}}} = \frac{-1}{\frac{\partial c_j^1}{\partial \hat{B}}} \tag{4.43}$$

In comparison with the equation (4.27), the left side of the equation (4.43) has changed from $(-1)$ to the term $\frac{-1}{\frac{\partial c_j^1}{\partial \hat{B}}}$. It is because the deficit ceiling $\hat{B}$ not only influences the revenue in the second period $R^{DO\,ODC}$, but it has also an impact on the consumption spending in the first period $c_j^1$ throughout the higher level of revenue in the first period. Hence, the optimal level of deficit ceiling must take it into account and the narrow interpretation of the relationship between the deficit $B$ and the productive spending $X$ in the former sections have disappeared.

#### Brief summary

The deficit ceiling, which is set before the fiscal decision making, does not achieve social optimum alone. We observe that 1) it has no impact on overspending in the second period relative to the tax revenue. 2) It increases the cost of productive spending in the first period. As a consequence, we observe too low productive spending captured by the equation (4.38). 3) It mitigates the uneven distribution of consumption spending across periods as captured by the equation (4.37) in contrast to the equation (4.19).

## 4.3 Centralized productive spending

Now assume that the interest groups set the productive spending $X$ in the first period centrally and each group funds an equal share $X_j = \frac{X}{n}$ from its budget. All the interest groups are symmetric and they, therefore, prefer the same level of productive spending $X$.

### 4.3.1 Unconstrained outcome

The optimization process of interest group $j$ is very similar to the optimization process in a completely decentralized fiscal environment. In the second period, the tax revenue $T^{2\,CPS}$ is given residually and the interest group $j$ decides on its consumption spending $c_j^{2\,CPS}$. The superscript $CPS$ refers to "Centralized productive spending". The outcome of the optimization is captured by the equation (4.15) and it is not directly influenced by the centralization of productive spending decision. The solution to the system of the best response functions (4.15) is a function

$$g_j^{2\,CPS} = g_j^2(\sum_{i=1}^{n} g_i^1,\, X_j)$$

where we have that:

$$\frac{\partial g_j^{2\,CPS}}{\partial X_j} = -n\frac{\partial g_j^{2\,CPS}}{\partial g_j^1}\theta'(X) \tag{4.44}$$

In the first period, the interest group $j$ decides simultaneously on the productive spending $X_j^{CPS}$ and its budget size $g_j^{1\,CPS}$. The first-order condition for the budget size $g_j^{1\,CPS}$, which refers to the best response function of group $j$, is given by the equation (4.17). Analogically as in the decentralized environment, we can derive a relation between $c_j^{1\,CPS}$ and $c_j^{2\,CPS}$ and between $c_j^{1\,CPS}$ and $T^{2\,CPS}$ captured by the equations (4.18) and (4.19) respectively. The condition for productive spending $X_j^{CPS}$ changes since we have due to the centralization that:

$$\theta(X) = \theta(nX_j)$$

The first-order condition is then given as

$$\frac{\partial W}{\partial T^2}(T^{2,CPS})\left[n\sum_{i=1}^{n}\frac{\partial g_i^{2\,CPS}}{\partial X_j} - n\theta'(X^{CPS})\right] - \frac{\partial h}{\partial c_j^1}(c_j^{1\,CPS}) + n\frac{\partial h}{\partial c_j^2}(c_j^{2\,CPS})\frac{\partial g_j^{2\,CPS}}{\partial X_j} = 0 \quad \text{for } j = 1, ..., n \tag{4.45}$$

since the effect of interest group $j$'s productive spending on the common productive spending $\frac{\partial X}{\partial X_j}$ has changed from 1 under the decentralized outcome to $n$. Similarly as under the decentralized fiscal policy we have from the equations (4.45), (4.44), (4.15) and (4.17) that:

$$\theta'(X^{CPS}) = 1 \tag{4.46}$$

This result is the same as in the decentralized outcome or the social optimum. The higher return from interest group $j$'s productive spending $X_j$ compensates a proportional increase in its cost.

### Consumer choice parallel

From the consumer choice perspective, the productive spending again serves only as a transmitter of resource from the first period to the second period and satisfies the condition:

$$MRT^{CPS}_{X_j,B} = MRTS^{CPS}_{X_j,B}(R^{2\,CPS}_j)$$

The productive spending level is simply a result of optimization between the deficit $B$ and the productive spending $X_j$ where we have that:

$$c^1_j - B + nX_j \leq T^1 + \gamma - c^1_{-j} \tag{4.47}$$

$$c^2_j - T^2 \leq \theta(nX_j) - B - \gamma - c^2_{-}j = R^{2\,CPS}_j \tag{4.48}$$

Then

$$MRTS^{CPS}_{X_j,B}(R^{2\,CPS}_j) = \frac{\frac{\partial R^{2\,CPS}_j}{\partial X_j}}{\frac{\partial R^{2\,CPS}_j}{\partial B}} = \frac{\frac{\partial R^{2\,CPS}_j}{\partial \theta} n\theta(X)}{\frac{\partial R^{2\,CPS}_j}{\partial B}}$$

and

$$MRTS^{CPS}_{X_j,B}(R^{2\,CPS}_j) = -n$$

The increase in the return to productive spending cancels out with the increase in the cost of productive spending. As a result, the level of productive spending again satisfies the socially optimal condition.

#### Brief summary

The centralization of productive spending has no impact on the outcome of *unconstrained* fiscal policy.

### 4.3.2 Optimal deficit ceiling

The effect of productive spending centralization occurs when it is introduced together with a deficit ceiling as it is shown in the following analysis. To determine the socially optimal deficit ceiling $\hat{B}^{CPS}$ for the centralized productive spending decision we proceed the analogical optimization process as in Subsection 4.2.2.

#### Fiscal policy given a deficit ceiling

The optimization of group $j$ in the second period is not affected neither by the centralized productive spending nor by the optimal deficit ceiling and it is captured by the equations (4.31) and (4.30) with the solution

$$g_j^{2\,CPSODC} = g_j^2(\hat{B}, X_j)$$

.

In the first period, the budget size $g_j^{1\,CPSODC}$ is given by the deficit ceiling residually as it is captured by the equation (4.33) and the interest group $j$ decides on the level of productive spending $X_j^{CPSODC}$. The condition for productive spending is captured by the equation (4.45) where the solution is a function

$$X_j^{CPSODC} = X_j(\hat{B})$$

### Optimal deficit ceiling choice

Knowing the outcome of fiscal policy we can set the deficit $\hat{B}^{CPS}$ similarly as in the case of decentralized fiscal policy by maximizing the social utility:

$$W(T^{2,CPSODC}) + h(g_j^{1\,CPSODC} - X_j^{CPSODC}) + h(g_j^{2\,CPSODC})$$

From FOC w.r.t $\hat{B}$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2,CPSODC})\Big[\sum_{i=1}^{n}\Big(\frac{\partial g_i^{2\,CPSODC}}{\partial X_j}\frac{\partial X_j^{CPSODC}}{\partial \hat{B}} + \frac{\partial g_i^{2\,CPSODC}}{\partial \hat{B}}\Big) + 1 - \theta'(X^{CPSODC})\frac{\partial X_j^{CPSODC}}{\partial \hat{B}}\Big] + \frac{\partial h}{\partial c_j^1}(c_j^{1\,CPSODC})\Big[\frac{1}{n} - \frac{1}{n}\frac{\partial X_j^{CPSODC}}{\partial \hat{B}}\Big] + \frac{\partial h}{\partial c_j^2}(c_j^{2\,CPSODC})\Big[\frac{\partial g_j^2}{\partial X_j}\frac{\partial X_j^{CPSODC}}{\partial \hat{B}} + \frac{\partial g_j^{CPSODC}}{\partial \hat{B}}\Big] = 0 \quad \text{for } j = 1, ..., n \quad (4.49)$$

From the equation (4.49) together with the conditions (4.45), (4.44) and (4.31) we can derive a relation between the consumption spending in each period.

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,CPSODC})\frac{1}{n} = \frac{\partial h}{\partial c_j^2}(c_j^{2\,CPSODC})\Big[1 + \frac{\partial g_{-j}^{2\,CPSODC}}{\partial \hat{B}}(\hat{B}^{CPS})\Big] \quad \text{for } j = 1, ..., n \quad (4.50)$$

In comparison with the equation (4.37), optimal deficit ceiling combined with the centralized productive spending shifts the consumption spending from the first period to the second period and mitigates the uneven distribution. From the conditions (4.49), (4.45), (4.44) and (4.34) we can derive for the productive spending that

$$\theta'(X^{CPS\,ODC}(\hat{B}^{CPS})) = 1$$

Compared to the level of productive spending $X^{DO\,ODC}$, we observe the optimal level of productive spending. The negative effect of optimal deficit ceiling on the productive spending is overcome when the centralized decision is implemented. The reason is that the optimal deficit ceiling changes the cost of productive spending in the first period, as shown in Subsection 4.2.2, without compensating for a higher return to productive spending $X_j$ in the second period, that the interest group $j$ perceives. The higher return to productive spending $X_j$ in the second period is then given solely by the centralized decision. To avoid the reduction in the productive spending the introduced deficit ceiling should be followed by the productive spending centralization.

The optimal productive spending can be also shown from the general condition for the optimal deficit ceiling (A.10) where we have that:

$$\frac{\partial X}{\partial X_j} = n$$

and

$$\frac{\partial (n c_j^1 + X)}{\partial \hat{B}} = 1$$

#### Brief summary

1) The centralization of productive spending together with deficit ceiling has no impact on the overspending in the second period relative to the tax revenue.
2) The centralization of productive spending compensates the increase in the cost of productive spending stemming from the presence of the deficit ceiling in the first period. As a result, the productive spending is at the optimal level.
3) We observe that the consumption spending in the second period exceeds consumption spending in the first period as captured by the equation (4.50) (in contrast to the equations (4.37) and (4.19))

## 4.4 Centralized tax revenue

Now assume that the government decides centrally on the tax revenue $T^2$ in the second period. The centralization can be perceived as if the tax revenue is set jointly before the groups set their consumption spending.

### 4.4.1 Unconstrained outcome

In the second period, the fiscal decision is changed due to the centralized decision on tax revenue. The consumption spending of interest group $j$ is given residually as

$$g_j^{2\,CT} = \frac{T^2 + T^1 + \theta(X) - \sum_{i=1}^{n} g_i^1}{n} \tag{4.51}$$

and the interest groups jointly decide on a level of tax revenue $T^{2\,CT}$. Since all the interest groups are symmetric, they prefer the same level of tax revenue. The interest group $j$ maximizes the expression:

$$W(T^2) + h(g_j^{2\,CT}) \tag{4.52}$$

and from FOC w.r.t. $T^2$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2\,CT}) + \frac{1}{n}\frac{\partial h}{\partial c^2}(c_j^{2\,CT}) = 0 \tag{4.53}$$

The centralized tax decision results in the same condition as for the socially optimal outcome captured by the equation (4.2). The solution to the system of equations (4.53) is a function

$$T^{2\,CP} = T^2(\sum_{i=1}^{n} g_i^1, X_j) \tag{4.54}$$

where we have that:

$$\frac{\partial T^{2\,CP}}{\partial X_j} = -\frac{\partial T^{2\,CP}}{\partial g_j^1}\theta'(X) \tag{4.55}$$

From the consumer choice perspective, the centralized decision changes the interest group $j$'s budget constraint (captured by the equation (4.22)) to

$$nc_j^2 - T^2 \leq \theta(X) - B - \gamma = R_j^{2\,CT} \tag{4.56}$$

It changes the marginal rate of transformation between consumption spending and tax revenue from 1 to its socially optimal level $n$.

In the first period, the interest group $j$ decides on size of its budget $g_j^1$ and a level of productive spending $X_j$ and maximizes the expression:

$$W(T^{2\,CT}) + h(g_j^1 - X_j) + h(g_j^{2\,CT}) \tag{4.57}$$

From FOC w.r.t. $g_j^1$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2\,CP})\frac{\partial T^{2\,CP}}{\partial g_j^1} + \frac{\partial h}{\partial c_j^1}(c_j^{1\,CP}) + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CP})\Big[\frac{\partial T^{2\,CP}}{\partial g_j^1} - 1\Big] = 0 \tag{4.58}$$

From the equations (4.58) and (4.53) we can derive a relation between consumption spending $c_j^{1\,CP}$ and $c_j^{2\,CP}$ and between tax revenue $T^{2\,CP}$ and $c_j^{1\,CP}$ captured in the equations (4.59) and (4.60) respectively.

$$\frac{\partial h}{\partial c^1}(c_j^{1\,CT}) = \frac{1}{n}\frac{\partial h}{\partial c^2}(c_j^{2\,CT}) \tag{4.59}$$

$$\frac{\partial W}{\partial T^2}(T^{2\,CT}) + \frac{\partial h}{\partial c^1}(c_j^{1\,CT}) = 0 \tag{4.60}$$

Comparing the equations (4.59) and (4.60) to the equations (4.5) and (4.1) respectively, we can see that the term $\frac{\partial g_{-j}^2}{\partial g_j^1}$ disappeared. It is due to the change in resource constraint (4.54) caused by the tax centralization. The centralization eliminated the dynamic aspect of common pool in the first period and the relative costs between consumption spending $c_j^1$, $c_j^2$ and the tax revenue $T^2$ are now straightforward. They are given from the intertemporal resource constraint:

$$c_j^1 + nc_j^2 + X_j - T^2 \leq T^1 + \theta(X) - c_{-j}^1 - X_{-j} \tag{4.61}$$

which is composed from the separate resource constraints (4.56) and (4.24). The relative costs stem from the sequential decision over the consumption spending as discussed in Subsection 3.2.3. From FOC w.r.t. $X_j$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2\,CP})\frac{\partial T^{2\,CP}}{\partial X_j}\theta'(X) - \frac{\partial h}{\partial c_j^1}(c_j^{1\,CP}) + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CP})\Big[\frac{\partial T^{2\,CP}}{\partial X_j} + \theta'(X)\Big] = 0 \tag{4.62}$$

From the equations (4.62), (4.55), (4.58) and (4.53) we can derive that

$$\theta'(X) = 1 \tag{4.63}$$

This is because the return to productive spending and its cost that group $j$ realizes are not affected by the centralized decision on tax revenue in the second period. The level of productive spending is the same as in the case of decentralized unconstrained fiscal policy. In comparison with the productive spending centralization, the centralized tax revenue has an impact on fiscal policy even without the presence of the deficit ceiling.

#### Brief summary

1) The centralization of decision on taxes increases the cost of consumption spending in the second period and eliminates the negative externality. As a result, the consumption spending in the second period is at the optimal level (this is captured by the equation (4.53)) 2) The productive spending is unaffected and remains at the socially optimal level. 3) The tax centralization eliminates the dynamic aspect of the model and increases the cost of consumption spending in the second period. As a consequence, we observe even stronger overspending in the first period relative to the second period. It follows from the comparison of the equations (4.59) and (4.19).

### 4.4.2 Optimal deficit ceiling

#### Fiscal policy given a deficit ceiling

Again, to set the optimal deficit ceiling we need to determine the outcome of fiscal policy with the centrally given tax revenue. In the second period, the budget size is given by the tax revenue and the binding deficit. For the group $j$ we have that:

$$g_j^{2\,CT\,ODC} = \frac{T^2 - \hat{B} - \gamma + \theta(X)}{n} \tag{4.64}$$

The condition for optimal tax revenue $T^{2\,CT\,ODC}$ is captured by the equation (4.53) and the solution is a function

$$T^{2\,CT\,ODC} = T^2(\hat{B}, X_j) \tag{4.65}$$

where we have that:

$$\frac{\partial T^{2\,CT\,ODC}}{\partial X_j} = -\frac{\partial T^{2\,CT\,ODC}}{\partial \hat{B}}\theta'(X) \tag{4.66}$$

In the first period, the budget size is given by the binding deficit ceiling and we have that:

$$g_j^{1\,CT\,ODC} = \frac{\hat{B} + T^1 + \gamma}{n} \tag{4.67}$$

Given that, the interest group $j$ decides on the productive spending $X_j$ and maximizes the expression:

$$W(T^{2\,CT\,ODC}) + h(g_j^{1\,CT\,ODC} - X_j) + h(g_j^{2\,CT\,ODC}) \tag{4.68}$$

From FOC w.r.t $X_j$ we have that:

$$\begin{aligned}\frac{\partial W}{\partial T^2}(T^{2\,CT\,ODC})\frac{\partial T^{2\,CT\,ODC}}{\partial X_j}(\hat{B}, X_j^{CT\,ODC}) - \frac{\partial h}{\partial c_j^1}(c_j^{1\,CT\,ODC}) \\ + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CT\,ODC})\Big[\frac{\partial T^{2\,CT\,ODC}}{\partial X_j}(\hat{B}, X_j^{CT\,ODC}) + \theta'(X^{CT\,ODC})\Big] = 0\end{aligned} \tag{4.69}$$

As a solution of the equation (4.69) we have a function of the optimal deficit:

$$X_j^{CT\,ODC} = X_j(\hat{B}) \tag{4.70}$$

#### Optimal deficit ceiling choice

Knowing the outcome of fiscal policy, where the tax revenue is centrally given, we can set the optimal level of deficit $\hat{B}^{CT}$, which maximizes the social utility:

$$W(T^{2,\,CT\,ODC}) + h(g_j^{1\,CT\,ODC} - X_j^{CT\,ODC}) + h(g_j^{2\,CT\,ODC})$$

From FOC w.r.t. $\hat{B}$ we have:

$$\begin{aligned}\frac{\partial W}{\partial T^2}(T^{2,\,CT\,ODC})\Big[\frac{\partial T^{2,\,CT\,ODC}}{\partial \hat{B}} + \frac{\partial T^{2,\,CT\,ODC}}{\partial X_j}\frac{\partial X_j^{CT\,ODC}}{\partial \hat{B}}\Big] \\ + \frac{\partial h}{\partial c_j^1}(c_j^{1\,CT\,ODC})\Big[\frac{1}{n} - \frac{\partial X_j^{CT\,ODC}}{\partial \hat{B}}\Big] + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CT\,ODC})\Big[(\frac{\partial T^{2,\,CT\,ODC}}{\partial \hat{B}} \\ + \frac{\partial T^{2,\,CT\,ODC}}{\partial X_j}\frac{\partial X_j^{CT\,ODC}}{\partial \hat{B}}) - 1 + \theta'(X^{CT\,ODC})\frac{\partial X_j^{CT\,ODC}}{\partial \hat{B}}\Big] = 0 \\ \text{for } j = 1, ..., n\end{aligned} \tag{4.71}$$

From the equations (4.71), (4.69), (4.66) and (4.53) we eventually get a relation between the consumption spending in each period:

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,CT\,ODC})\left[\frac{1}{n}+\frac{\partial X_{-j}^{CT\,ODC}}{\partial \hat{B}}(\hat{B}^{CT})\right]=\frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CT\,ODC})$$

$$\text{for } j=1,...,n \quad (4.72)$$

Compared with the equation (4.37), the term $1+\frac{\partial g_{-j}^2}{\partial g_j^1}$ has changed to the term $\frac{1}{n}$. From the section Subsection 4.2.2, we have that:

$$\frac{1}{n}<1+\frac{\partial g_{-j}^2}{\partial g_j^1}<1 \quad (4.73)$$

The tax centralization then strengthens the uneven distribution of consumption spending across periods either with or without deficit ceiling. On the other hand, it reduces the excessive tax revenue $T^2$.

The condition for productive spending is

$$\theta'(X^{CT\,ODC}(\hat{B}^{CT}))=\frac{1}{\frac{1}{n}+\frac{\partial X_{-j}^{CT\,ODC}}{\partial \hat{B}}(\hat{B}^{CT})} \quad (4.74)$$

The outcome is the same as in the case of decentralized fiscal policy with the deficit ceiling rule. It is because tax centralization has no impact on the cost or the return to productive spending that the group $j$ perceives.

### Brief summary

1) The consumption spending in the second period is at the socially optimal level regardless of the deficit ceiling (it is captured by the equation (4.53)). 2) The tax centralization has no impact on the level of productive spending even when the deficit ceiling takes place. The productive spending is due to the deficit ceiling below its optimal level (as captured by the equation (4.74)). 3) The optimal deficit ceiling mitigates overspending in the first period relative to the second period (as is captured in the equation (4.72) in contrast to the equation (4.59)).

## 4.5 Centralized taxes and productive spending

Now assume that the government can centrally decide on both the productive spending in the first period and on the tax revenue in the second period.

### 4.5.1 Unconstrained outcome

In the second period, the consumption spending of group $j$ are given residually as in the equation (4.51), where we have that $X = nX_j$. The condition for tax revenue $T^{2\,CO}$ (where the superscript $CO$ stands for "centralized outcome" to avoid superscript CTPS) is captured by the equation (4.53), where the solution is a function

$$T^{2\,CO} = T^2(\sum_{i=1}^{n} g_i^1, X) \tag{4.75}$$

We have that:

$$\frac{\partial T^{2\,CO}}{\partial X_j} = -\frac{\partial T^{2\,CO}}{\partial g_j^1}\theta'(X) \tag{4.76}$$

In the first period, the interest group $j$ decides on the productive spending $X_j$, where we have that $X = nX_j$ due to the centralization of productive spending, and its budget size $g_j^1$. The condition for budget size is captured by the equation (4.58) and for the productive spending we have that:

$$\frac{\partial W}{\partial T^2}(T^{2,CO})\frac{\partial T^{2,CO}}{X_j}(X_j^{CO}) - \frac{\partial h}{\partial c_j^1}(c_j^{1\,CO}) + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CO})\Big[\frac{\partial T^{2,CO}}{X_j}(X_j^{CO}) + n\theta'(X^{CO})\Big] = 0 \quad \text{for } j = 1, ..., n \tag{4.77}$$

From the equations (4.77), (4.58), (4.53) and (4.76) we can derive that

$$\theta'(X^{CO}) = 1 \tag{4.78}$$

and that

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,CO}) = \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CO}) \tag{4.79}$$

The outcome of unconstrained centralized fiscal policy is the same as in the case of only tax centralization since the productive spending centralization influences fiscal policy only when the deficit ceiling is introduced.

### Brief summary

There is no interaction between the productive spending and the tax centralization. 1) The consumption spending in the second period is at the socially optimal level. 2) The productive spending is at the socially optimal level. 3) There is the same overspending in the first period compared to the second period as when only tax centralization took place.

## 4.5.2 Optimal deficit ceiling

### Fiscal policy given a deficit ceiling

The budget size of group $j$ in the first and the second period is given as

$$g_j^{2\,CO\,ODC} = \frac{T^2 - \hat{B} - \gamma + \theta(X)}{n} \tag{4.80}$$

$$g_j^{1\,CO\,ODC} = \frac{T^1 + \hat{B} + \gamma}{n} \tag{4.81}$$

The outcome of fiscal policy in the second is captured by the equation (4.53), where the solution is again a function

$$T^{2\,CO\,ODC} = T^2(\hat{B}, X_j) \tag{4.82}$$

Knowing the outcome of the second period the group $j$ chooses the optimal level of productive spending $X_j = \frac{X}{n}$ and maximizes the expression:

$$W(T^{2\,CO\,ODC}) + h(g_j^{1\,CO\,ODC} - X_j) + h(g_j^{2\,CO\,ODC})$$

From FOC w.r.t. $X_j$ we have that:

$$\frac{\partial W}{\partial T^2}(T^{2\,CO\,ODC})\frac{\partial T^{2\,CO\,ODC}}{\partial X_j}(\hat{B}, X_j^{CO\,ODC}) - \frac{\partial h}{\partial c_j^1}(c_j^{1\,CO\,ODC}) + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CO\,ODC})\Big[\frac{\partial T^{2\,CO\,ODC}}{\partial X_j}(\hat{B}, X_j^{CO\,ODC}) + n\theta'(X_j^{CO\,ODC})\Big] = 0 \tag{4.83}$$

As a solution we have:

$$X_j^{CO\,ODC} = X_j(\hat{B}) \tag{4.84}$$

### Optimal deficit ceiling choice

Knowing the outcome of fiscal policy we can set the optimal level of deficit ceiling $\hat{B}^{CO}$ maximizing the expression:

$$W(T^{2\,CO\,ODC}) + h(g_j^{1\,CO\,ODC} - X_j^{CO\,ODC}) + h(g_j^{2\,CO\,ODC})$$

From FOC w.r.t. $\hat{B}$ we have that:

$$\begin{aligned}\frac{\partial W}{\partial T^2}(T^{2,CO\,ODC})\Big[\frac{\partial T^{2\,CO\,ODC}}{\partial \hat{B}} + \frac{\partial T^{2\,CO\,ODC}}{\partial X_j}\frac{\partial X_j^{CO\,ODC}}{\partial \hat{B}}\Big] \\ + \frac{\partial h}{\partial c_j^1}(c_j^{1\,CO\,ODC})\Big[\frac{1}{n} - \frac{\partial X_j^{CO\,ODC}}{\partial \hat{B}}\Big] \\ + \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^{2\,CO\,ODC})\Big[\frac{\partial T^{2\,CO\,ODC}}{\partial \hat{B}} + \frac{\partial T^{2\,CO\,ODC}}{\partial X_j}\frac{\partial X_j^{CO\,ODC}}{\partial \hat{B}} \\ - \frac{1}{n} + n\theta'(X^{CO\,ODC})\frac{\partial X_j^{CO\,ODC}}{\partial \hat{B}}\Big] = 0 \\ \text{for } j = 1, ..., n \quad (4.85)\end{aligned}$$

From the equations (4.53), (4.83), (4.85) and (4.76) we can derive that:

$$\frac{\partial h}{\partial c_j^1}(c_j^{1\,CO\,ODC}) = \frac{\partial h}{\partial c_j^2}(c_j^{2\,CO\,ODC}) \quad (4.86)$$

and for the productive spending it holds that:

$$\theta'(X^{CO\,ODC}(\hat{B}^{CO})) = 1$$

The outcome of the centralized fiscal policy constrained by the optimal deficit ceiling reaches the social optimum. It is because the deficit ceiling $\hat{B}$ eliminates the intertemporal distortion and the tax distortion by increasing the relative cost of consumption spending $c_j^1$ with respect to the consumption spending $c_2^j$ and the tax revenue $T^2$ to its socially optimal level. As a side effect, it also increases the cost of productive spending $X_j$ in the first period. However, the increase in the cost is compensated by the higher return to productive spending $X_j$ stemming from the productive spending centralization. The tax centralization eliminates the dynamic aspect of the common pool and it increases the relative cost of consumption spending $c_2^j$ to tax revenue $T^2$ to its socially optimal level. The combination of the centralized tax revenue, the centralized productive spending and the optimal deficit ceiling eliminates all externalities in the model and sets incentives of groups so that they coincide with the social utility maximizing preferences.

#### Brief summary

The centralization of productive spending and tax revenue combined with the optimal deficit ceiling reaches the socially optimal fiscal policy.

## 4.6 Budget size agreement

Another form of the fiscal policy centralization is an agreement of the groups in both periods on the overall budget size. The optimization process is analogical to all the previous cases and to determine the solution it is sufficient to look only at the budget constraints that the interest group $j$ faces.

In the first and the second period, the budget size agreement determines the budget size $g_j^1$ and $g_j^2$ as in the equations (4.81) and (4.80) respectively and the resource constraints for the group $j$ are given as

$$nc_j^1 + nX_j - B \leq T^1 + \gamma \tag{4.87}$$

$$nc_j^2 + T^2 \leq \theta(X_j + X_{-j}) - B - \gamma \tag{4.88}$$

The outcome of fiscal policy is the same as in the case of centralized tax revenue policy with the implemented deficit ceiling except for the level of productive spending where we have that:

$$\theta'(X) = \frac{1}{n} \tag{4.89}$$

We observe the same outcome as in Subsection 3.2.2 due to the distorted cost of the productive spending $X_j$ (which is n) compared to its return $\theta(X_j + X_{-j})$. After introducing the deficit ceiling the outcome of fiscal policy is identical to the centralized tax revenue policy with implemented deficit ceiling and the condition for productive spending is captured by the equation (4.74).

When the centralized decision on the productive spending is introduced, the outcome of fiscal policy coincides with the socially optimal fiscal policy regardless of the optimal deficit ceiling.

## 4.7 Centralized consumption spending

Finally, we have the most extreme case of fiscal centralization. In case that it would be possible to create an agreement on the consumption spending in both periods a core of the overall common pool would be completely eliminated. It is not surprising since the "tax" and "intertemporal" distortion described in Subsection 3.2.1 and Subsection 3.2.3 respectively are associated with the fragmented decision on consumption spending. The budget constraints are directly given from the agreement as

$$nc_j^1 + X_j - B \leq T^1 + \gamma - X_{-j} \tag{4.90}$$

and

$$nc_j^2 + T^2 \leq \theta(X_j + X_{-j}) - B - \gamma \tag{4.91}$$

and no additional fiscal constraint is necessary to reach the socially optimal fiscal policy.

## 4.8 Tables with overview of results

On the next two pages, there are two tables summarizing the fiscal policy in different fiscal frameworks. In Table 4.1, there is an overview of resource constraints that the representative group $j$ faces in the different fiscal frameworks in the first and the second period. From the resource constraints we can observe relative costs between the fiscal variables. In Table 4.2, there is an overview of equations describing the outcome of fiscal policy. The level of productive spending is captured in the second column. The level of consumption spending with respect to the tax revenue in the second period is captured in the third column. The intertemporal choice of consumption spending is captured in the fourth column.

Table 4.1: Resource constraints based on the fiscal framework

| Fiscal framework | First period | Second period |
|---|---|---|
| SO, CO ODC, BSA + CPS, CCS (+CPS) | $nc_j^1 - B + nX_j \leq T^1 + \gamma$ | $nc_j^2 - T^2 \leq \theta(nX_j) - B - \gamma$ |
| DO | $c_j^1 - B + X_j \leq T^1 + \gamma - c_{-j}^1 - X_{-j}$ | $c_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma - c_{-j}^2$ |
| DO ODC | $nc_j^1 + nX_j \leq T^1 + \gamma + B$ | $c_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma - c_{-j}^2$ |
| CPS | $c_j^1 - B + nX_j \leq T^1 + \gamma - c_{-j}^1$ | $c_j^2 - T^2 \leq \theta(nX_j) - B - \gamma - c_{-j}^2$ |
| CPS ODC | $nc_j^1 + nX_j \leq T^1 + \gamma + B$ | $c_j^2 - T^2 \leq \theta(nX_j) - B - \gamma - c_{-j}^2$ |
| CT | $c_j^1 - B + X_j \leq T^1 + \gamma - c_{-j}^1 - X_{-j}$ | $nc_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma$ |
| CT ODC, BSA ODC | $nc_j^1 + nX_j \leq T^1 + \gamma + B$ | $nc_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma$ |
| CO | $c_j^1 - B + nX_j \leq T^1 + \gamma - c_{-j}^1$ | $nc_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma$ |
| BSA | $nc_j^1 + nX_j \leq T^1 + \gamma + B$ | $nc_j^2 - T^2 \leq \theta(X_j + X_{-j}) - B - \gamma$ |

SO - Social optimum, DO - Decentralized outcome, CPS - Centralized productive spending, CT - Centralized tax revenue, CO - Centralized outcome (centralizad tax revenue and productive spending), BSA - Budget size agreement, CCS - Centralized consumption spending, ODC - Optimal deficit ceiling

Table 4.2: Outcome of fiscal policy based on the fiscal framework

| Fiscal framework | Productive spending | Second period outcome | Across period consumption |
|---|---|---|---|
| SO, CO ODC, CCS, BSA + CPS | $\theta'(X) = 1$ | $n\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{\partial h}{\partial c_j^2}(c_j^2)$ |
| DO | $\theta'(X) = 1$ | $\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{\partial h}{\partial c_j^2}(c_j^2)(1 + \frac{\partial g_{-j}^2}{\partial B})$ |
| DO ODC | $\theta'(X) = \frac{1}{\frac{1}{n} + \frac{\partial X_{-j}}{\partial B}}$ | $\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1)(\frac{1}{n} + \frac{\partial X_{-j}}{\partial B}) = \frac{\partial h}{\partial c_j^2}(c_j^2)(1 + \frac{\partial g_{-j}^2}{\partial B})$ |
| CPS | $\theta'(X) = 1$ | $\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{\partial h}{\partial c_j^2}(c_j^2)(1 + \frac{\partial g_{-j}^2}{\partial B})$ |
| CPS ODC | $\theta'(X) = 1$ | $\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{1}{n}\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{\partial h}{\partial c_j^2}(c_j^2)(1 + \frac{\partial g_{-j}^2}{\partial B})$ |
| CT | $\theta'(X) = 1$ | $n\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^2)$ |
| CT ODC, BSA ODC | $\theta'(X) = \frac{1}{\frac{1}{n} + \frac{\partial X_{-j}}{\partial B}}$ | $n\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1)(\frac{1}{n} + \frac{\partial X_{-j}}{\partial B}) = \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^2)$ |
| CO | $\theta'(X) = 1$ | $n\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{1}{n}\frac{\partial h}{\partial c_j^2}(c_j^2)$ |
| BSA | $\theta'(X) = \frac{1}{n}$ | $n\frac{\partial W}{\partial T^2}(T^2) + \frac{\partial h}{\partial c_j^2}(c_j^2) = 0$ | $\frac{\partial h}{\partial c_j^1}(c_j^1) = \frac{\partial h}{\partial c_j^2}(c_j^2)$ |

# Chapter 5

# Discussion

## 5.1 Institutions that generate social optimum

The social optimum can be characterized by a full centralization of fiscal variables (as shown in Section 4.1). This result is not surprising since there are no costs connected to the centralization of fiscal variables in the model. Furthermore, the groups are assumed to be of the same size with the symmetric preferences. Hence, there is always an agreement among groups on a common level of fiscal variables without any trade-offs between efficiency and equity of the fiscal policy. Nonetheless, a full centralization of fiscal policy is not needed to attain socially optimal fiscal policy as shown in the analysis in Chapter 4. There are three fiscal frameworks that attain socially desirable fiscal policy in the context of a dynamic common pool problem and does not require the full centralization of fiscal policy:

- centralization of productive spending and tax revenue combined with an ex-ante set optimal deficit ceiling
- centralization of consumption spending in both periods
- centralization of productive spending and budget size in both periods

Hence, to achieve socially desirable fiscal policy, the full centralization of fiscal policy is not necessary.

## 5.2 Who sets the optimal deficit ceiling?

There are several ways how the optimal deficit ceiling can be determined. It can be delegated to a fiscal council, which decides on its value, or we can introduce a numerical rule stating its level. Nonetheless, in our model, the groups are of the same size with the symmetric preferences. Hence, it is sufficient to amend the fiscal process in a way that the groups decide on the deficit ceiling before other fiscal variables. In the context of our model, we can imagine that each group proposes its preferred level of deficit ceiling in the zero period. Due to the symmetry, all the proposals coincide with the socially optimal level of deficit ceiling and the level is accepted by all the groups. After the amendment of budgetary process, any fiscal council or numerical rule is redundant.

## 5.3 A reflection of KW model

The main result of the KW model is that the implementation of the ex-ante deficit ceiling alone cannot achieve social optimum. The reason is that it creates a distortion in productive spending. The implication of the KW model is that the deficit ceiling should always be introduced with centralization of productive spending (in the KW model we speak of a credible pre-commitment of groups to the level of productive spending, which is an equivalent to centralization in our model) In our model, we observe the same issue as well.

In contrast to the KW model, our model includes decision on the tax revenue, which is absent in KW model. As a consequence, we need to take it into account and we need the additional centralization of tax revenue in our model to achieve the social optimum. However, there is no interaction between the tax distortion and implementation of ex-ante deficit ceiling as it is in the case of productive spending and deficit ceiling.

# Chapter 6

# Conclusion

At the beginning of this thesis, the model of a dynamic common pool was constructed based on two models used by Persson & Tabellini (2000) and Krogstrup & Wyplosz (2009). In the models, I identified three core fiscal choices:

1. **Decision on a level of tax revenue**
2. **Decision on a level of productive spending (public investment)**
3. **Intertemporal choice of group consumption spending across two periods**

with three associated distortions stemming from the common pool problem:

1. **Excessive taxation**
2. **Suboptimal productive spending**
3. **Intertemporal distortion (excessive spending in early period)**

Firstly, I examined each distortion in a separate model without the presence of the other two fiscal choices. Secondly, I described the outcome of fiscal policy and the distortion in the general model by using two approaches: 1) Comparison of the outcome of fiscal policy (outcome of the three fiscal choices) with the socially optimal fiscal policy, which was given as a policy, that maximizes social utility given the constraints of a representative group. 2) Comparison of the costs of fiscal variables that the representative group faces with the socially desirably relative costs of fiscal variables that eliminate externalities among the groups. I conducted the analysis for various fiscal frameworks: 1) fully decentralized fiscal policy, 2) for fiscal policy with centralized decision on productive spending, 3) for fiscal policy with centralized decision on tax revenue,

4) for fiscal policy combining both centralization of productive spending and tax revenue, 5) for fiscal policy with centralized decision on budget size and 6) for fiscal policy with centralized consumption spending. For each form of the centralization, I examined the effect of ex-ante set deficit ceiling on the outcome of fiscal policy and the three distortions.

The main findings of the analysis are:

1. For **full decentralization** of fiscal policy, the productive spending are at their optimal level since there is **no distortion to the productive spending**.

2. The **deficit ceiling involves a trade-off**. On the one hand, it increases the cost of productive spending in the first period for the groups and **reduces** their contribution to **productive spending**. On the other hand, deficit ceiling shifts consumption from the first period to the second period and **mitigates overspending in the first period**.

3. The **centralization of productive spending has an impact only when a deficit ceiling is introduced**. Then it compensates the increased cost of productive spending in the first period that the groups face.

4. The **centralization of tax revenue eliminates the dynamic aspect** of the fiscal choice in the first period and **increases the cost of consumption spending in the second period** for the groups. This **involves a trade-off**. On the one hand, it **eliminates** the **excessive taxation** in the second period. On the other hand, it **strengthens overspending in the first period** compared to the second period due to the increased cost of consumption spending in the second period. Next, the tax distortion does not interact with the deficit ceiling.

5. The **socially optimal fiscal policy** is attained when the **centralization of tax revenue**, the **centralization of productive spending** and the ex-ante set **optimal deficit ceiling** are introduced **jointly**.

6. In addition, the **socially optimal fiscal policy** can be attained **without** a presence of optimal **deficit ceiling**. It can be achieved by the centralization of consumption spending or by the joint centralization of budget size and productive spending.

At the end of this thesis, I discussed that the full centralization of fiscal policy is not needed to attain socially desirable fiscal policy. Next, due to the symmetry of groups, there is no conflict among the groups on the preferred level of fiscal variables when the variables are set centrally. To implement the optimal deficit ceiling it is sufficient to amend the budgetary process such that the deficit ceiling is agreed on before the whole fiscal process. In this particular case of symmetric groups, no decision making fiscal authority or numerical fiscal rules are needed to determine a level of the optimal deficit ceiling.

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# Appendix A

# Comparison of optimal deficit ceilings

The aim of this section is to compare optimal deficit ceilings calculated in the fiscal frameworks from Chapter 4 with the socially optimal level of deficit $B^{SO}$. The question is whether a different fiscal framework requires a different level of ex-ante set optimal deficit ceiling. It is whether we have to know what fiscal variables are centralized when we want to determine an optimal level of deficit ceiling.

### General condition for optimal deficit ceiling

To compare $B^{SO}$ with the other deficit ceilings $\hat{B}^{DO}, \hat{B}^{CPS}, \hat{B}^{CT}, \hat{B}^{CO}$ we can look at $B^{SO}$ as an deficit ceiling $\hat{B}^{SO}$ that is set before the fiscal decision making in the socially desirable fiscal framework. Both the deficit and the optimal deficit ceiling in the socially desirable fiscal environment coincide. The general condition that all the deficit ceilings satisfy is

$$\frac{\frac{\partial \omega_j^1}{\partial \hat{B}}}{\frac{\partial \omega_j^2}{\partial \hat{B}}} = \frac{\frac{\partial \omega_j^1}{\partial R^{1\,ODC}} \frac{\partial R^{1\,ODC}}{\partial \hat{B}}}{\frac{\partial \omega_j^2}{R_j^{2\,ODC}} \frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}} = \frac{MUI^{1\,ODC} \frac{\partial R^{1\,ODC}}{\partial \hat{B}}}{MUI^{2\,ODC} \frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}} = (-1) \tag{A.1}$$

which is a generalized version of the condition (4.42). The resource constraints $R^{1\,ODC}$ and $R^{2\,ODC}$ denotes general resource constraints when the deficit ceiling is introduced regardless of centralization of fiscal policy. The general condition for the productive spending and the consumption good in the second period (captured in the equation (4.41)) is

$$MRS_{c_j^1, X_j} = \frac{\frac{\partial h^1}{\partial c_j^1}}{MUI^{2\,ODC}\frac{\partial R_j^{2\,ODC}}{\partial X_j}} = 1 = MRT_{c_j^1, X_j} \tag{A.2}$$

Always when the deficit ceiling is introduced we have that $MRT_{c_j^1, X_j} = 1$. For the marginal utility of income in the first period we have that:

$$MUI^{1\,ODC} = \frac{\partial h^1}{\partial c_j^1}\frac{\partial c_j^1}{\partial R^{1\,ODC}} \tag{A.3}$$

Hence, we obtain a relation between the marginal utility of income in the first and the second period:

$$MUI^{1\,ODC} = MUI^{2\,ODC}\frac{\partial R_j^{2\,ODC}}{\partial X_j}\frac{\partial c_j^1}{\partial R^{1\,ODC}} \tag{A.4}$$

and we can generally write that:

$$\frac{\frac{\partial \omega_j^1}{\partial \hat{B}}}{\frac{\partial \omega_j^2}{\partial \hat{B}}} = \frac{\frac{\partial R^{1\,ODC}}{\partial \hat{B}}}{\frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}}\frac{\partial R_j^{2\,ODC}}{\partial X_j}\frac{\partial c_j^1}{\partial R^{1\,ODC}} = (-1) \tag{A.5}$$

The effect of the deficit ceiling on the resources in the first period is straightforward:

$$\frac{\partial R^{1\,ODC}}{\partial \hat{B}} = 1 \tag{A.6}$$

and we can write that:

$$\frac{\partial c_j^1}{\partial R^{1\,ODC}} = \frac{\partial c_j^1}{\partial \hat{B}}$$

We can also decompose the effects on resource constraint $R_j^{2\,ODC}$:

$$\frac{\partial R_j^{2\,ODC}}{\partial X_j} = \frac{\partial R_j^{2\,ODC}}{\partial \theta}\theta'\frac{\partial X}{\partial X_j} \tag{A.7}$$

and

$$\frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}(\hat{B}) = \frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}(\theta, \hat{B}) + \frac{\partial R_j^{2}}{\partial \theta}\theta'\frac{\partial X}{\partial \hat{B}} \tag{A.8}$$

Also notice that regardless of the centralization of fiscal policy we have that:

$$\frac{\partial R_j^{2\,ODC}}{\partial \theta}(\theta, \hat{B}) = -\frac{\partial R_j^{2\,ODC}}{\partial \hat{B}}(\theta, \hat{B}) \tag{A.9}$$

Even though that the personal resource constraints $R_j^{2\,SO\,ODC}$, $R_j^{2\,DO\,ODC}$, $R_j^{2\,CPS\,ODC}$, $R_j^{2\,CT\,ODC}$ and $R_j^{2\,CO\,ODC}$ are different in the term $c^2_{-j}$, the equilibrium value of $c^2_{-j}$ in the second period is always a function of the aggregate resource constraint $R^2$. The aggregate resource constraint $R^2$ is the same in all the fiscal environments and we clearly have that:

$$\frac{\partial R^2}{\partial \theta}(\theta, \hat{B}) = -\frac{\partial R^2}{\partial \hat{B}}(\theta, \hat{B})$$

As a consequence, the equation (A.9) holds for any form of fiscal centralization in the model. (It can be shown for all the forms of centralization as we showed in Section 4.2) The equation (A.5) can be simplified by using the equations (A.6), (A.7), (A.8) and (A.9) to a general condition. Given any fiscal centralization the optimal deficit ceiling satisfies:

$$\theta'(X)\left(\frac{\partial X}{\partial X_j}\frac{\partial c_j^1}{\partial \hat{B}} + \frac{\partial X}{\partial \hat{B}}\right) = 1 \tag{A.10}$$

Unfortunately, to be able to compare the deficit ceilings directly from the condition A.10 we would need to know the relationship between the equilibrium functions $X_j^{ODC}(\hat{B})$ and between the equilibrium functions $c_j^{1\,DO\,ODC}(\hat{B})$ for each fiscal framework. However, these are due to the general form of the utility functions $h$ and $W$ unknown and goes beyond the scope of this thesis.

### Optimal deficit ceiling for quasi-linear preferences

To compare the deficit ceilings, we can change our initial assumption that the utility from consumption in the first and the second period has the same functional form and we can assume quasi-linear preferences. Specifically, if we assume that $h(c_j^1) = log(c_j^1)$, $h(c_j^2) = c_j^2$, $W = log(1 - T^2)$ and $\theta(X) = log(X)$ then the model can be quickly solved with an explicit solution:

$$\hat{B}^{SO} = \hat{B}^{DO} = \hat{B}^{CPS} = \hat{B}^{CT} = \hat{B}^{CO} = n + 1 - T^1 - \gamma \tag{A.11}$$

The level of optimal deficit ceiling in all the forms of centralization of fiscal policy is the same as the socially optimal deficit $\hat{B}^{SO}$. In the special case of quasi-linear preferences, the centralization of fiscal policy does not change the requirements for the optimal deficit ceiling.