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**FACULTY OF SOCIAL SCIENCES**

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**Using CAPM for assessment of efficiency of  
managed portfolios-mutual funds**

*Bachelor thesis*

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## **Declaration of Authorship**

1. The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.
2. The author hereby declares that all the sources and literature used have been properly cited.
3. The author hereby declares that the thesis has not been used to obtain a different or the same degree.

Prague, May 6, 2019

**David Pergl**

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## **Abstract**

This bachelor thesis tested hypothesis if 30 randomly selected equity funds outperformed the market systematically in the time period 2003-2018. Funds were divided into two groups with respect to their investment strategies (Small caps and Large caps) and were tested in periods of Bull and Bear markets. As a theoretical concept the Capital Asset Pricing model (CAPM) was used. Two parameters of its equation were tested, alpha coefficient as an indicator of managers' skills and fund expenses and beta coefficient as an indicator of level of risk. The CAPM equation was expanded by dummy variables to measure the effects of different investment strategies and market conditions. The thesis used panel data analysis as an approach of estimation of the parameters with Fixed and Random Effects models. Funds invested mainly on the U.S. market. Their prices were transformed to fund returns as required by the CAPM model and compared with returns of S&P500. Statistically significant results confirmed that the CAPM fitted the expected relationship of market and fund returns. It showed that the funds taking higher risk were rewarded by higher expected returns expressed by beta greater than 1. It also showed that the managers invested more carefully in the periods of Bear market. Values of alphas revealed that Large cap fund managers invested more efficiently and cheaper than Small cap managers. It was recommended to focus on the selection of funds and fulfilment of all statistical assumptions in further analysis.

**Keywords** performance of mutual funds, Capital Asset Pricing model, Large cap funds, Small cap funds, Bull market, Bear market, Fixed and Random Effects model

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## Abstrakt

Tato bakalářská práce testuje hypotézu, zda 30 náhodně vybraných akciových fondů překonalo systematicky trh ve sledovaném období 2003-2018. Fondy byly rozděleny do dvou skupin podle investiční strategie a zkoumány v obdobích poklesu a růstu. Teoretickým konceptem práce je Capital Asset Pricing model (CAPM), kde byl zkoumán koeficient alfa, jako ukazatel umění manažera a nákladů spojených s fondem a koeficient beta, jako ukazatel míry rizika. Klasická rovnice CAPM byla rozšířena o dummy proměnné k měření efektů spojených s různou investiční strategií a tržními podmínkami. Práce používá metodu panelových dat k odhadu parametrů rovnice prostřednictvím Pevných a Náhodných efektů. Fondy investují hlavně na americkém trhu. Jejich ceny byly přetransformovány na výnosy, jak to vyžaduje CAPM model, a srovnány s výnosy indexu S&P500. Statisticky významné výsledky potvrzují, že se koncept CAPM hodí k vyjádření vztahu mezi výnosy trhu a fondů. Výsledky ukazují, že fondy, které přijímají vyšší riziko, jsou odměněny vyšším očekávaným výnosem vyjádřeným koeficientem beta větším než jedna. Práce také ukázala, že manažeři investovali opatrněji v obdobích poklesu. Hodnoty koeficientů alfa prozradily, že manažeři fondů Large cap investovali efektivněji než manažeři Small cap. Pro další analýzu bylo doporučeno se zaměřit na výběr podílových fondů a na splnění všech ekonometrických předpokladů.

### Klíčová slova

výkonost podílových fondů, Capital Asset Pricing model, Large cap fondy, Small cap fondy, období poklesu, období růstu, model fixních a náhodných efektů

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# Bachelor Thesis Proposal

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Author: David Pergl

Supervisor: PhDr. Petr Gapko Ph.D.

Proposed topic: Using CAPM for assessment of efficiency of managed portfolios-mutual funds

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## **Preliminary scope of work:**

### ***Research question and motivation***

The research question of the thesis is to explore is the comparison of performance of actively managed portfolios with the performance of their benchmarks. In particular, we will compare the returns of mutual funds with the returns of market indices or other indices, which the funds' portfolio manager chose as a benchmark. The popularity of mutual funds is growing as the economic situation is positive and the population is able to execute more investments due to growing savings. At the same time the number opportunities to invest directly to the whole markets or classes of assets, which in some cases may be much cheaper, is growing as well. With the thesis we will answer a question whether it is worth to invest into actively managed mutual funds by statistically proving whether the actively managed portfolios are systematically able to outperform their benchmarks. Additionally, we will be interested in comparing the results for different phases of the economic cycle. There is plenty of literature dealing with comparison of performance of mutual funds and ETF's or market benchmarks (e.g. Measuring Performance of Exchange Traded Funds (2013), Marlène Hassine and Thierry Roncalli)

### ***Contribution***

We will test the hypothesis that actively managed funds are systematically able to outperform their benchmarks.

The outcome of the thesis will be interesting particularly when compared to the results of other researches, mainly because of different data sets used. Also, the comparison of the performance in different phases of economic cycle might shed additional light to the question whether it is really worth to invest into actively managed portfolios.

### ***Methodology***

The basic concept of the thesis will be the CAPM model, which was and still is commonly used to measure potential systematic premiums. The CAPM model will be constructed in such a way that it will

be estimable by the OLS regression. In order to perform a correct estimation, we will need to test all OLS assumptions as well as the stability of the dataset.

***Outline***

Abstract

Literature and mutual funds overview

Methodology description

Empirical statistical analysis of the actively managed mutual funds performance with respect to their benchmark

Results (including comparison to findings of other researchers)

Conclusion

References

**List of academic literature:**

***Bibliography***

Amenc N. and Martellini L. (2002), The Brave New World of Hedge Fund Indices, EDHEC-Risk Working paper.

William F. Sharpe and Gordon J. Alexander (1995), Investments, Prentice Hall, Englewood Cliffs, New Jersey

Edwin J. Elton and Martin J. Gruber, Modern portfolio theory and investment analysis (1991), John Wiley and Sons

Janette Rutterford (1993), Introduction to stock exchange investment, The Macmillan press LTD

Marlène Hassine and Thierry Roncalli(2013), Measuring Performance of Exchange Traded Funds

Eugene F. Fama and Kenneth R. French(2004), The Capital Asset Pricing Model: Theory and Evidence

Sharpe, William F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk". Journal of Finance. 19 (3): 425–442.

Markowitz, Harry . (1999). "The early history of portfolio theory: 1600–1960". Financial Analysts Journal. 55 (4).

Lintner, John (1965). "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets". Review of Economics and Statistics. 47 (1): 13–37.

Treynor, Jack L. (1962). Toward a Theory of Market Value of Risky Assets.

# 1. Introduction

Many sources state that in the long-run it is the most profitable to invest in stocks in comparison with other securities. Pozen and Hamacher (2015) suggest that the U.S. stock market has yielded a total return of 8% per year on average over the last 100 years. It seems as a good opportunity for investors where to expand their wealth in comparison with current low level of interest rates.

Nevertheless, if we check the historical chart of one of the most popular market indices S&P 500 (*see Figure 1*), the index exhibits high volatility and even periods with declining trend. It is possible to trace events like world wars, financial crises, oil prices shocks and terrorist attacks there. These events had an impact on the periods of Bull and Bear markets.

The profit on the equity market is reward for its high volatility, in other words for high risk. The relationship of higher expected reward for taking higher risk is the cornerstone of the Capital Asset Pricing model that will be used in this thesis. We should stress the word expected in statistical meaning and probably to remind that it must be the period of sufficient length.

The easiest way for non-professional investors how to get access to the market is to buy some mutual fund. Their experienced managers invest on behalf of all fund holders. They try to be better than others to attract more investors and to increase their own reward, which is dependent on the volume of the fund. The most natural benchmark is, however, the market itself. They sell their ability to outperform their benchmarks and make higher profits for holders of their fund.

Fund managers have different tools how to outperform the benchmarks. They use different types of strategic and tactical asset allocation. However, the study (Millard and Power, 2004) shows that almost 80% of the price movements of stocks are caused by the movements of the whole market. This fact implies that: if the market rises prices of all equities in the fund will probably rise and vice versa, if the market declines mutual fund's assets value will decrease too. Giving that, the target of the managers is to try to increase the value of managed portfolio more than the market during periods of boom and to suffer lower losses of the portfolio value than the market during recessions.

*Figure 1: Description of S&P 500-time development*



Note: Figure 1 includes month-end closing values, source: Macrotrends.net. (2019)

There are fund managers able to outperform the market. They are on the list of Morningstar and other journals. But the question is if they are able to outperform their benchmarks in the long-run. Active fund management usually means making use of deviations of the fund portfolio structure from the benchmark. It is costly. Would not it be more efficient to invest directly to the market and thus always to keep its structure? There are so called passively managed funds in the form of index funds or ETFs available on the market.

We decided to test whether the random selection of actively managed funds was able to outperform their benchmark systematically. Therefore, we selected pure equity funds and compared their performance with market index over last fifteen years. We wanted to test our hypothesis that active management really pays.

Performance of fund managers was studied many times making use of different tools. It is very attractive topic for researches as it relates to the huge sums of money that ordinary people put to mutual funds in hope to earn reasonable margin. We will compare the approach of some of them with our findings.

The research shows that the equity mutual funds cover 53% of the whole mutual funds' world. To test our hypothesis, we also decided to use the observations of equity mutual funds. To simplify the comparison, we chose mutual funds, which invest mainly on the U.S. market. This market is the biggest one and the most developed in the world.

There are many funds that invest on the U.S. market only. Selection of these helped me to avoid the problem with exchange rates and calculation of Net asset value.

There are many categories of equity funds. We selected Small Cap and Large Cap funds from the family of the Growth funds. These two types differ by their investment strategies to invest in large matured companies and in smaller developing companies. We wanted to compare the performance in periods of booms and recessions, too.

Capital Asset Pricing Model suites very well for testing of our hypothesis. The CAPM describes the relationship between systematic risk and expected return for assets, particularly stocks. CAPM is widely used throughout finance for pricing risky securities and generating expected returns for assets given the risk of those assets and cost of capital. (Sharpe, Alexander and Bailey, 1995)

Capital Asset Pricing Model provides a theoretically-based single index model, which is defined as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i,$$

where  $R_i$  represents return of i-th security and  $R_m$  is return of the market.

As usual, all economic models are based on some assumptions. We discussed them in detail in chapter *Literature review*. The basic assumption of the CAPM regards the investor's behaviour and the set of equilibrium conditions that allows to predict the return of assets for a given level of risk.

These assumptions are the most contested objects of the model, because some of them are very difficult to fulfil in the real world like one-period investment horizon. All CAPM's assumptions were studied by academics a lot and they came with ideas how to move them closer to reality. Using the CAPM allows me to compare my results with other researchers.

We have used a panel data structure to form our data set. Given this data structure, we have estimated parameters of the CAPM using Fixed and Random Effects models. These statistical models also require fulfilment of their econometrics assumptions for validity of the estimates. Mainly the problem of autocorrelation present in time series may question plausibility of results. We used transformation of data to avoid this danger. To be able to distinguish the impact of a fund group and to separate the influence of Bull and Bear markets we used a set of dummy variables. All models'

remaining assumptions were also tested and evaluated to ensure that estimated values are statistically significant.

The structure of the thesis is as follows:

Chapter 2 provides a brief review of literature covering the topics outlined above. The first section is focused on the development of the CAPM. The second section describes empirical and theoretical tests of the validity of this model. The last part is devoted to other studies on funds' performance that used CAPM for examination.

Chapter 3 brings brief discussion on description of mutual fund concept and their categories by investment strategies and asset selection.

Chapter 4 is devoted to the methodology and data quality. It includes description of CAPM, Fixed and Random Effects models, way of data collection, and criteria used for specification of periods of Bull and Bear markets. This chapter also includes a brief evaluation of all econometrics assumptions.

The last chapters are reserved for presentation of results and my opinions and conclusions.



## **2. Literature review**

In this chapter, we are going to provide brief overview of literature related to the CAPM. The section 2.1. is focused on the historical development of the CAPM, some key studies are mentioned as well. The section 2.2. provides the empirical and historical tests of the model and an examination of the model's assumptions. Last section 2.3. describes the examination of mutual funds' performance by the CAPM.

### **2.1 Historical development of the CAPM**

Since the period when the mutual funds had become the most popular financial instrument in the countries across the whole world, especially in the U.S. academics started thinking about the best choice of the portfolio with an efficient relationship between the risk and the return.

The basic elements of the modern portfolio theory were proposed by Dr. Harry M. Markowitz in 1952. The central idea is that the rational investors will prefer a portfolio with minimum expected deviation of returns around the mean. So the risk can be measured by the standard deviation (SD) of returns around the mean. Having two portfolios with the same expected return rational investors would prefer that with lower standard deviation. On the basis of this assumption Harry Markowitz came with the first justification for diversifying and selecting an efficient portfolio in his study Portfolio selection. This was one of the first studies, which described behaviour of investor's selection of the portfolio.

Markowitz suggested that the process of portfolio selection may be divided into two stages. The first stage devotes to the observations and experiences which would end with beliefs about the future performances of available securities. The second stage of the portfolio choice comes from the belief about portfolio future performances. These two stages enable the investor to decide which combinations of the securities give him the best return for the risk involved. This fact means that each investor will end up with a portfolio called efficient. Rutterford (1993) defined the efficient portfolio as a portfolio, which has the highest expected return at a given level of risk.

Markowitz understood that the risk-return trade-off of the investments could be improved by the risk diversification. Rutterford (1993) suggested that risk diversification of the portfolio is defined as the process of allocating capital to the securities in a portfolio in a way that securities are not positively correlated. Generally, it means that

the more diversified a portfolio is in the sense of weak relation of securities the better risk-return relationship is obtained.

The risk of the portfolio is influenced by the number of securities and correlation of their returns. For example, correlation can be decreased by the selection of suitable industries. This kind of intentional diversification was later called Markowitz diversification.

Another type of diversification is called Naïve. Its key concept is based on strategy of investments into the randomly selected securities of the same amounts of capital. Rutteford (1993) said that Naïve diversification would reduce risk substantially, but will create a portfolio with more securities and a worse risk-return ratio than the more rigorous Markowitz diversification.

These theories established one of the pillars of financial economics. Markowitz's works and studies present central part of the CAPM. Markowitz also influenced later studies of his fellow William Sharpe, one of the founders of the CAPM theory by mean-variance efficiency model in which risk-averse investors with one period horizon care only about expected returns and risk. William Sharpe used this model to develop his part of the CAPM in 1964.

Another researcher, who followed Markowitz's ideas, was James Tobin. Tobin (1958) added a concept of the capital market line and efficient frontier. He suggested that investors, regardless their risk tolerance, will hold stock portfolio in the same proportion so long as they hold identical expectations regarding the future. The consequence of Tobin's idea is that investor's portfolios will differ only in their relative proportions of stocks and bonds.

In the middle of 60's 3 academics William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) independently developed a theoretical concept of market prices equilibrium model called the Capital Asset Pricing Model (CAPM). The CAPM should be called as the variant of the efficient market hypothesis that provides such an efficient allocation.

The term of efficient market hypothesis came from the study of Modigliani and Miller (1958), who defined efficient market hypothesis as the condition when prices of securities fully reflect all available information at the time. Sharpe, Litner and Mossin worked individually and developed the CAPM in a slightly different way. The discovery of the CAPM means an important evolutionary step in the theory of capital

market's equilibrium. It provides better valuation of securities as a function of systematic risk.

Sharpe suggested in his study in 1963 that the returns of securities, which are usually positively correlated, were only so related because of their common market response. Rutterford (1993) assumed that this led him to the thought that the expected return of any security could be expressed as a linear function of the expected return of the market as a whole. Sharpe (1964) improved the Tobin's idea of the efficient frontier and the capital market line concept and included them in the CAPM. He also assumed that each investor is a mean-variance portfolio selector. In other words, it means that investors share the same expectations about the returns, variances and covariances. This fact leads them to having the same inputs, i.e. exactly the same portfolio, which means that investors would hold the market portfolio. In 1965, John Lintner introduced his idea of using the CAPM from the perspective of a corporation issuing shares.

The Sharpe-Lintner CAPM theory changed the mean-variance model into a market-clearing asset-pricing model. In other words, all investors agreed on the risk-free rate of lending and borrowing and the distributions of returns. A year after, Jan Mosin (1966) specified quadratic utility functions in the context of the model.

Despite independent development of the model, the form of the CAPM is similar for all of them:

$$E(R_p) = R_F + \beta_p(E(R_m) - R_F)$$

where  $E(R_p)$  is the expected return of the portfolio,  $R_F$  is the return of the risk-free investment and  $E(R_m)$  is the return of the whole market

Over time, the discovered parts of the model started being questioned. This fact led to the testing of a validity of the CAPM. The first test was carried out in 1970.

The most questioned part of the CAPM are model's assumptions. Rutterford (1993) provides their overview:

- According to Markowitz the investors produce a portfolio to be on the efficiency frontier based on expected rate of return and standard deviation; they combine a risk-free asset with the set of effective portfolios on efficiency portfolio frontier
- All the investors have the same investment horizon-one period model rather than multi-period model

- All investors can lend and borrow for the same risk-free rate existing on the capital market
- There are no transaction costs
- There are no income taxes, investors are neutral regarding the form of income (dividend, capital gain,..)
- All the assets are infinitely divisible
- Investors have homogenous expectations, meaning that they have the same attitude with regard to the expected returns, standard deviations and covariances of securities
- There is no inflation
- All the investors are price takers
- Capital market is efficient, information is freely and instantly available to all investors
- The market of securities is perfect, there is perfect competition in a frictionless securities market, which is in equilibrium

Some assumptions seem to be unrealistic or far from the “real world”. For instance, no distorting factors as taxes and transactions costs. So, the validity of the model was tested empirically. The empirical tests are described in the following section.

## 2.2 Empirical and historical tests of the CAPM

At the first time, Jensen (1968) thought that the relationship between the expected return and systematic risk of a large sample of mutual funds can be adequately expressed in the form:

$$E(R_j) = \beta_j E(R_m)$$

However, the study presented by Lintner (1965) said that Jensen’s model does not provide a total description of the structure of security’s returns. They suggested that the missing variable alpha describes particular relationship: high-beta assets tend to have negative alpha and low-beta assets tend to have positive alpha. This idea was confirmed by Miller and Scholes in their study Rates of Return in Relation to Risk: A Re-examination of Some Recent Findings from 1972.

The next set of CAPM's tests were estimation the cross-sectional relationship between average return on assets and their betas over some period of time. The first academics, who examined this problem were Fischer Black. In 1972, Black developed his own CAPM later called after him Black's CAPM. He questioned the unrealistic assumption of unlimited risk-free borrowing and lending and created his version of the CAPM without this preposition. In the empirical tests he got the results that market portfolio was mean-variance efficient by allowing for unrestricted short sales of risky assets.

Black continued in this study considering a model with risk-free rate of lending but not borrowing and showed that the CAPM model does not require the existence of a risk-free assets and provided the fact that variable alpha has a zero covariance with the return of the market portfolio. In other words, it means that alpha represents added value of the manager (skills of manager). In conclusion of Black's work, he showed that we can still obtain a linear relationship between an asset return and its risk. This model is sometimes called zero-alpha CAPM.

Despite all of these findings and studies, the effort to test the CAPM empirically and theoretically continued. Researches and academics started analysing model's assumptions more and in greater detail, which stated a validity and functionality of the model.

Another academic after Black, Jensen and Scholes (1972), who doubted one of the model's assumptions was Robert Merton in 1973 in his study Optimum Consumption and Portfolio Rules in a Continuous-time Model. Merton concerned with the assumption of one-period investment horizon, which means that the investors invest just in one-period rather than in multi-period horizon. He refused this assumption and showed in number of examples that the CAPM behaves as an intertemporal model.

Breeden (1979) described an intertemporal choice as an economic term explaining how an individual's current decisions affect what options will become available in the future. Theoretically, by not consuming today, consumption levels could increase significantly in the future, and vice versa.

Merton's idea was later supported by Breeden (1979), who developed this idea of Intertemporal Capital Asset Pricing model and defined more precisely functionality and form of the model

Two years after John Merton's study two academics Elton and Gruber (1975) examined under which conditions the problem of multi-period investment horizon of the model would be reduced to a single period CAPM, where the effort of all individuals is to maximize single-period utility function. The set of conditions were described in their study "Finance as a dynamic process" as follows: "Consumers will act as if the one-period returns are not state dependent, the consumption opportunities are not dependent on the state and consumers' tastes are independent of the future events."

In the following years researches showed that the market beta does not cover the risk premium, e.g. Reinganum (1981) in his study.

More recent tests, both cross-sectional and time series, found that variables such as market value of equity ratio (MVE), the stock price to earnings (P/E), debt to equity ratio, and the book-to-market equity ratio (B/M) provide variables, which have an explanatory power beyond market beta. (Basu, 1983)

These studies confirmed the now-recognized empirical flaws in both the Sharpe–Lintner and the Black versions of the CAPM. Behaviourists interpret the results as an evidence of irrational pricing caused by investor overreaction. In these days, these variables have no clear role inside established asset pricing models and are considered as anomalies.

The development of the CAPM represented really long process, which haven't been finished yet. During this long period, many studies and tests about the CAPM were performed. Academics focused on the validity of the model, tested the assumptions and also tried to change its mathematic form. They used many ways e.g. omission of some model's assumptions, adding some proxy variables or changes in the sample spaces. However, even though these trials comprised a big progress in the research of the CAPM, they were criticized by other academics e.g. Solnik, Adler Dumas or Stulz. But the famous model's critique came in 1977. Richard Roll claimed that the CAPM can hold theoretically but it is difficult to test the model empirically since stock indices and other measures of the market are poor proxy variables for the CAPM. This critique became later known as the Roll's critique.

In conclusion, many academics tested the Capital Asset Pricing Model's assumptions. The fact that these assumptions underlying the CAPM are demanding and represent most criticized part of the model does not mean that they are inflexible. The period full of the technological advanced markets have tendency to operate as if these

assumptions are satisfied. Due to this fact, we decided not struggle with verifying all CAPM assumptions in this study.

The next section of this chapter is devoted to the other studies based on the same principle.

### **2.3 Examination of mutual funds' performance by the CAPM**

The core of this study was to examine the performance of mutual funds in relation to the market performance by means of the CAPM. Some academics and researchers tried to evaluate and summarize theoretical validity of the model but others focused on its practical use for the real stock markets. We provided the list of academics, who examined and evaluated performance of mutual funds making use of the CAPM concept e.g. Jensen (1968) and Malkiel (1995). Other studies are focused on suitable choice of benchmark. These secondary studies suggested that the actively managed mutual funds are not able to outperform various market benchmarks in the long-run.

One of the first practical use of the CAPM applied on the set of real data provided academic M. Jensen in 1968. With the help of the CAPM he tried to measure absolute performance of mutual funds. He used CAPM as a model that statistically measures fund's performance in relation to its benchmark. His model assumed that all investors are risk averse, have homogenous expectations and are able to choose among portfolios on the basis of their risk and return preference. He basically used following equilibrium model:

$$E(R_j) = R_f + \beta_j(R_M - R_f)$$

where  $E(R_j)$  is an expected return of portfolio j,  $R_f$  represents risk-free rate,  $\beta$  is a coefficient of the systematic risk,  $R_M$  is a return of the market

Jensen's model allowed additionally heterogeneous horizon investment period and continuous trading of securities. Therefore, he used the set of transformations and determined another model in this form:

$$R_{jt} - R_{Ft} = \alpha_j + \beta_j(R_{Mt} - R_{Ft}) + u_{jt}$$

where  $(R_{jt} - R_{Ft})$  represents the risk premium,  $(R_{Mt} - R_{Ft})$  is a risk premium of the market portfolio,  $u_{jt}$  is the random error term

The constant  $\alpha$  was marked as Jensen's alpha and has following characteristics. If alpha is positive, it is an indicator of the ability of the manager to generate better forecasts of security prices. On the other hand, negative alpha indicates poor security selection and generation of high expenses connected with the frequent trading.

For the purpose of his study, he used the data for 115 mutual funds from the time period 1945-1964 and as a benchmark he took S&P 500 index. He concluded that the funds in average earned 1.1% less than they should earn given the level of risk and the study of alpha showed that majority of funds had alpha smaller than zero. It means that funds were not able to produce returns high enough to cover or balance the fund expenses and management fees.

Another researcher, who examined mutual fund's performance, was R. Carlson (1970). Carlson studied the impact of different market indices, time periods of interest and fund type on the performance. He used the data for the period 1948-1967 and selected three different types of mutual funds (diversified stock funds, balanced funds and income funds) with different market indices. Carlson suggested that mutual funds should be grouped by investment objectives before examining their performance against the market.

Carlson also studied some potential determinants of mutual fund performance and found that past performance played small role in the prediction of future performance. He also found that net returns in the observed period were influenced neither by the fund size nor by the expense ratios and fund's performance was positively related to the amount of new cash available for investment purposes.

Academic S. Kon (1983) focused on the measurement of mutual fund manager's performance. He claimed that managers try to make use of market-timing and stock selection. The belief that managers are able to earn an above average forecasts of portfolio market stems from the idea that they will be able to adjust their portfolio risk level before market movements. But doing this the fund managers increase systematic risk above the portfolio target level. Then managers may earn an additional return by means of taking higher risk. S. Kon examined this idea on a data set from the period 1960-1976 with a sample of 37 mutual funds.

In another paper, two academics Lehmann and Modest researched whether the choice of a benchmark has some effect on the portfolio performance. To evaluate fund performance, they worked with an equation:



$$\tilde{R}_{pt} = \beta_{pt}\tilde{R}_{mt} + \varepsilon_{pt}$$

where the estimated beta represents 2 factors, i.e.: target sensitivities of the fund to the common factors and the deviation from the targeted sensitivities influenced by the fund manager at any given time. Lehmann and Modest (1987) added the second equation in the spirit of Jensen's previous work (Jensen, 1968) with the parameter alpha, which helped to indicate the manager's ability of stock selection:

$$E(R_{pt}) = \tilde{\alpha}_p + \hat{\beta}_{pt}\tilde{R}_{mt}$$

They explained alpha in this way. If the manager does not have any extra skills, the parameter alpha is negative or equal to zero, on the other hand if the manager is able to display superior skills, the alpha would be greater than zero.

In the second part of their study Lehmann and Modest created benchmark portfolio suitable for the CAPM and concluded that the choice of appropriate benchmark may significantly influence the fund performance. Thus, the crucial step of measuring performance of mutual funds is to choose appropriate benchmark.

The listed studies did not confirm that mutual funds are able to outperform their benchmarks. Taking into account the outcomes of these academic analyses there is a question why the reality is so different. Many people want to invest their money, but they are not able to do that on their own. Generally, they don't have needed knowledge, information, analytical skills or theoretical background. Therefore, they utilize the skills and knowledge of portfolio managers. Managers offer different investment strategies and aim to choose the most suitable securities and to create a profitable portfolio accordingly. These activities contributed to the creation of wide range of financial instruments.

Stock selection and portfolio management is not for free. Investment companies charge management fees that are higher for portfolios with more volatile assets. Management of these portfolios probably requires more analyses and higher portfolio turnover. All this increases costs.

One recent study shows that 86.7 percent of US managed funds underperformed their benchmark on a net-of-fees basis, over the 10-year period ending in 2017. (2019 Investment Management Outlook A mix of opportunity and challenge, 2019). Under the assumption that the market is efficient and the theoretical background of the CAPM is valid, the cheapest alternative to the managed investment portfolio would be to invest directly to the market and to save expenses on the portfolio manager

For that reason, a new investment opportunity exists, which offers to invest directly to the whole market. They are called Exchange Traded Funds. ETFs do not need any analyses of fund managers, because their portfolios just copy market indices. Deloitte's study shows that this investment strategy is more profitable in the long-run.

The aim of our study is to compare the returns of one of the best performing equity funds with the market index. However, the universe of mutual funds is very broad and we would like to structure fund types and division of them in the next paragraph.

### **3. Mutual funds**

In this chapter, the concept of mutual funds is explained. We try to provide some theoretical overview of mutual funds' form and explain the concept of different investment strategies.

Jeffrey M. Laderman (1997) defined a mutual fund as an investment company that pools the money of many individual investors. The principle of investment into funds is really easy. The investor comes with the money and puts this money into the certain mutual fund with an investment strategy he or she likes. The fund pays dividends or earnings based on the funds' strategy.

The concept of mutual funds represents one of the most popular investment opportunities around the world especially in the U.S. One study shows that over 90 million people own mutual fund in the USA (Investment Company Fact Book, 2018).

Each year the number of mutual funds increases around the world. The ease and efficiency make mutual funds perfect investment opportunity for long-term savings programs like retirement accounts. The investors have perfect overview of the actual value of their investment.

The price of a mutual funds' share is calculated every day as the value of total fund's assets after deduction of total fund's liabilities divided by the number of outstanding shares. This formula represents fund's net asset value, commonly labelled as NAV. The assets of the fund are valued on the basis of market closing prices of all securities in their portfolio.

Another important feature of mutual funds is the label "open-end", which refers to the funds' liquidity. Jeffrey M. Laderman (1997) claimed that this liquidity equals to the

ability investors to get in and out of the fund easily and at little or no cost, is one of the most important features that the funds bring to the financial system. Research showed that open-end funds covers c. 83% of total net assets of US-registered companies on the contrary to closed-end funds, which covers only around 2% (Investment Company Fact Book, 2018). Therefore, we chose the group of Open-end funds.

By the way of portfolio management, mutual funds can be grouped into actively and passively managed. Active mutual funds implement active management approach with an effort to outperform a specific market index by buying and selling stocks in context of funds investment strategy. Passively managed index funds or ETFs try to copy in its portfolio the structure of the whole market as much as possible.

An active fund manager decides which stocks should be bought and sold and tries to beat the market by deviations from the market structure. These activities makes active funds more expensive than passive funds.

All investors have different expectation and perspective about risk, returns, type of funds and basically in two words investment strategy. The main advantage of active mutual funds is that an investor can chose from a great number of funds with different investment strategies. The investment strategy is described in the fund's status. It is possible to invest in different mixtures of equities and bonds in so called mixed funds. He or she can invest even in so called green funds that invest in environmentally clean companies only. Therefore, actively managed equity funds with different investment strategies can satisfy many investor's requirements.

We chose for this study the equity funds. Equity funds belongs to the most popular and most traded mutual funds. The equity funds invest principally only in stocks. We selected randomly 30 funds from the list of Morning Star Journal. As a benchmark we chose index S&P 500.

There are 500 stocks in a portfolio of S&P 500. They are recognized U.S. companies with large and small capitalization created by Standard&Poor's. The main aim of the S&P 500 is to capture a large portion of the total value of the U.S. stock market, currently it is 80%. Thus, it serves as an economic indicator in its own right.

We chose two different groups of funds. The funds that invest in companies with large market capitalization (Large Cap) and the funds that invest in companies with small market capitalization (Small Cap). According to a definition Large cap funds refer to set

of companies with a market capitalization value of more than \$10 billion. Small cap funds invest in companies with market capitalization value between \$300 million and \$2 billion.

Large cap funds provide better capital appreciation over a long term and distribute dividends fairly regularly. They are also more stable in the Bear (falling) market. In the view of risk, they are less risky than Small caps. But there is a probability of underperforming market in the Bull (rising) market. According to the expectation of individuals Large cap funds are suitable for risk-averse investors, who want to invest in high-quality stocks and have long-term investment perspective.

Small cap funds are characteristic by investment in small-cap companies, generally firms with higher growth potential. Basically, these companies represent some start-ups. In the view of risk, these companies represent high risk investment, caused by greater volatility of prices and unstable market conditions. Therefore Small cap companies are able to significantly outperform market in the period of both recession and growth. These funds are ideal for investor with high risk preference, which means higher returns expectations.

The index S&P 500 covers both Large and Small caps. In the context of using the CAPM, we were able to formulate our expectation about values of beta coefficients. The fact that Large cap companies are more stable and exhibit lower risk should mean that the coefficient of beta would be lower than 1. In other words, the ability of outperforming benchmark would be lower than the ability of Small cap funds. On the other hand, beta coefficient of Small cap funds should be higher than 1.

## 4. Measurement of fund performance

### 4.1 Methodology

This chapter explains the way of estimation of CAPM parameters by means of panel data technique, introduction of dummy variables, and merging parameters of appropriate factors.

The key equation of the CAPM explains the returns of mutual funds by means of returns of the whole market. The model is a simple linear regression:

$$R_{pi} = \alpha + \beta_p R_{Mi} + \varepsilon_i$$

where

$R_{pi}$  represents the returns of p-th funds of portfolio in time i,

$R_{Mi}$  is the return of the market of i-th observation and

$\varepsilon_i$  is the error term of i-th observation

To be able to distinguish different groups of funds and the periods of Bull and Bear market we tried to get time series as long as possible. However, some of 30 funds in our sample did not have equally long history. But we did not want to lose information using just the shortest period and estimation of  $\alpha$  and  $\beta$  by simple OLS. So, we organized our data set in the form of panel data. Cross-sectional members are represented by 30 funds where the groups of Large cap and Small cap are distinguished. The time span ranges from 02.01.2004 to 04.01.2019. In this way, we were able to utilize in total 21 618 observations. Of course, the panel is unbalanced, because of missing observations for certain funds in certain time points. The data set is explained in detail in the *DATA section*.

Panel data methods use either Fixed Effects (FE) or Random Effects (RE) model for estimation of the parameters. The unbalanced panel causes no problem for either of them.

We supposed that the factors of different period and investment strategy would play significant role in the estimation of parameters. Making use of the panel data we expanded CAPM linear regression with dummy variables to be able to estimate the impact of different factors as follows:

$$Returns\_Fund_{it} = \beta_0 + \beta_1 Returns\_Benchmark_{it} + \beta_2 Dummy\_Decline_{it} + \beta_3 Dummy\_Largecap_{it} + \beta_4 Returns\_Benchmark * Dummy\_Decline_{it} + \beta_5 Returns\_Benchmark * Dummy\_Largecap_{it} + \epsilon_{it}$$

where  $Returns\_Fund_{it}$  represents our dependent variable of returns of i-th fund in time t and  $Returns\_Benchmark_{it}$  is our explanatory variable of returns of the market index S&P 500 (*more in DATA section*). We added in the equation 4 extra dummy variables:

$Dummy\_Decline_{it}$  is used to distinguish the Bear and the Bull market.

$Dummy\_Largecap_{it}$  separates Large and Small cap funds.

$Returns\_Benchmark * Dummy\_Decline_{it}$  is showing the difference in returns for period of decline

$Returns\_Benchmark * Dummy\_Largecap_{it}$  is showing the difference in returns for Large cap funds.

The last two artificial variables are called interaction terms as they are products of benchmark returns and dummies for period and fund type.

It is important to realize which coefficients belong to which parameter of the CAPM equation. When dummies are applied estimated parameters  $\hat{\beta}_2$  and  $\hat{\beta}_3$  become constants, which will shift market line intercept. So, these two coefficients belong to CAPM alpha. Coefficients  $\beta_4$  and  $\beta_5$  increase or decrease the increment of fund returns in relation to the benchmark returns so they should be associated with the value of CAPM beta.

The impact of factors can be expressed as different combinations of coefficients. They are summarized in the following table:

*Table 1: Calculation of Alpha and Beta coefficients*

<b>Period of Decline and Large cap funds</b>	
$\hat{\alpha} = \hat{\beta}_0 + \hat{\beta}_2 + \hat{\beta}_3$	$\hat{\beta} = \hat{\beta}_1 + \hat{\beta}_4 + \hat{\beta}_5$
<b>Period of Decline and Small cap funds</b>	
$\hat{\alpha} = \hat{\beta}_0 + \hat{\beta}_2$	$\hat{\beta} = \hat{\beta}_1 + \hat{\beta}_4$
<b>Period of Growth and Large cap funds</b>	
$\hat{\alpha} = \hat{\beta}_0 + \hat{\beta}_3$	$\hat{\beta} = \hat{\beta}_1 + \hat{\beta}_5$
<b>Period of Growth and Small cap funds</b>	
$\hat{\alpha} = \hat{\beta}_0$	$\hat{\beta} = \hat{\beta}_1$

Note: Table was created in the Excel file as an overview of calculation of Alpha and Beta coefficients.

The parameters can be estimated either by means of Fixed Effects (FE) or by Random Effects (RE) models. Random Effects model assumes that unobserved error  $a_i$  is uncorrelated with each explanatory variable. RE model is defined as:

$$y_{it} = \beta_1 x_{it1} + \dots + \beta_k x_{itk} + a_i + u_{it},$$

where  $Cov(x_{itj}, a_i) = 0$  for all  $t = 1, \dots, T$ , and  $i = 1, 2, \dots, k$

FE model eliminates the unobserved errors  $a_i$  by the subtraction of the averages from original observations.

Both models should fulfil all assumptions for the consistency of the results.

The first assumption regards linearity. It is fulfilled automatically, if the regression and CAPM model hold. The second assumption regards the sample of data, which should be random. Both returns of funds and returns of the index are random variables. As well as, the third assumption of no perfect linear relationship between explanatory variables and their variability in time is also fulfilled. It should be also ensured by right way of data set collection.

The assumptions 4 through 7 that regard random variables  $a_i$  and  $u_{it}$  have to be tested by the appropriate statistical tools. We used Breusch-Pagan test for heteroscedasticity. The Durbin-Watson statistic was used to detect the autocorrelation in the time series. The test for autocorrelation gave satisfactory result but heteroskedasticity and the normality of residuals showed problem, which will be discussed later.

The detail overview of assumptions is in the *Tables 9 and 10 (Appendix)*

In the empirical studies, many authors try to distinguish between the use of FE or RE model. They attempt to determine whether the unobserved error  $a_i$  is a parameter, which can be estimated or an outcome of a random variable. Taking  $a_i$  as a parameter is appropriate for cases, where the observations were not randomly collected from a large population. In these cases, it is better to think about  $a_i$  as a parameter. Then the Fixed Effect estimation is suitable.

We believe that in the case of our study, our model and with our transformed data there are no the unobserved stable part of the error term in the sense of FE consideration. On the other hand, if we decide to treat the  $a_i$  as random, we have to check whether the  $a_i$  is uncorrelated with the explanatory variables. In the case that this assumption is fulfilled, we should use RE model for estimation

Two tests can be used to select FE or RE model. They check whether the unobserved error  $a_i$  is uncorrelated with explanatory variables or if the observations have the same intercept.

The first test is known as Hausman test, which checks if the covariance between unobserved error  $a_i$  and variable  $x_{it}$  is zero. Under the null hypothesis of independence both FE and RE are consistent, but RE is asymptotically more efficient. If the null hypothesis is rejected using FE is recommended because FE is still consistent but RE is not.

The second Test checks possible intercept of the variables included in the model. If we cannot reject null hypothesis of having such intercept RE model would be more suitable.

Conclusions of the Hausman test and Test for different intercepts among groups imply that results of estimations by both methods will be similar.

As seen from the *Table 7*, Test for different intercepts among groups shows us that the use of FE model is not appropriate, which is supported by p-value 0.99. This fact indicates that more suitable model is RE. The most persuading evidence should provide Hausman test. *Table 8* shows that we are able to reject null hypothesis at the level of p-value 0.16. It indicates that under the null hypothesis both RE and FE methods are consistent, but the RE method is asymptotically more efficient. As seen from the analysis of the results, tests provide the evidence that RE model is more



suitable again. For that reasons, we took into account and commented results of RE model only.

## 4.2 Data

We collected randomly mutual funds from the list of funds in the web of Morning Star and asked Reuters for help with collection of data. They were kind in providing us with the time series of weekly unit prices of selected mutual funds and of weekly quotes of our benchmark S&P 500 in dollars.

The CAPM model used for our research describes relationship between returns of the market and returns of the respective mutual fund. The returns were derived from the weekly prices as follows:

$$r_t = \frac{(y_t - y_{t-1})}{y_{t-1}}$$

The mutual funds in the data set were chosen to satisfy our selection criteria:

Firstly, we were aware of possible bias in the weekly returns caused by exchange rates. Therefore, we selected mutual funds, which invest mainly (85-99%) in the U.S. market and are quoted in the U.S. dollars. So, we avoided otherwise necessary data recalculation.

Secondly, to be able to compare funds with significantly different investment strategies. We were interested in how big the differences may be.

Thirdly, we intended to compare the performance of funds in relation to the market in the periods of Bull and Bear market and to test our hypothesis about different relationships in these different market conditions. So, we needed time series of sufficient length to be able to explore these periods separately. And, of course, a big data set provides statistically more significant results of my measurements.

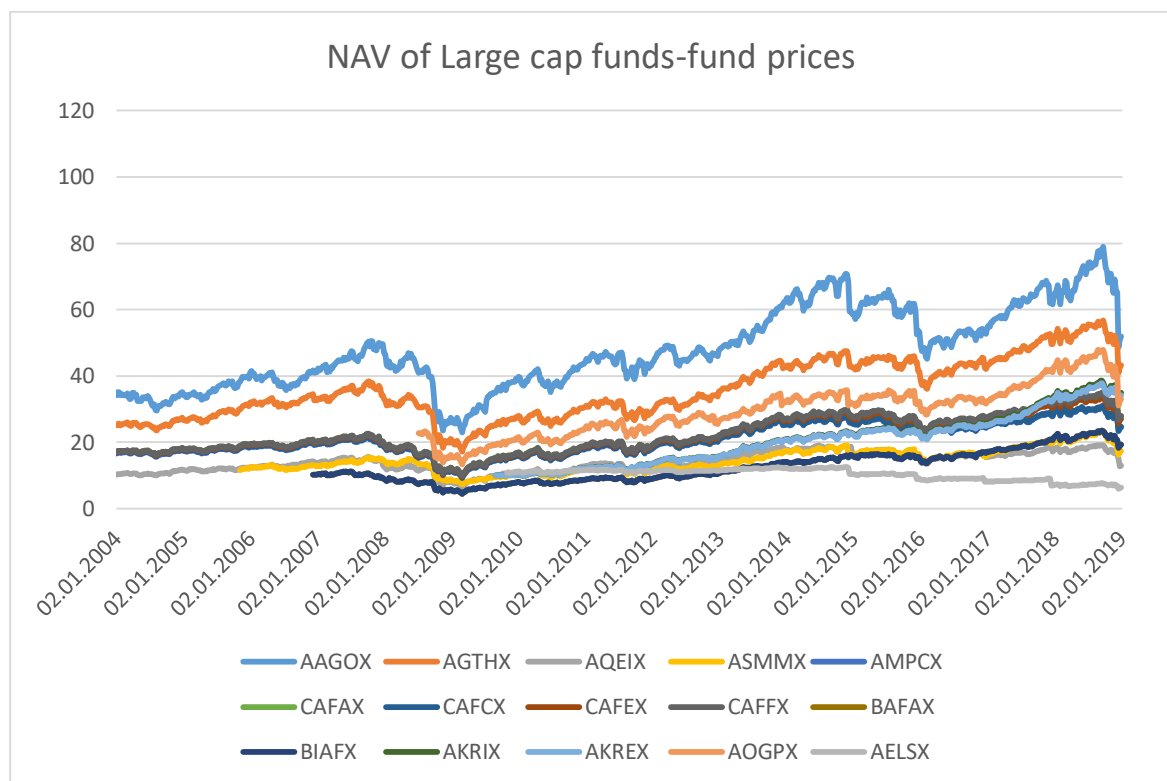
We picked the set of funds from the webpage of Morningstar.com/Funds category performance. It is very good source of information, where an investor can find an overview of mutual funds by types with tickers, names, basic information and returns for last years. We chose Growth funds, which has capital appreciation as its primary goal and their stocks are typical by higher potential to grow and develop. (Babson, 1951) From the three types of Growth funds Large cap, Mid-cap and Small cap we

selected two subcategories Large cap and Small cap. (Brief description of these investment strategies is in the chapter *Mutual funds*).

We picked for each subcategory 15 funds. Unfortunately, the history of their prices based on different dates of their registration was of different length.

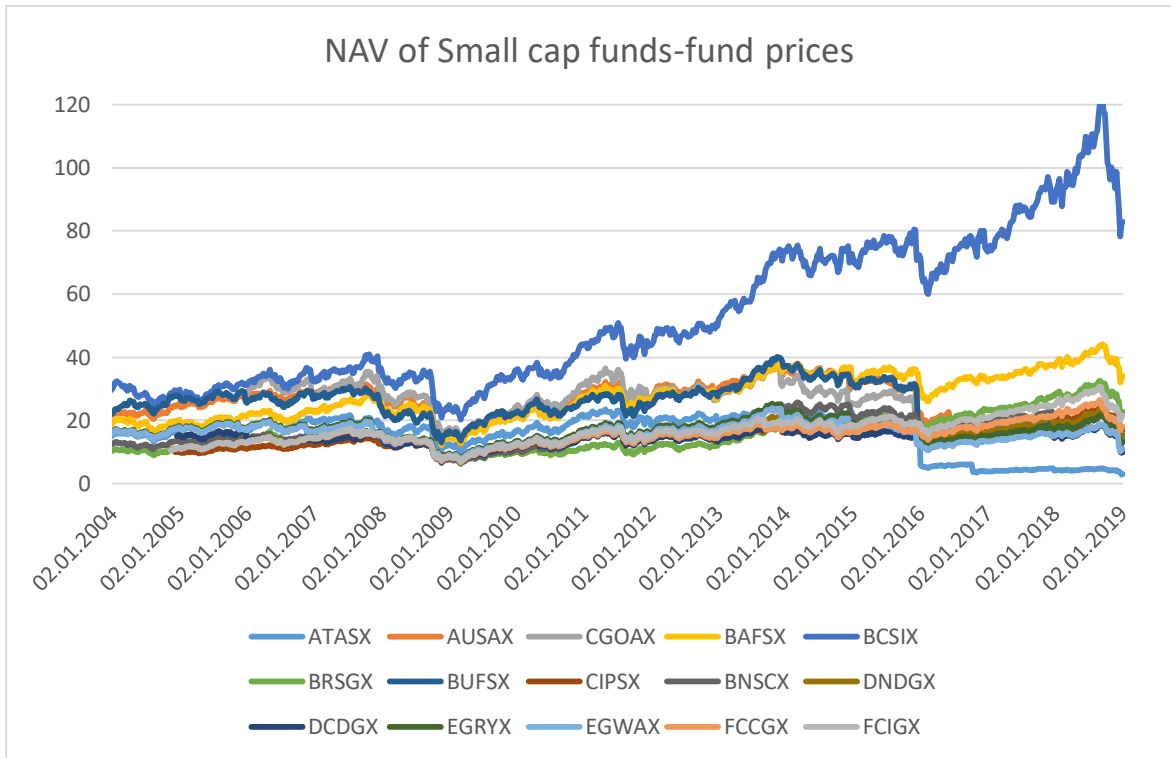
As seen from *Figures 2 and 3*, we can summarize some basic information about collected funds. Both groups of funds include funds which are more and less volatile. In addition, it is seen that each fund reacts to market changes in the same direction but with different volatility.

*Figure 2: NAV of Large cap funds-fund prices*



Note: Graph was created in the Excel file from the data set of Net asset values. The prices are in dollars on the vertical axis. The funds are labelled by their tickers.

Figure 3: NAV of Small cap funds-fund prices

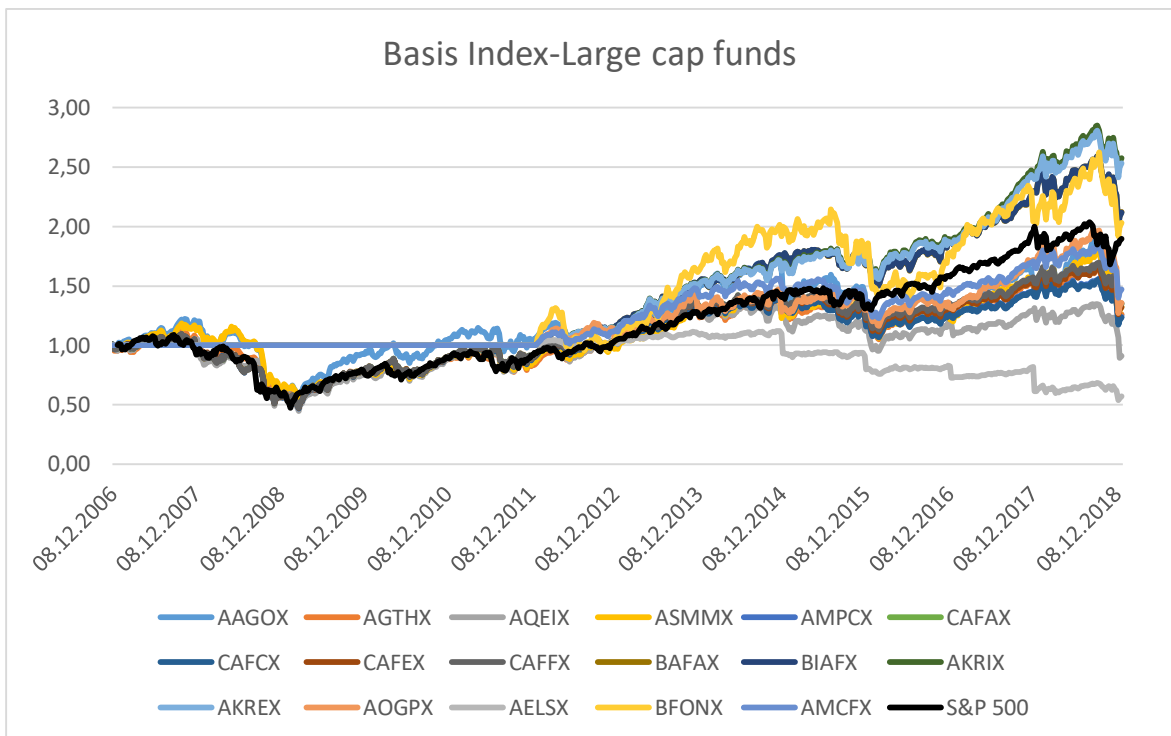


Note: Graph was created in the Excel file from the data set of Net asset values. The prices are in dollars on the vertical axis. The funds are labelled by their tickers.

Fund prices in the *Figures 2 and 3* do not have the same basis. Therefore, they cannot be compared. To be able to compare the performance of funds we used the same index basis for all of them. As seen from *Figures 4 and 5* some funds outperform others underperform the benchmark. It is visible that the range between the best performer and the worst loser and volatility of returns are bigger in the group of Small cap funds.

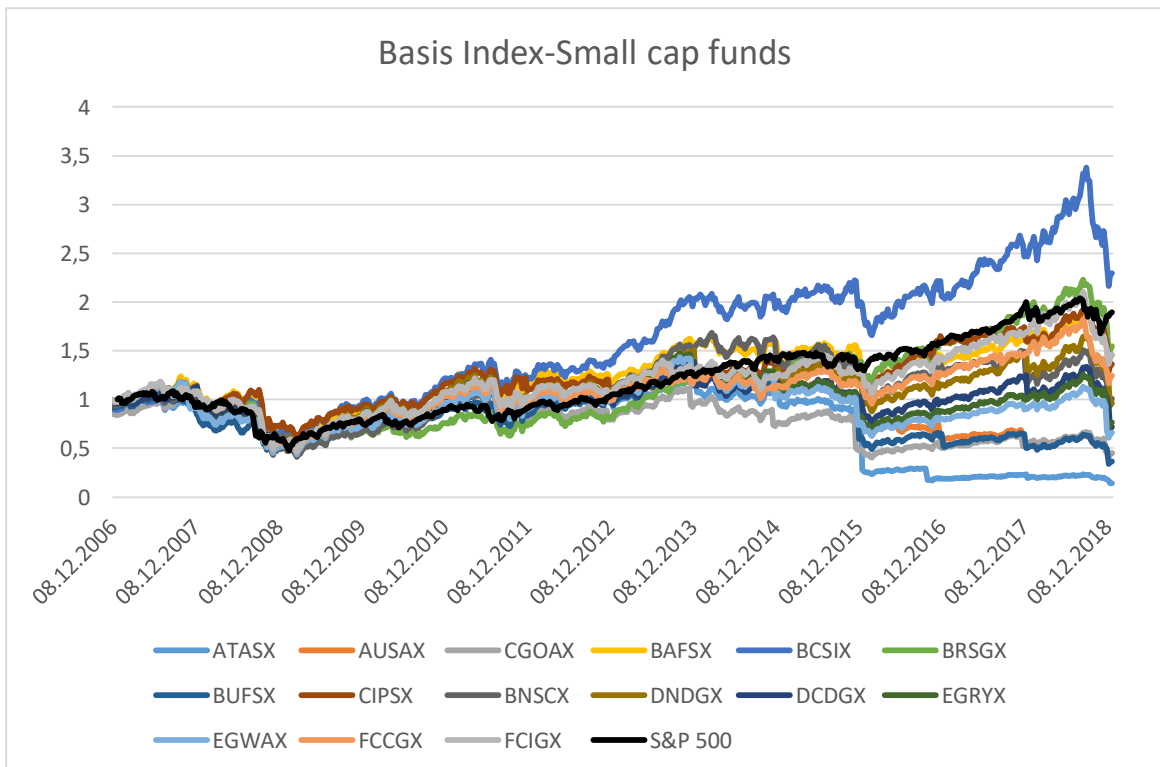
It is clear that the fund selection will have considerable impact on our results.

Figure 4: Basis Index-Large cap funds



Note: Graph was created in the Excel file from the data set of calculated index basis. The values of basis indices are on the vertical axis. The funds are labelled by their tickers.

Figure 5: Basis Index-Small cap funds

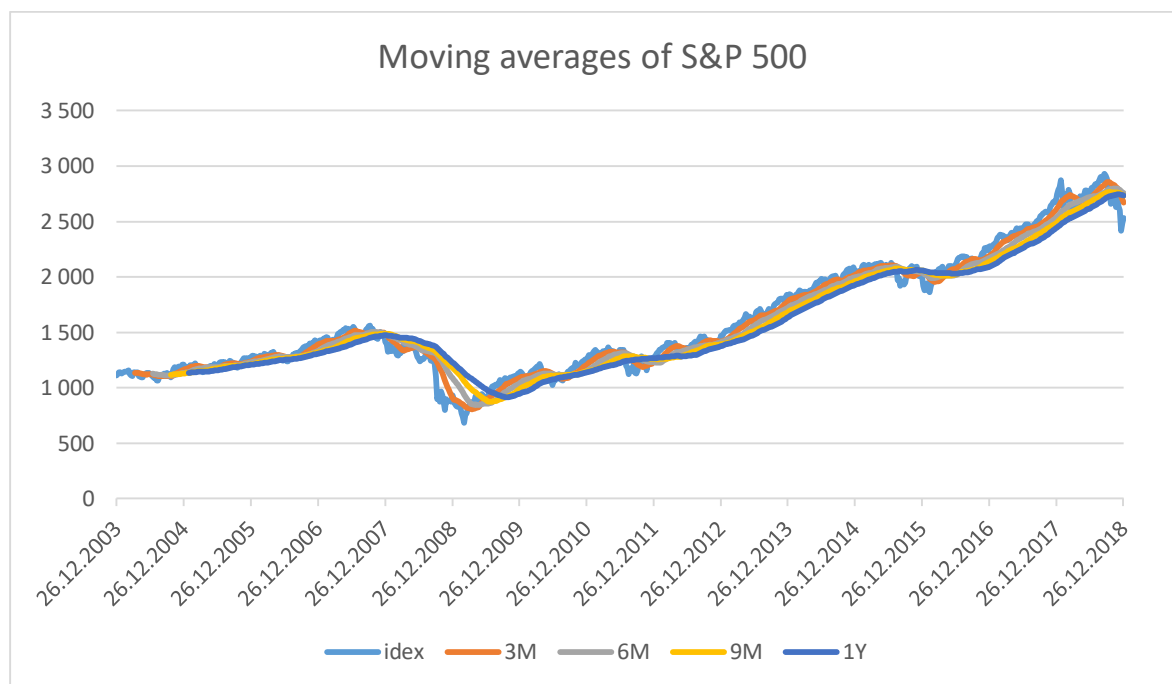


Note: Graph was created in the Excel file from the data set of calculated index basis. The values of basis indices are on the vertical axis. The funds are labelled by their tickers.

### Determination of Bull and Bear Markets:

As mentioned earlier, one part of our study was to compare performance of funds during different time periods. I was not able to identify Bull and Bear market periods in literature. Therefore, I used the method of moving averages that smoothed volatility and revealed the trends. We selected one-year moving average from the *Figure 6* for making decision. We assumed that if five values of smoothed data in a row were decreasing the period was showing the decline trend of the market or the Bear market. Otherwise, it determined the period of growth or the Bull market.

*Figure 6: Moving averages of S&P 500*



Note: Graph was created in the Excel file from calculated moving averages + S&P 500. The prices are in dollars on the vertical axis.

In this way we have separated three periods of Bear market and three periods of Bull market. It is encouraging for the investors in stocks that the periods of Bull market are much longer than the Bear market periods. Based on the *Table 2*, we can conclude that the variance of returns is higher in the periods of Bear market.

*Table 2: Variability of returns of S&P 500*

Bull market periods	12/2003 – 12/2007	11/2009 – 7/2015	7/2016 – 11/2018
Standard deviation	2.8%	3.6%	2.6%
Bear market periods	1/2008 – 10/2009	8/2015 – 6/2016	12/2018
Standard deviation	8.12%	3.8%	2.4%

Note: Table was created in the Excel file from periods of Bull and Bear markets. Standard deviation was calculated from returns of S&P 500 expressed in percent.

The basic characteristics of data set can be found in the following table:

*Table 3: Table of Descriptive statistics*

Variables	Min	5% Percentile	Mean	Median	95% Percentile	Interquartile range	Max	SD
Small cap funds	-31.54%	-5.27%	0.09%	0.36%	4.58%	3.34%	17.31%	3.28%
Large cap funds	-25.40%	-3.90%	0.10%	0.25%	3.42%	2.39%	16.50%	2.44%
Overall returns of funds	-31.54%	-4.57%	0.10%	0.30%	4.13%	2.88%	17.31%	2.92%
Returns of benchmark	-18.20%	-3.79%	0.13%	0.21%	3.22%	2.21%	12.03%	2.30%

Note: The table of descriptive statistics covers four variables and eight descriptive indicators. It is quoted in the percent as the returns. The values were generated by statistical software Gretl

Indicator of minimal and maximal value shows that the capital market can fall about 30% and rise about 18% in one week. The range of 5% percentile and 95% percentile indicates where 90% of weekly returns fall. It is interesting to compare these ranges for the Small cap funds (-5.27% – 4.58%), the Large cap funds (-3.90% - 3.42%) with the range for the benchmark where the returns fluctuate between (-3.79% – 3.22%). The same is confirmed by the values of standard deviation, of returns, which is the lowest for the benchmark, i.e. for the whole market. It can be explained by the fact that the Benchmark includes 500 companies including both Large caps and Small caps. We know that if we add more stocks to the portfolio its variance, i.e. the level of its risk will decrease, assumed that there is no perfect correlation with the added security.

The mean return denotes that Small cap funds generated in average the lowest mean return in comparison with the market and Large Caps. This may seem

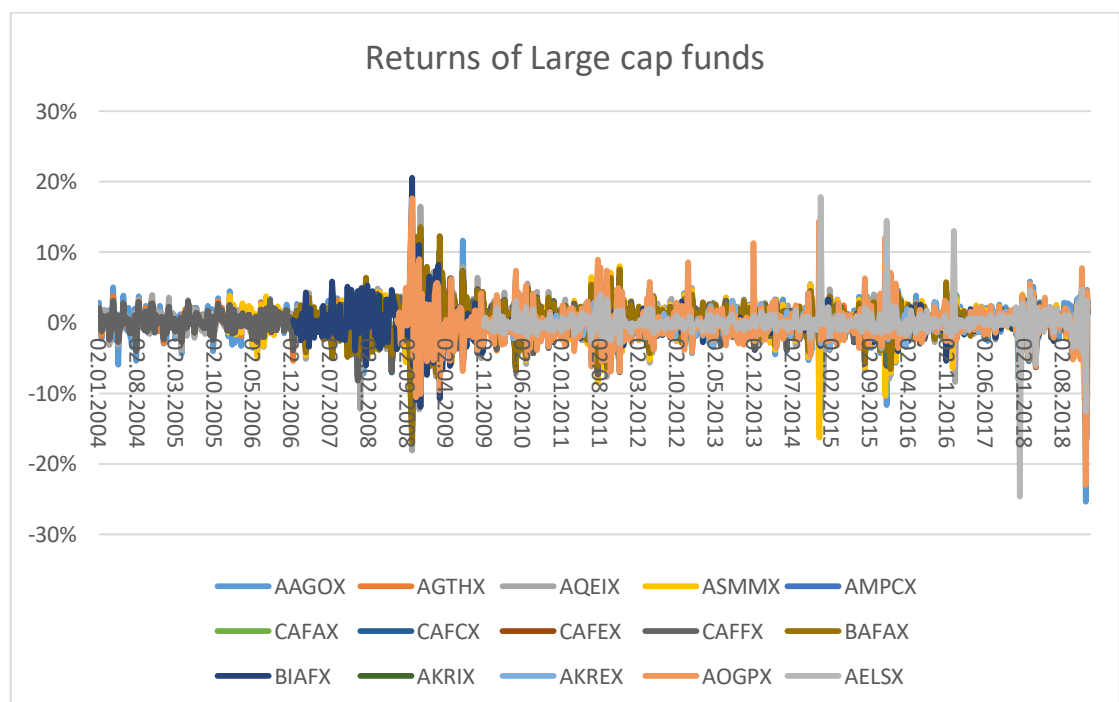
contradictory as people invest to the Small cap funds with the expectation of the higher returns as a reward for taking higher risk.

But we should take into consideration that Small cap funds are characteristic by higher fluctuation of returns with many negative values, which decreases the mean value. In this case the median is a better measure. Its value for Small Caps tells us that more than 50% of funds generate returns greater than 0.36%. It is the highest value as expected.

Last two indicators (Interquartile range and Standard deviation) support the idea that Small cap funds are riskier than the Large cap funds and Benchmark. Interquartile range tells us the fact that 50% of Small cap funds deviate about 3.34%, which is the highest value in the table.

Note: The usual problem of time series data is the autocorrelation, which can be detected in the *Figures 2 and 3*. However, instead of weekly prices, we used weekly returns. This transformation reduced the problem of autocorrelation. *Figure 7* depicts an example of time series of returns of Large cap funds. Values of their returns fluctuate around zero randomly without any visible trend. Measures of autocorrelation are discussed in detail in the section Results, where the Durbin-Watson test was applied.

*Figure 7: Returns of Large cap funds*



Note: Graph was created in the Excel file from calculated returns of Large cap funds. The returns are in on the vertical axis.

## 5. Results

As described in previous sections, the key objective was to analyse whether the managers are able to outperform the benchmark in the long-run. Extra factors, which should also influence the managers' performance were added. They are different investment strategies and different market conditions.

Just to summarize what has been discussed before: The CAPM equation formulates dependence of portfolio returns on the market returns in a simple way as

$$R_{pi} = \alpha + \beta_p R_{Mi} + \varepsilon_i,$$

Alpha and Beta coefficients in the CAPM have two different meanings.

Beta measures volatility of portfolio returns against volatility of the market.

Values of beta greater than one are typical for portfolios that take on higher risk than the market. Values of beta lower than one are typical for less risky portfolios.

Alpha measures if the return of portfolio is higher or lower than the expected return of the market adjusted for portfolio risk Beta. It will be typical for mutual funds that their Alpha is negative. The reason is the cost of fund management and the limited skills of the fund manager. (Remember the Jensen's study.) Alpha is typically higher for actively managed funds. Funds with high turnover might increase their returns but they would lose some part of their performance in higher trading costs.

Available data gave us an opportunity to look at the basic relationship from two different perspectives

Investment strategy:

The first perspective regards the risk. As mentioned in the *chapter Mutual funds*, the funds differ by the level of accepted risk. Consequently, we expected that the value of estimated coefficient  $\beta$  would be lower than one in the case of Large cap funds as share prices of Large cap companies are less volatile than the market. So, these funds take on lower risk compared to the whole market. On the other hand, the Small cap funds prefer investments to the more volatile shares of developing companies with higher expected return. This fact implies higher volatility and higher level of risk compared to the whole market. So, we expected that estimates of  $\beta$  would be higher than one.

Different market conditions:

There are periods of Bull market with growing trend of prices and periods of Bear market with the reverse trend. We guessed that in periods of Bear markets volatility of



prices would be generally higher than in periods of Bull markets. This assumption is supported by the comparison of standard deviation of market returns (*see DATA section, Table 2*).

As for the coefficients  $\beta$ , they measure the risk of funds vs. market risk. We expected that fund managers would be more cautious when the market falls. If it was true the values of  $\beta$ s should be lower in the Bear market periods than in the Bull market periods. (This does not mean that volatility of their prices should be lower.)

As for the coefficients  $\alpha$ , Jensen (1968) claimed that the coefficient alpha indicates how well an investment performed compared to its benchmark. Jensen tagged a negative alpha as an indicator of poor security selection and generator of high expenses connected with the frequent trading. However, in case of mutual funds, the investors pay voluntarily for portfolio management of their fund and these costs are deducted in calculation of Net asset values. We found the values of Total expense ratio (TER) as high as to 2.5%. Therefore, it can be expected that fund performance will be lower than the market potential by these costs. The rest of the difference measures the ability of the manager to manage his fund portfolio.

We have discussed the use of Fixed effects and Random effects models for estimation of parameters in the *Section 4.1*.

It is known, that the regressions with dummy variables are generally estimated by RE model, because the FE model may eliminate some of them. It would cause a problem with estimation of some parameters included in our model. As the assumptions for RE model were not violated, we opted for this model. However, the outcomes of both models are very similar. (Estimations of RE and FE panel data models are reported in *Tables 7 and 8*, respectively.)

Random Effects model used 21 618 observations in total, which included 30 cross-sectional units. The length of time series ranged from 718 to 724 observations. RE model estimated the linear regression as follows:

$$\begin{aligned}
 & Returns\_Funds_{ti} \\
 & = -0.00068 + 1.16297Returns\_Benchmark_{ti} \\
 & - 0.00070Dummy\_Decline_{ti} + 0.00042Dummy\_Largecap_{ti} \\
 & - 0.03905\beta_4Returns\_Benchmark * Dummy\_Decline_{ti} \\
 & - 0.18637Returns\_Benchmark * Dummy\_Largecap_{ti}
 \end{aligned}$$

Discussion on Alpha values:

With costs of the funds, i.e. with negative alphas, the funds were not able to produce returns high enough to cover or to balance funds expenses and management fees. The skills of the fund manager are measured by the difference between the value of alpha and the declared fund expenses (TER).

The calculation of final values **of alphas** is based on the summations for different combinations of factors in the *Table 1*. Due to this calculation, we can summarize the results as follows:

*Table 4: Weekly base Alphas*

	Large cap	Small cap	Both strategies
Decline	-0.10% ***	-0.14%***	-0.12%***
Growth	-0.03%***	-0.07%***	-0.05%***
Full length	-0.03%***	-0.08%***	-0.06%***

Note: Table was created in the Excel file from estimated values of parameter Alpha calculated in Gretl.

The values of estimated coefficients express weekly returns. We can estimate alpha returns on the one-year basis by multiplying those for weekly by 52, i.e. by the number of weeks per year.

*Table 5: One-year basis Alphas*

	Large cap	Small cap	Both strategies
Decline	-5.01%***	-7.17%***	-6.12%***
Growth	-1.35%***	-3.52%***	-2.50%***
Full length	-1.72%***	-3.92%***	-2.92%***

Note: Table was created in the Excel file from estimated values of parameter Alpha calculated in Gretl.

The numbers in tables are showing that the managers were less successful in the periods of decline. It is seen that alphas in the Bear markets are lower than in the Bull markets. It might be argued that it is more difficult to manage the portfolio when the market is falling and outflows of the fund are increasing. It pays regardless of investment strategy.

The Large cap funds' managers seem to be able to invest in a more efficient way than Small cap funds' managers, both on the Bull and the Bear markets and regardless of the situation on the market. This may indicate that management of riskier portfolios is more difficult.

Both of these results were in line of our expectations.

Discussion on Beta values:

The calculation of final values **of betas** is again based on the summations of different combinations of factors in *Table 1*. Due to this calculation, we can summarize the results as follows:

*Table 6: Final values of Beta coefficients*

	Large cap	Small cap	Both strategies
Decline	0.94***	1.12***	1.04***
Growth	0.98***	1.16***	1.07***
Full length	0.96***	1.14***	1.06***

Note: Table was created in the Excel file from estimated values of parameter Beta calculated in Gretl.

The coefficient  $\beta$  for the Large cap funds is significantly lower than 1, which means that their portfolios or their strategy is less risky than the market. On the other hand, the coefficient of beta for the Small cap strategy funds is significantly greater than 1, which means that their portfolios or their strategy is riskier than the market.

Higher values of betas in periods of growth indicate that managers of both fund groups take on higher risk in the periods of Bull markets. Maybe, that when everything growth it is easy to be successful even with riskier stakes.

Values of betas are in line with our expectations that Large cap strategies are less risky than Small cap strategies. It seems that fund managers invest less risky in periods of decline. This may be also the outcome of less risky structure of their portfolios as it can be expected that they would be forced to keep higher proportion of liquidity for increased redemptions.

Significance of the results:

Using classical t-tests, the estimates of particular coefficients are statistically significant with high p-values. The details are depicted in the *Table 8, Appendix*. Each

parameter is significant at least at 5%-level of significance. The coefficient of determination or the coefficient of overall significance is high with 0.73. This coefficient also measures the level of Goodness of fit of our linear model. It says that the variables included in regression explain 73% of the overall volatility of the observed returns of the examined funds. The values and details on correlations between dependent variable, fitted values and coefficient of determination can be found in the *Table 8, Appendix*. As we can summarize, the significance of the model doesn't make any problem in all respects.

#### Comments on the fulfilment of model assumptions

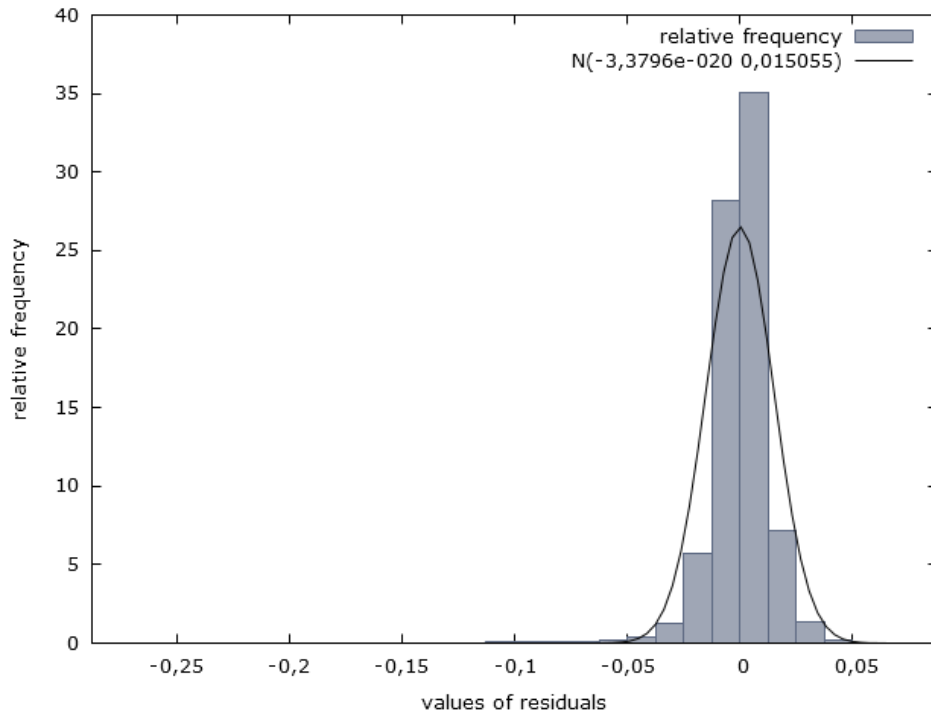
Fulfilment of model assumptions is important for interpretation of results and truthfulness of conclusions. The assumptions were discussed in detail in the chapter *Methodology* with their overview in *Appendix*.

The first tested assumption is the problem of **heteroscedasticity**. Breusch-Pagan test was used and allowed us to reject the null hypothesis about homoskedasticity with p-value 0.04. The heteroskedasticity of the market returns could have been expected, because the volatility of prices and returns is different in different market periods (see *Table 2*). This heteroskedasticity should not cause bias or inconsistency in the estimated parameters, but invalidates the use of t statistics and F statistics. (Wooldridge, 2012) The problem of heteroskedasticity might be reduced if the Sharpe ratios (returns divided by standard deviations) were used instead of market returns.

More serious could have been the problem of **autocorrelation**. The source data are prices of funds and index values that are as such highly correlated. However, the CAPM uses returns rather than prices or index values. As we explained in the section 4.2. Data this transformation reduces autocorrelation of the first order. The Durbin-Watson test confirmed that the autocorrelation was not present in transformed data with the value of Durbin-Watson statistic 1.96. Very low value -0.0002 of the autocorrelation coefficient  $\rho$  supports this statement as well.

The violation of assumption of **normality of residuals**. Test on normality confirmed deviation from normal distribution. Statistical test for normality provided in more detail in *Appendix*. The empirical distribution is nearly symmetrical but with higher kurtosis and thicker tails (*see Figure 8*). It means that critical values for a given significance are in general higher than for normal distribution. This might doubt the validity of statements about statistically significant estimates of parameters.

Figure 8: The distribution of residuals



Note: Graph was created in the statistical programme Gretl. The relative frequency is on the vertical axis and values of residuals are on the horizontal axis.

However, after the analysis of the empirical distribution of residuals we found that the value of 95% percentile of this distribution was only by 0.2 greater than that of normal distribution (1.64 vs. 1.84). This allows us to infer that the distribution of residuals is not so different from the normal distribution. In the context of p-values we can also argue that these values indicate high significance of estimated parameters. Therefore, we conclude that we were able to reject the hypothesis about insignificance of our estimates even with the empirical distribution that is not quite normally distributed.

## 6. Conclusions

The easiest way for non-professional investors how to get access to the market of securities is to buy some mutual fund. Their experienced managers invest on behalf of all fund holders. They try to be better than others to attract more investors and to increase their own reward, which is dependent on the volume of the fund. They sell their ability to outperform their benchmarks and make higher profits for holders of their fund. The most natural benchmark for their success is, however, the market itself.

Investors can choose between passively and actively managed funds. Managers from the first group keep the structure of their portfolios as close as possible to the structure of the market index. Managers from the second group believe that they are able to offer higher returns than the market by means of tactical asset allocation, i.e. by deviations from market structure. The second strategy is more costly and it is the investor who pays for it.

Equity market is heralded as the most profitable of all assets. The profit on the equity market is the reward for its high volatility, in other words for high risk. The relationship of the higher expected reward for taking higher risk is the cornerstone of the Capital Asset Pricing model (CAPM) that we used as a tool for our study.

The main objective of our thesis was to analyse the ability of mutual fund managers to outperform the market index in the long-run. We selected two groups of equity funds with different investment strategies, namely Large Cap and Small Cap funds. We noticed that there are periods of growth and periods of decline on the equity market. So, we tried to measure performance of funds in both periods apart. The key question was if the investors really got extra reward for higher costs of the actively managed funds.

There are many equity markets in the world with huge number of traded equities and nearly unlimited number of different mutual funds. To be able to focus on our topic and to avoid distortions we selected U.S. market represented by its index S&P500 and 30 funds that invest on the U.S. market mainly. We managed to obtain time series of weekly returns with 15 years history.

The results of our thesis are in line with the theory of investments. The Small cap funds take higher risk than the whole market portfolio and with beta higher than one are rewarded by higher expected return. The Large cap funds invest with lower risk than the

market and their beta is lower than one. Greater betas in the periods of Bull markets suggest that managers invest surprisingly more carefully and conscientiously in the period of Bear market.

Nevertheless, the final return of the fund is not dependent on the risk only but on the fund expenses and the skills of the manager of his actively managed fund. Both these items are very low. Alphas of actively managed funds measured in both of our groups decreased the returns more than expenses declared in their KIIDs. By their alphas Large cap fund managers invest more efficiently and cheaper than the Small cap fund managers. In other words, it suggests that Large cap fund managers are more skilful.

Values of all estimated parameters were statistically significant and high value of coefficient of determination confirmed that CAPM model describes the examined relationships very well. However, we were not able to reject the hypothesis about heteroskedasticity. We really observed different variability of market returns in different periods. Another deviation from model assumptions was the shape of empirical distribution of residuals. Even if it is symmetrical it differs from the normal distribution. However, very low p-values for all estimates of parameters indicate that they are statistically significant regardless of heteroskedasticity and deviation of normality.

Our findings could serve as an advice for ordinary investors, who do not know which mutual fund type to choose. If someone is ready to take the risk of the equity market the passively managed fund should be the first choice. Actively managed funds are more costly and underperform the market. Even the experienced American managers of actively managed funds make no exception. There are many options to choose from, mainly from different ETFs strategies.

We are aware that our random selection of 30 mutual funds may not be a representative sample. As seen in the Jensen's study (1968), he picked 300 funds and came to slightly different values of alphas. (However, he did not examine different strategies and different conditions on the market.) It is not so easy to get data and we managed to get them only with the courtesy of Reuters. In case of further and more thorough research it would be vital to get access to the reliable source of data.

We have measured the performance against the whole U.S. market and used only one index as the benchmark. It would be possible to choose funds with more investment strategies and to compare their performance against their specific benchmarks. This may

increase the overall significance of regressions. It would be interesting to compare the best performing funds with less successful funds. Periods of Bull and Bear markets could be also precisely determined and examined. More space for further research provides also the empirical distribution of residuals and confirmation of statistical significance of results.



## **Bibliography**

- Babson, D. The case of growth vs. income stocks on a yield basis. Babson and Company, Inc., Weekly Staff Letter, 1951. pp. 15-30
- Basu, S. The Relationship between Earning Yields, Market Value, and Return for NYSE Common Stock: Further Evidence?, *Journal of Financial Economics*, 1983, pp. 129-156
- Black, F.; Jensen M. C.; Scholes M. The Capital Asset Pricing Model: Some Empirical Tests, 1972, in *Studies in the Theory of Capital Markets*, M. C. Jensen, pp. 250-267
- Black, F. Capital Market Equilibrium with Restricted Borrowing.” *Journal of Business*. 45:3, 1972, pp. 444-454
- Breeden, D., T. An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. *Journal of Financial Economics*, 7, 1979, pp. 265-96
- Carlson, R. Aggregate Performance of Mutual Funds, 1948-1967, *Journal of Financial and Quantitative Analysis*, 1970, pp. 1-32.
- Elton, J., Edwin; Gruber, J. Martin. *Modern portfolio theory and investment analysis*, Wiley, New York, 1991, vol. 5, pp. 45-68
- Investment Company Fact Book-A Review of Trends and Activities in the Investment Company Industry. (2018). 58th ed. [ebook] Investment Company Institute, pp.15-45. Available at: [https://www.ici.org/pdf/2018\\_factbook.pdf](https://www.ici.org/pdf/2018_factbook.pdf)
- Jensen, C., Michael. The Performance of Mutual Funds in the Period 1945-1964, *Journal of Finance*, 1967, Vol. 23, No. 2, pp. 389-416
- Kenton, W. (2018). Intertemporal Choice.[online] Available at: <https://www.investopedia.com/terms/i/intertemporalchoice.asp>
- Kon, S. The Market-Timing Performance of Mutual Fund Managers, *The Journal of Business*, 56, 1983, pp. 323-347.
- Laderman, Jeffrey, M. *Business week, Guide to Mutual funds*, seventh annual edition, 1997, pp. 5-12
- Lehmann, B.; Modest D. Mutual Fund Performance Evaluation: A Comparison of Benchmarks and Benchmark Comparisons," *The Journal of Finance*, 42, 1987, pp. 233-265
- Lintner, John. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 1965, 47 (1), pp. 13-37.

Macrotrends.net. (2019). S&P 500 Index - 90 Year Historical Chart. [online] Available at: <https://www.macrotrends.net/2324/sp-500-historical-chart-data>

Malkiel, B. Returns from Investing in Equity Mutual Funds: 1971 to 1991," *The Journal of Finance*, 50, 1995, pp. 549-572

Markowitz, Harry, M. Portfolio selection. *The Journal of Finance*, 1952, vol. 7, No. 1, pp. 77-91

Merton, C., Robert. An Intertemporal Capital Asset Pricing Model, *Econometrica*, Vol. 41, No. 5, 1973, pp. 867-887

Merton, Miller, H.; Scholes, Myron. Rates of Return in Relation to Risk: A Reexamination of Some Recent Findings, 1972, pp.78-80 in ed. Michael C. Jensen, *Studies in the Theory of Capital Markets*. New York: Praeger

Modigliani, Franco; Merton, Miller, H. The Cost of Capital, Corporation Finance and the *The American Economic Review*, 1958, Vol. 48, No. 3, pp. 261-297 Published by: American Economic Association

Mossin, J. Equilibrium in a Capital Asset Market, *Econometrica*, 1966), Vol. 34, No. 4, pp. 768-783.

Pozen, R; Hamacher, T. *The Fund Industry: How Your Money is Managed*, Hobokom, NJ: Wiley Finance, 2015

Reinganum, M., R. The Arbitrage Pricing Theory: Some Empirical Results, *Journal of Finance*, 36, 1981, pp. 337 - 349

Roll, R. A Critique of the Asset Pricing Theory's Tests; Part I: on Past and Potential Testability of the Theory. *Journal of Financial Economics* 4, 1977, pp. 129-176

Rutterford, J. *Introduction to stock exchange investment*, The Macmillan press LTD 1993, second edition, pp. 238-276, ISBN 0-333-59387-1

Sharpe, William, F. Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance*. 19 (3), 1964, 425–442.

Sharpe, William, F.; Gordon, J.; Alexander; Bailey, Jeffery, V. *Investments*, Fifth Edition, Prentice Hall, Englewood Cliffs, New Jersey, 1995. pp. 262

Tobin, J. Liquidity preference as behaviour toward risk. *Review of Economic Studies*, 1958, pp. 65-85.

Wooldrige, Jeffrey, M. *Introductory econometrics, A modern approach*, Fifth Edition, 2012, pp. 340-400

2019 Investment Management Outlook A mix of opportunity and challenge. (2019).  
[ebook] Deloitte center for Financial services, pp. 5-15. Available at:  
<https://www2.deloitte.com/content/dam/Deloitte/us/Documents/financial-services/us-fsi-dcfs-investment-management-2019-outlook-rev.pdf>

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# Appendix A

## Tables and model assumptions

Table 7: A.1 - The results of Fixed Effects estimation

Source: Statistical program Gretl, own calculation

<b>Fixed Effects</b>				
Model 1: Fixed effects (GLS), using 21618 observations				
Included 30 cross-sectional units				
The length of time series: min. 718, max 724				
Dependent variable: returns_fund				
	Coefficient	Standard error	t-statistics	p-value
Const.	-0,00067	0,000149747	-4,518	6,26e-06 ***
returns_benchmark	1,16357	0,00759958	153,1	0,0000 ***
Decline	-0,00070	0,000271664	-2,597	0,0094 ***
Large_cap	0,000415	0,000205554	2,023	0,0431 **
DeclineXBench.	-0,03907	0,00893832	-4,372	1,24e-05 ***
Large_capXBench.	-0,18633	0,00891598	-20,90	4,89e-096 ***
Mean value of dependent variable		0,000889		
Standard deviation of dependent variable		0,028728		
Sum of squares residuals		4,895154		
Standard error of regression		0,015060		
LSDV R-squared		0,725607		
Within R-squared		0,724626		
LSDV F(33, 21584)		1470.603		
P-value(F)		0.000000		
$\rho$ (coefficient of autocorrelation)		-0,000213		
Durbin-Watson statistics		1,967406		

### Test for different intercepts among groups

Null hypothesis: Groups have the same intercepts

Test statistics:  $F(28; 21584) = 0,490739$

with p-value =  $P(F(28; 21584) > 0,490739) = 0,990118$

Table 8: A.2 - The results of Random Effects estimation

Source: Statistical program Gretl, own calculation

### Random Effects

---

Model 1: Random effects (GLS), using 21618 observations

---

Included 30 cross-sectional units

---

The length of time series: min. 718, max 724

---

Dependent variable: returns\_fund

---

	Coefficient	Standard error	t-statistics	p-value
Const.	-0,00068	0,000149695	-4,517	6,28e-06 ***
returns_benchmark	1,16293	0,00758702	153,3	0,0000 ***
Decline	-0,00070	0,000271569	-2,589	0,0096 ***
Large_cap	0,00041	0,000205483	2,024	0,0430 **
DeclineXBench.	-0,03904	0,00893437	-4,370	1,24e-05 ***
Large_capXBench.	-0,18636	0,00891265	-20,91	4,29e-097 ***

Mean value of dependent variable            0,000889  
Standard deviation of dependent variable    0,028728  
Sum of squares residuals                        4,898382  
Standard error of regression                   0,015055  
ρ (coefficient of autocorrelation)            -0,000213  
Durbin-Watson statistics                        1,967406

Mean theta = 0.23617

Corr (y,yhat)^2 = 0,725427

Breusch-Pagan test – Test for Heteroscedasticity

Null hypothesis: The variance of errors is 0

Test statistics: Chi-square = 4,15555

with p-value = 0,0414983

Hausman test – Test for consistent use of FE and RE

Null hypothesis: GLS estimations are consistent or  $Cov(a_i, x_{it}) = 0$

Test statistics: Chi-square = 7,93433

with p-value = 0,159892

Test for normality of residuals

Null hypothesis: the residuals are normally distributed

Test statistics: Chi-square = 29413.6

with p-value = 0

*Table 9: A.3 - Assumptions of Random Effects model*

Source: (Wooldridge, 2012)

Assumptions of Random Effects model:

RE1=FE1: For each I, the model is  $y_{it} = \beta_1 x_{it1} + a_i + u_{it}$ ,  $t = 1, \dots, T$ , where parameter  $\beta$  is to be estimated and  $a_i$  is the unobserved error of fixed effect

RE2=FE2: We have a random sample in the cross-sectional dimension

RE3: There are no perfect linear relationship among the explanatory variables

RE4: in addition to FE4, the expected value of  $a_i$  given all explanatory variables is constant:  $E(a_i|X_i) = \beta_0$

RE5: in addition to FE5, the variance of  $a_i$  given all explanatory variable is constant  $Var(a_i|X_i) = \sigma_a^2$

RE6=FE6: For all  $t \neq s$ , the idiosyncratic errors are uncorrelated (conditional on all explanatory variables and  $a_i$ ):  $Cov(u_{it}, u_{is}|X_i, a_i) = 0$

*Table 10: A.4 - Assumptions of Fixed Effects model*

Source: (Wooldridge, 2012)

Assumptions of Fixed Effects model:

The assumptions of the FE estimator are:

FE1=RE1

FE2=RE2

FE3: Each explanatory variable changes over time (for at least some i) and there are no perfect linear relationships among the explanatory variables.

FE4: For each t, the expected value of the idiosyncratic error given the explanatory variables in all time periods and the unobserved effect is zero:

$$E(u_{it}|X_i, a_i) = 0$$

Under these 4 assumption we can say that the estimator is unbiased.

FE5:  $Var(u_{it}|X_i, a_i) = Var(u_{it}) = \sigma_u^2$  for all  $t = 1, \dots, T$

FE6=RE6

FE7: Conditional on  $X_i$  and  $a_i$ , the  $u_{it}$  are independent and identically distributed as  $Normal(0, \sigma^2)$