Let R be a commutative 1-Gorenstein ring. Our main result characterizes all tilting and cotilting R-modules: up to equivalence: they are parametrized by subsets of the set of all prime ideals of height one. More precisely, every tilting (cotilting) R-module is equivalent to some Bass tilting (cotilting) module. This characterization was known in the particular case of Dedekind domains: Chapter 4 contains a new and simpler proof of this fact. Our main result is proved in Chapter 5, while Chapter 6 deals with the cotilting case. In Chapter 4, there is also a proof of the less well-known fact that all finitely generated tilting modules over commutative rings are projective.