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Diplomová práce

VOLATILITA AKCIOVÉHO TRHU V ČESKÉ REPUBLICE: VZESTUPY A PÁDY

Diploma thesis

THE STOCK MARKET VOLATILITY IN THE CZECH REPUBLIC: RISES AND FALLS

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Prohlášení

Prohlašuji, že jsem diplomovou práci vypracoval samostatně a použil pouze uvedené prameny a literaturu.

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Poděkování:

Rád bych tímto poděkoval panu PhDr. Martinu Netukovi za cenné rady a připomínky při tvorbě této diplomové práce.

Abstract:

A stock market came through a significant development in the Czech Republic; from its artificial beginning, through a fierce decline in listed companies, to a gradual rise in the market capitalization, which was suddenly turned off by a global financial crisis in 2008. The diploma thesis concentrate on a volatility analysis of a stock market in the Czech Republic in years 1994-2009 including a comparison with a data available from world developed stock markets - namely European region, USA and Japan. The most important and influential events concerning world markets and also a development of Prague Stock Exchange are included in the analysis. Econometric tools includes GARCH model and its most popular derivatives and generalisations i.e. IGARCH, EGARCH and APARCH processes.

The thesis is split into two main parts. The first part is devoted to a PSE volatility analysis based only on domestic data series involving GARCH class models estimations, a forecasting abilities comparison and also a structural-break analysis based on the ICSS algorithm including the Inclan-Tiao test and its successors. Next part involves a dynamic analysis based on the DCC MVGARCH model, which describes a change in a volatility spillover effect during the time. It is furthermore supported by the Granger causality estimation, which reveals a real direction of noticed interdependences between PSE and other markets. The result shows a long-lasting unidirectional dependence of PSE on other developed markets.

The result of the analysis shows that the stock market in the Czech Republic came through three main phases. The first phase started from its establishment in 1994 and ended in 1998, when an integration with other markets remained very low. Then the market shifted to a intermediate stage lasting to 2004, during this period the market is characterised by a mediocre financial integration. The Czech stock market in a final stage starting in 2004 can be denoted as a developed market, which includes a henceforth rising integration with other developed European stock markets. The goal of the thesis is also to uncover important events, which could affect a development at the Czech stock market. This means that an accession of the Czech Republic into European Union coincides with a shift in a development stage of the Czech stock market and it indicates that EU enlargement was a triggering event that allowed a further development and an increase in a degree of integration of the Prague Stock Exchange.

Abstrakt:

Akciový trh v České republice prošel významným vývojem, od svého umělého začátku, přes prudký pokles počtu emitentů, po postupný nárůst kapitalizace, který byl ovšem náhle ukončen globální finanční krizí v roce 2008. Diplomová práce se zabývá analýzou volatility českého akciového trhu v letech 1994 až 2009 včetně srovnání s vyspělými světovými akciovými trhy - konkrétně se jedná o evropský region, USA a Japonsko. Analýza obsahuje nejdůležitější a nejvlivnější události týkající se světových trhů a také vývoje Pražské burzy cenných papírů. Nástroji ekonometrické analýzy jsou často užívané modely odvozené od původního procesu GARCH tzn. IGARCH, EGARCH a APARCH procesy.

Diplomová práce je rozdělena do dvou hlavních částí. První část je věnována analýze volatility Burzy cenných papírů Praha založené pouze na domácích informacích. Analýza obsahuje odhady modelů GARCH, srovnání jejich schopností předpovídání a rovněž část věnovanou strukturálním zlomům založené na ICSS algoritmu, Inclan-Tiao testu a jeho upravených verzích. Další část se zabývá dynamickou analýzou založenou na DCC MVGARCH modelu, který popisuje vývoj volatility spillover efektů během pozorovaného období. Analýza je dále podpořena výpočty Grangerovy kausality, která odhaluje skutečný směr působení vzájemných vztahů mezi BCPP a ostatními trhy. Výsledek ukazuje na dlouhodobou jednosměrnou závislost BCPP na ostatních vyspělých trzích.

Výsledek analýzy ukazuje, že Český akciový trh prošel třemi fázemi vývoje. První fáze začala od jeho založení v roce 1994 a skončila v roce 1998, kdy byla integrace s ostatními trhy na velmi nízké úrovni. Poté akciový trh postoupil do přechodné fáze trvající až do roku 2004 během níž zaznamenal průměrnou úroveň integrace do ostatních trhů. Konečná fáze začala rokem 2004, od něhož lze český trh považovat za rozvinutý, což s sebou nese i nadále rostoucí míru integrace s ostatními vyspělými evropskými akciovými trhy. Záměrem analýzy je rovněž prozkoumat významné události, které by mohly ovlivnit vývoj českého akciové trhu. To znamená například, že vstup České republiky do Evropské unie koinciduje se změnou vývojové fáze českého akciové trhu a to ukazuje, že rozšíření EU bylo spouštěcí událostí, která umožnila další vývoj a zvýšení míry integrace Burzy cenných papírů Praha.

Table of Contents

I. Introduction

There are many circumstances, which affected a development of the Czech stock market. There was also a change in a degree of interconnection of the Prague Stock Exchange with other markets, the relation between markets was in early 90's definitely different from a state at the beginning of the 3rd millennium. A structure of investors trading on PSE has changed through an existence of PSE. At first a majority of shareholders were represented only by home investors, who participated in a coupon privatisation, represented by local shares funds or minority shareholders, while later came also a foreign investors - directly or indirectly through local daughter companies; who added Czech shares to their global portfolios. Also a structure of stock issues has changed from an instantaneous outcome of a coupon privatisation, through a stabilization of the market, to a developed international cross-listing with other foreign equity markets. From 1st May 2004 the Czech Republic became a member of European Union which significantly deepened an ongoing integration and can be regarded as one of the most important events in an economic history of the Czech Republic.

The thesis will research all the available data[1] of PSE from its beginning until the global financial crisis in years 2008/2009 to uncover a breakpoints of PSE's development to match them with important events and milestones. The goal is to determine important stages of development of Czech capital market and reveal the unique characteristics typical for particular proposed stages, which would be based on the empirical econometric modelling.

At first a brief history of a stock market in the Czech Republic will be sketched for a purpose of finding significant events, which can be further tested in proposed models. This means events arising from changes in PSE's functioning and also globally important events originating from financial crises, which were important for the European region, or a strengthening international integration, which is mainly affected by an existence of European Union and its own development.

The following parts are devoted to two main themes involving different volatility testing methods. It namely means the national[2] and the international volatility analyses from a point of view of the Czech Republic. This brings an opportunity to compare outcomes from a local analysis to global figures and events and answer, which events were more important for a development of the Czech stock market.

A compact summary of financial data modelling is proposed. It tackles a possible methods

1 Only data series from 1st May 1994 was available.
2 National volatility testing incorporates methods, which analyze solely a time series from the Czech Republic.

involving a national index analysis, in this case represented by PX index of PSE. Definitions of generalized conditional heteroskedasticity processes are a core tools used in further estimations i.e. GARCH, IGARCH, EGARCH and APARCH models.

The first part of the research involves methods, which analyse an internal structure of the Czech stock market and namely PX index of the Prague Stock Exchange. The analysis aims at first at GARCH class models i.e. GARCH, IGARCH, EGARCH, APARCH; in order to find, which model fits the data best and thus also describes the underlying structure of the Czech market. The GARCH models are capable of incorporating a number of widely observed features of stock prices behaviour such as leptokurtosis, skewness, and a volatility clustering. Although the models are mostly used as descriptive tools, there is also a possibility to use them as predictive measures and this propose a question, which of the chosen models has the best abilities to predict a probable development of the Czech market. These issues will be also tested in the chapter using several defined quality criteria.

When a proper description of the market is finished using particular models for the whole period of time, there can be tested a possibility for an existence of structural breakpoints. The structural breakpoints cluster the whole time series into shorter periods of time and also indicate that there is either a way to gain significantly better outcomes using multiple estimations instead of a single one or show differentiated capabilities of estimated models among newly defined periods. There are econometric procedures, which can be employed in order to find out these structural breakpoints. This namely means the Inclan-Tiao test and its successors, which find breakpoints according to the ICSS algorithm and also its redesigned test statistics. The results of the breakpoints estimations are thoroughly tested against a quality of forecasts obtained from new subsamples bordered by structural breakpoints.

In a next part of the thesis there are solved questions involving international interconnections and relations between the Czech stock market and other developed equity markets. This includes DCC MVGARCH model, which is capable of a dynamical approach to conditional correlations among researched markets. Estimations of the DCC MVGARCH are made for daily returns computed only from foreign index data series and also for daily net returns including exchange rate effects, when the CZK is set as a basis for all observations. Although the DCC MVGARCH model is capable to estimate a correlation between particular markets it cannot reveal a direction of the information relay and thus a different econometric tool have to be employed.

This results into a usage of the Granger causality test, which can find directions of volatility flows across the world from a point of view of the Czech Republic. For a purpose of a higher precision also the Akaike information criterion is combined with the Granger causality test, which allows to choose an appropriate number of variables needed for estimations. The outcome of Granger causality test is then confronted against DCC MVGARCH results, which leads to a final synthesis of the models.

The final chapter concludes results from all sections in order to find common elements and recapitulate the most important findings.

II. Historical Preview

A historical preview is presented in order to find suitable events possibly influencing the evolution of PSE. The chapter is spilt into two subsections, the first is devoted to national events summarized into the Czech stock market overview, the second part is describing important international events denoted as exogenous events.

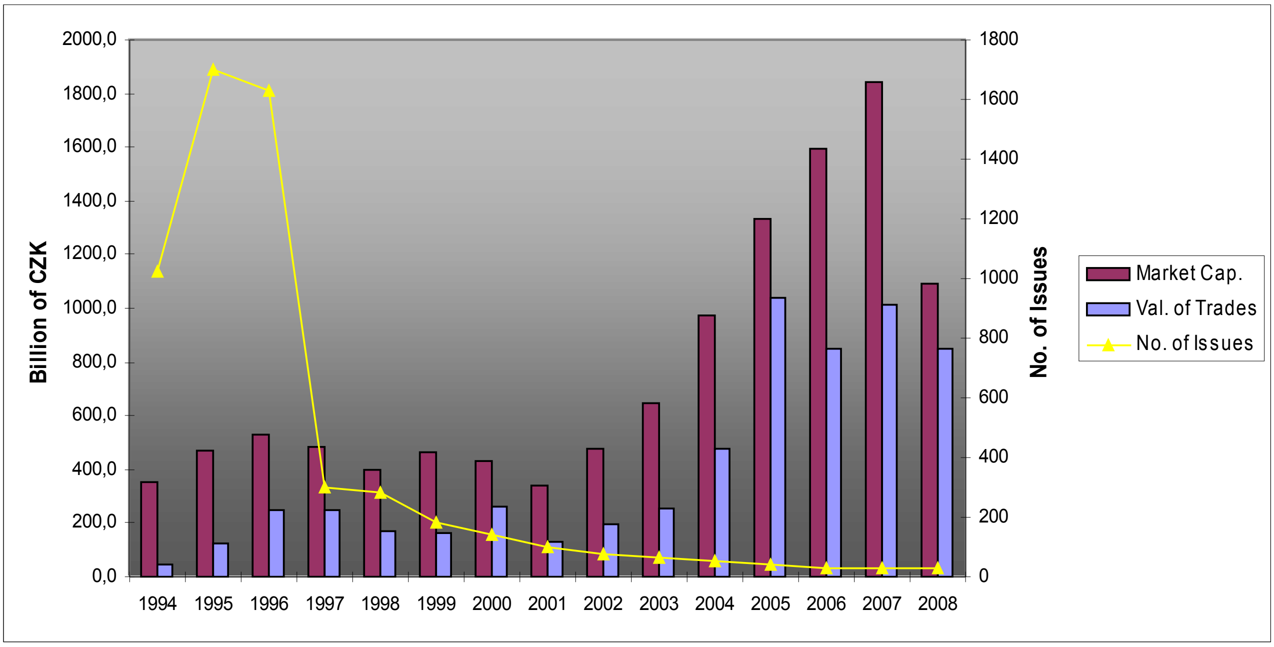
2.1. Czech Stock Market Overview

Although a market started its way of liberalization early after a fall of a communist era, a self-transformation process was not so intensive to support a spontaneous massive demand for an establishment of a stock market in the Czech Republic. Prague Stock Exchange was established on 24th November 1992 and attached an interest of issuers, which resulted in a start of trading involving 7 stock issues. There was early an artificial initial public offering in the Czech republic in years 1993 and 1995, which introduced more than 1600 individual shares. It was rather a political decision than a natural evolution of the financial market to constitute a Prague Stock Exchange and thus motives of issuers were not consistent with a long-term participation in the stock exchange resulting in huge delisting in 1997.

Czech capital market passed through a very important milestones in its quick development: starting at an abolition of centrally planned economics through a phase of liberalization to the economic integration into European Union resulting to a full membership of Federation of the European Securities Exchanges. Namely the PX[3] index, which is a basis for further analysis (PSE) experienced its artificial birth in 1994, then era of steady development from 1996 to 2001, followed by a booming increase and development, which was unfortunately broken in 2008, because of a global financial crisis. The best picture of the development can be perceived through a quantitative summary of PSE described by next Graph 1, which shows a market capitalization and a value of trades in CZK and also a number of traded issues.

3 Formerly PX 50 index

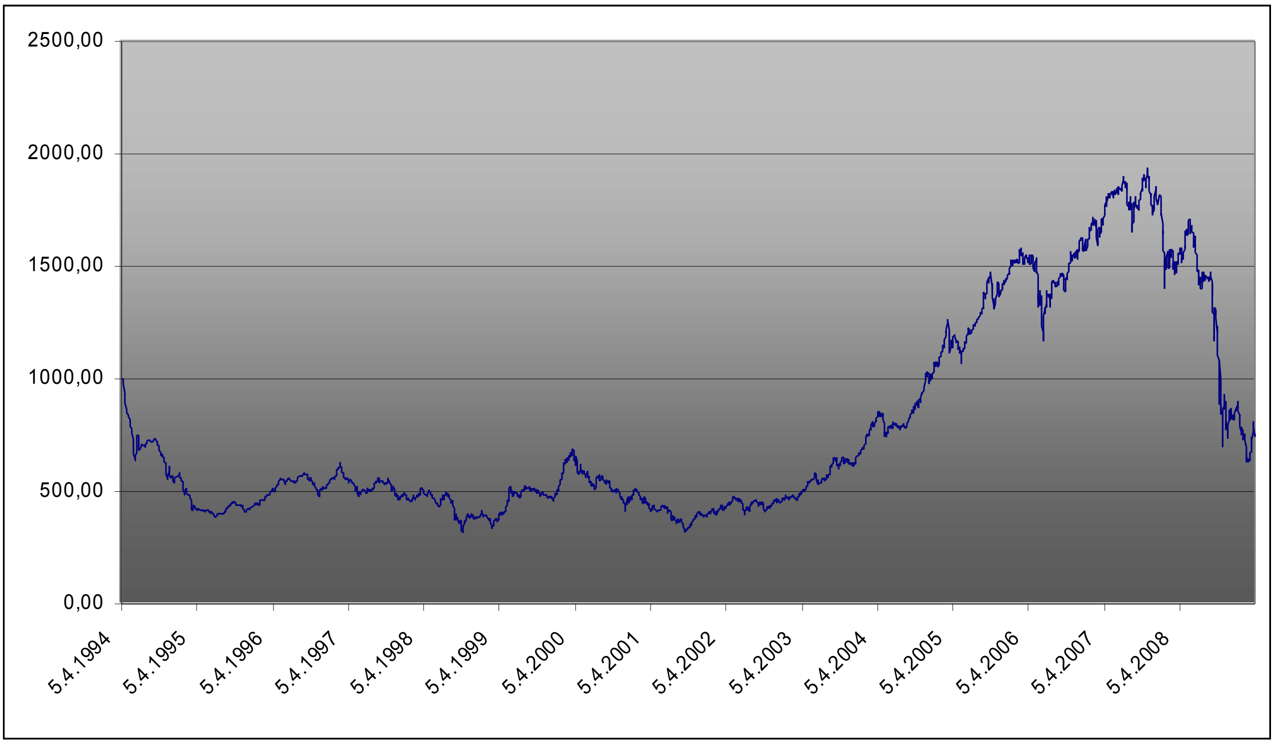
Graph 1: Development of Prague Stock Exchange



Source: PSE Fact Books

During the development of PSE also a value of PX index has changed, which describes following Graph 2, which data series will be used in the next chapter devoted to an analysis of the Czech market volatility. It describes an initial downfall during first two years, a steady value during years 1996 to 2003, a huge increase from year 2003 to 2007, which is stopped by a steep fall caused by a global financial crisis in a period 2008/2009.

Graph 2: Development of PX index



Source: PSE

Finally it is possible to summarize all important events of PSE to a single table, which will connect all important events with appropriate dates. The information is summarized in Table 1. Alas it is not possible to examine events before 5th April 1994, because data series was not available for this period[4]. Events in years 1992/1993 are described in order to offer a whole picture of PSE history.

4 5th April 1994 is a date of PX 50 establishment, thus data series before the date would be compared with rest of the sample only with great problems, because new 'artificial index' had to be employed.

TABLE 1: SUMMARY OF PSE DEVELOPMENT

24/11/92	Establishment of Prague Stock Exchange
06/04/93	Begin of trading with 7 stock issues
22/06/93	Enlisting of 622 stock issues from 1st wave of coupon privatisation
13/07/93	Enlisting of 333 stock issues from 1st wave of coupon privatisation
05/04/94	Initial computation of official PSE index PX 50
01/03/95	Enlisting of 674 stock issues from 2nd wave of coupon privatisation
01/09/95	Change of PSE structure – main, minor and free markets established
15/03/96	KOBOS established - continuous trading with variable pricing
1997	Delisting of 1301 illiquid stock issues from free market
05/01/98	35 stock issues transferred from main market to minor, because of unfulfilled criteria
25/05/98	SPAD trading established – instantaneous trading
04/01/99	Continual computation of PX 50
20/09/99	Delisting of 75 stock issues from free market
14/06/01	PSE was affiliated as the Associate member of the FESE
01/10/02	First foreign stock issues accepted to PSE – ERSTE BANK
01/05/04	PSE became the full member of FESE in connection with accession of the Czech Republic into EU
May – 2004	U.S. Securities and Exchange Commission officially granted the status of a "designated offshore securities market" to PSE
28/06/04	IPO of Zentiva stock issue
17/03/06	Indices PX 50 and PX-D were replaced by index PX
04/10/06	Established trading with investment certificates
05/10/06	Established trading with futures
07/12/06	IPO of ECM stock issue
11/12/06	Established trading with warrants on free market
18/12/06	IPO of Pegas Nonwovens stock issue
01/07/07	Merger of minor and main markets

Source: PSE website

2.2. Exogenous Events

This chapter will summarize a list of the most important events, who affected financial markets. The nature of the events can split into two major groups of events. There are incidents, which were caused by 'bad events' such an Asian crisis, and there are also events influenced by 'good events' as European Union enlargement.

The first important international event, which could affect a PSE development from a global perspective, can be perceived in the Asian crisis, which started in 1997 and affected a volatility spillovers among many markets, which is described by HYDE ET AL. (2007), KHALID AND RAJAGURU (2007) or WORTHINGTON AND HIGGS H., (2004). The studies confirm a commonly agreed opinion, that during crises there are significant increases in conditional correlations amongst financial markets, which is proved by various dynamical models based GARCH processes.

Moreover the crisis spread all over the world and fiercely affected Russian equity markets in year 1998, which is described by SALEEM (2008) using GARCH - BEKK model. The study revealed that Russia was directly affected by close Asian markets, which resulted in an "avalanche" effect further influencing USA, EU and also European emerging markets[5]. Thus these results suggest to examine the development of PSE in terms of international relations to other equity markets using GARCH dynamic models, which are capable of an analysis of revealing evidence of a contagion. The results in mentioned studies confirmed that periods crises led to an increased contagion amid financial markets, which should be similar in a case of a global financial crisis in 2008.

Events, which can be regarded as very significant for a development of the Czech Republic, are also closely linked with evolutionary processes in the European Union, because of a great dependence of the Czech Republic on international trade with its neighbouring countries. CAPPIELLO ET AL. (2006) revealed that an increase in correlations between equity markets can be also associated with a deepening integration. It was proved on example of a Euro adoption in 1999, which exhibited even earlier in May 1998 because of an assessment of irrevocable fixed exchange rates between Euro and integrating national currencies. The result suggests that PSE should be also affected by the most important event of an integration of the Czech Republic, which was an accession to European Union. In addition the accession was related in case of PSE with a full membership in FESE and a granted status of a "designated offshore securities market" from U.S. Securities and Exchange Commission.

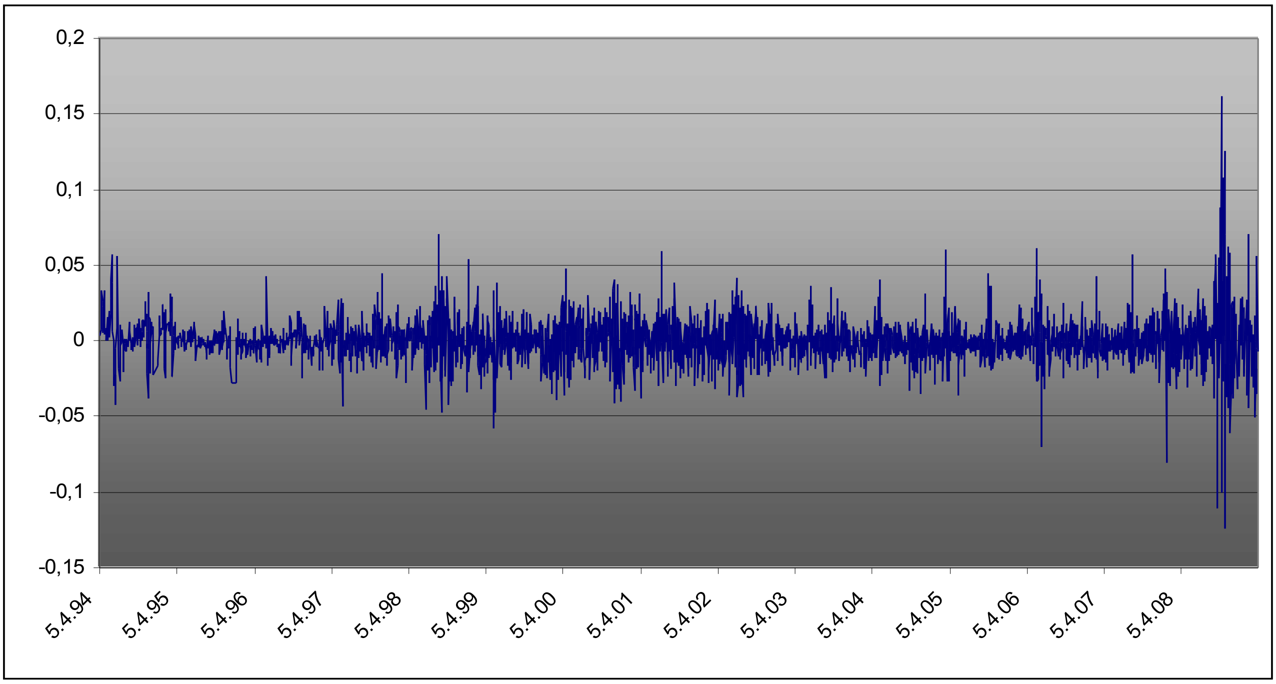
5 In that time the Czech republic was denoted as an emerging market.

III. Czech Market Volatility Analysis

There are two main purposes of time series modelling. First of all the models are built to fit data sets and thus describe the underlying nature[6] of the data. This knowledge of a time series behaviour is used in a next step of the econometric analysis, which tries to forecast a future development of researched variables. And thus a structure of volatility modelling in the Czech Republic will also be devoted to these two ways of analysis. At first a theoretical background, based on descriptive methods, will be set in order to prepare a groundwork for a usage of econometric models in practice, which will result in a quality comparison of forecasting abilities. The analysis of the volatility will use the daily frequency data with estimations of various models. These basic facts sketch the final outcome of the analysis, which will also try to figure out whether more complex models pay out in a superior quality in a comparison to more simple models.

The first graph, which is a result of basic data analysis, shows an intensity of volatility during the existence of PX index on Prague Stock Exchange. The Graph 3 shows daily net returns of PX index.

Graph 3: Daily Rate of Return - Index PX[7]



Source: Prague Stock Exchange

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6 e.g. leptokurtosity, conditional heteroskedasticity, leverage effects

7 The graph includes data series from 7.4.1994 to 1.4.2009 (3678 samples) in form $R_t = \log(P_t / P_{t-1})$.

3.1. Basic Concept

A volatility modelling became a widely used part in research of financial markets. The methods give opportunity to search through structure and characteristics of markets. At this stage I would like to prepare a theoretical background for my further more complex models.

The basic approach, which can be used in a case of analysis of a single variable, represent autoregressive processes. The most simple model, which is a predecessor of all other derived and more sophisticated models, is AR(1) process[8]. It assumes a linear dependence of variable on previous observations, which means that variable Y_t depends linearly upon its shifted value Y_{t-1} as is described in following form:

$$Y_t = \delta + \theta Y_{t-1} + \varepsilon_t$$

where Y_1 ,..., Y_T is assumed to be a time series of observations and ε_t denotes a serially uncorrelated residual with a mean of zero and a constant variance over a time. The stationary condition implies that $|\theta| < 1$ and thus a simple adjustment can be made in order to simplify proposed model.

When expected value of Y_t is computed

$$E\{Y_t\} = \delta + \theta E\{Y_{t-1}\}$$

and under assumption that $E\{Y_t\}$ does not depend upon time *t*, it can be written

$$\mu \equiv E\{Y_t\} = \frac{\delta}{1-\theta} \quad ,$$

with definition of $y_t \equiv Y_t - \mu$ it result in final form of the model

$$y_t = \theta y_{t-1} + \varepsilon_t \quad ,$$

which can be further generalized to AR(p) process

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_p y_{t-p} + \varepsilon_t$$

8 For further details I refer to Verbeek (2008) chapter 8.1.

A next stage of more general econometric modelling can be captured in the ARMA process, which is a compilation of a general autoregressive and moving average processes, which has following form for MA(1) representation

$$y_t = \varepsilon_t + \alpha\,\varepsilon_{t-1} \quad ,$$

which can be generalized into MA(q) process

$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \ldots + \alpha_q \varepsilon_{t-q}$$

And this leads to a simple collection of previously mentioned AR(p) and MA(q) processes, which can be summarized into one equation describing the ARMA(p,q) model

$$y_t = \theta_1\, y_{t-1} + \theta_2\, y_{t-2} + \ldots + \theta_p\, y_{t-p} + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \ldots + \alpha_q \varepsilon_{t-q}$$

However solely the ARMA process did not provide sufficient outcomes, when used for financial data series and thus more sophisticated models were proposed such a concept of autoregressive conditional heteroskedasticity (ARCH).

3.2. ARCH Class models

In order to capture a real behaviour on financial markets and describe a common event called volatility clustering, which means that big shocks tend to be accompanied by another big shocks in historical data sets and also small shocks incline to be followed by small shocks[9], ENGLE (1982) proposed the ARCH process, which allows that residuals resulting from different levels of volatility can shift during the time. The definition of the ARCH(1) model shows that the variance of the error term at time *t* depends on a squared error term from a previous period, which can be defined as follows:

$$\sigma_t^2 \equiv E\left\{\varepsilon_t^2 \middle| \Gamma_{t-1}\right\} = \omega + \alpha\,\varepsilon_{t-1}^2 \quad ,$$

9 In case of an estimation using AR processes the residuals would differ across the data series, because of its inability to capture different behaviour during "big shocks" and "small shocks" periods characterized by a different level of volatility.

where Γ_{t-1} stands for the information set, which includes residuals ε_{t-1} and its complete historical information[10].

In order to fulfil conditions emerging from a definition of a variance $\sigma_t^2 \geq 0$, it is necessary to hold $\omega \geq 0$ and $\alpha \geq 0$. The essence of the ARCH(1) process pronounce that the size of a shock in period *t-1* affect also a probability of occurrence of a similar shock in a next period *t*. Although in case of big shocks it is also more probable that a big shock will occur in a following period, it does not imply that the ARCH process for an error term ε_t is non-stationary, it only states that squared values ε_{t-1}^2 and ε_t^2 are correlated. The unconditional variance of ε_t is defined as

$$\sigma_t^2 = E\{\varepsilon_t^2\} = \omega + \alpha E\{\varepsilon_{t-1}^2\}$$

and it has a stationary solution

$$\sigma^2 = \frac{\omega}{1-\alpha} \quad ,$$

which imposes an additional condition $0 \leq \alpha \leq 1$. A definition of the ARCH(1) allows it to be extended to an ARCH(p) process, which is given by

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2 = \omega + \alpha(L) \varepsilon_{t-1}^2 \quad ,$$

where $\alpha(L)$ is a polynomial lag of order p-1. To ensure a necessary condition of a non-negativity for the conditional variance, $\omega \geq 0$ and also the coefficients in $\alpha(L)$ must be non-negative.

The stationary condition for the process require that $\alpha(1) < 1$. The outcome of a definition of ARCH(p) model is that shocks older than than *p* periods ago have no impact on current volatility in time *t*. Further generalisation of ARCH(p) model was proposed by Bollerslev (1986) and it led to well known and commonly used generalized ARCH model.

10 For further information I refer to Verbeek (2008) chapter 8.10.

3.2.1. GARCH Model

The GARCH model[11] approach allows for an empirical assessment of the relationship between risk and returns in a setting that is consistent with the characteristics of a leptokurtosis and a volatility clustering observed in the stock market data series. The meaning of the GARCH model can be shortly summarized into a statement that a model incorporates heteroskedasticity of the data sample and thus can describe changes in a volatility during the time in more general way than the ARCH process.

In an univariate GARCH model is assumed that residuals are denoted as ε_t , where $\varepsilon_t = \sigma_t z_t$ and $z_t \sim iid(0,1)$ and variance is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \quad , \quad p \geq 0, q > 0, i > 0$$

with following restrictions $\omega, \alpha_i \geq 0, \beta_i \geq 0$ which arise from a condition of non-negative variance σ_t^2 and also restrictions, which ensure a stationarity of the process $\alpha + \beta < 1$.[12] The most simple version of the model is GARCH (1,1), which has a following form

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

and after definition of $\mu_t \equiv \varepsilon_t^2 - \sigma_t^2$ it can be redefined as

$$\varepsilon_t^2 = \omega + (\alpha + \beta) \varepsilon_{t-1}^2 + \mu_t - \beta \mu_{t-1}$$

which results into an outcome that the squared error terms follow ARMA(1,1) process, which makes a close interlink with previously mentioned models and put them into one family. Also μ_t term is uncorrelated over the time and thus reveal the heteroskedasticity in the model.

11 In full name generalized autoregressive conditional heteroskedasticity model.

12 Values of $\alpha + \beta$ near to one imply that the persistence in volatility is high and this assumption is a basis for the IGARCH model.

3.2.2. IGARCH Model

As was proposed in a previous section the GARCH model impose a restriction $\alpha+\beta<1$ in order to maintain a stationarity of the process, however data series from financial markets tend to have $\alpha+\beta$ close unity, which implies that a volatility level persists for long periods of time. Thus the integrated GARCH(p,q) model was proposed in Bollerslev (1986) and its main feature is that it assumes and incorporates a unit root in the GARCH process. Therefore it is a restricted version of GARCH model, where the sum of the persistent parameters sum exactly to one. This condition is fulfilled for IGARCH(p,q)when:

$$\sum_{i=1}^{p}\alpha_i+\sum_{i=1}^{q}\beta_i=1$$

And moreover in a specific case of IGARCH(1,1):

$$\alpha_1+\beta_1=1$$

The result of the unit root existence is that impact of past shocks is persistent through the time and thus also an unconditional variance is not defined in the model. This all leads to a conclusion that IGARCH model involves a restricting rule in order to simplify its real-life interpretation, when it is properly used.

3.2.3. EGARCH Model

A modified specification of the GARCH model can be represented by exponential GARCH[13] process invented by Nelson (1991), which incorporates an idea of asymmetrical impacts on volatility based on a differentiation between unexpected drops in prices and also unexpected increases[14]. The definition of EGARCH (1,1) is following[15]

$$\log\sigma_t^2=\omega_t+\beta\log\sigma_{t-1}^2+\theta\frac{\varepsilon_{t-1}}{\sigma_{t-1}}+\alpha\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}$$

13 EGARCH

14 In a case of the classical GARCH model a price drop and an increase in price would be perceived as same events, because their only result is an common increase in a volatility.

15 The term "log" indicates a natural logarithm.

A feature capturing the asymmetry called also as a leverage effect is included in the model in case that $\theta \neq 0$, moreover in case of $\theta < 0$ a positive shock[16] generates less volatility than a negative shock. Because of an assumed logarithmic transformation there is no danger that the conditional variance σ_t^2 would be negative.

3.2.4. APARCH Model

Asymmetric power GARCH (p,q) model proposed in Ding, Granger, and Engle (1993) is defined in the form

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

with following conditions

$$\omega > 0, \delta \geq 0,$$

$$\alpha_i \geq 0, i = 1, \ldots, p,$$

$$|\gamma_i| < 1, i = 1, \ldots, p,$$

$$\beta_j \geq 0, j = 1, \ldots, q$$

It is a further generalization of the original GARCH model, moreover the APARCH(p,q) model is so effective that it includes seven other nested models as special cases[17], it namely means ARCH(p) model, GARCH(p,q) model, Taylor/Schwert's GARCH in standard deviation model, GJR model, Zakoian's TARCH model, Higgins and Bera's NARCH model, Geweke and Pantula log-ARCH.

For example APARCH(p,q) behaves as the previously mentioned ARCH(p) in case that $\delta = 2 \wedge \gamma_i = 0, i = 1, \ldots, p, \beta_j = 0, j = 1, \ldots, q$, similarly APARCH has the same features as GARCH(p,q) model in case that $\delta = 2 \wedge \gamma_i = 0, i = 1, \ldots, p$. This strength of the APARCH model indicates that it could be the best model for a fitting into data series or an estimation of forecasts, however it also has a drawback, which inheres in its complexity. Thus the model should be clearly superior to other models to prove its worthiness. The covariance stationarity condition can be

16 As a positive shock is regarded an event caused by the unexpected increase in price, in contrary a negative shock is an event involving a drop in price.

17 For further details see Ding, Granger, and Engle (1993)

written in a following form

$$\sum_{i=1}^{p} \alpha\left(|\varepsilon_t| - \gamma_i \varepsilon_t\right)^{\delta} + \sum_{j=1}^{q} \beta_j < 1$$

3.3. Forecasting Abilities

As was already mentioned one of the main goals of the econometric modelling is to forecast a future development based on historical data. A precision of forecasts can be regarded as a useful benchmark of a goodness of fit to researched data series, because it enables a comparison of real and estimated values. Thus in this chapter the forecasting abilities of previously mentioned models will be tested in order to compare their efficiency and bias, which can help to uncover the most suitable process for a further modelling.

Alas neither of previously defined models have any feature, which would allow to estimate a conditional mean and thus a real value of the researched index cannot be computed. The only available solution would be an upgrade of the models, which is commonly achieved with AR processes, e.g. h-step forecasts using AR (1):

$$\hat{y}_{t+h|t} = \hat{\mu} + \hat{\theta}_1\left(\hat{y}_{t+h-1|t} - \hat{\mu}\right)$$

However this kind of solution does not depend on a definition of the GARCH class models and thus an incorporation of the method would not improve results of the analysis and thus the only term, which can be forecasted, is a conditional variance.

3.3.1. Conditional Variance

An ability to forecast the conditional variance arises from a design of GARCH class models, which main purpose is to describe a nature of volatility as was already shown in previous chapters. In this section a characteristics of forecasting methods will be described for each model. Starting from GARCH(1,1) process the 1-step forecast of the conditional variance can be written as

$$\hat{\sigma}^2_{t+1|t} = \hat{\omega} + \left(\hat{\alpha} + \hat{\beta}\right)\hat{\sigma}^2_t \quad ,$$

which is a basis for other h-step forecasts calculated directly or recursively from original 1-step forecast. Analogously declared h-step forecast

$$\hat{\sigma}^2_{t+h|t} = \hat{\omega} + \left(\hat{\alpha} + \hat{\beta}\right)\hat{\sigma}^2_t \quad ,$$

can be adjusted to a final form, which will allow to directly compute h-step forecast without intermediate outcomes.

$$\hat{\sigma}^2_{t+h|t} = \frac{\hat{\omega}\left(1 - \left(\hat{\alpha} + \hat{\beta}\right)^h\right)}{1 - \left(\hat{\alpha} + \hat{\beta}\right)} + \left(\hat{\alpha} + \hat{\beta}\right)^h \hat{\sigma}^2_t$$

For the sake of simplicity[18] I will only mention 1-step forecasts of estimated models[19], which can be then used to h-step forecasts using recursive computations i.e. GARCH(p,q) process:

$$\hat{\sigma}^2_{t+1|t} = \hat{\omega} + \sum_{i=1}^{p} \hat{\alpha}_i \varepsilon^2_{t+1-i} + \sum_{j=1}^{q} \hat{\beta}_j \sigma^2_{t+1-j}$$

The form of 1-step forecast in case of IGARCH(p,q) is exactly the same as GARCH(p,q), because the only difference between models is an additional condition.

$$\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i = 1$$

18 The final estimation of forecasts will be made by OxEdit 5.10 using libraries G@RCH 4.2.

19 For further details I refer to Pasha et al. (2007)

The 1-step forecast for EGARCH(p,q), when defined $\eta = \frac{\varepsilon_t}{\sigma_t}$:

$$\hat{\sigma}_{t+1|t} = \exp\left(\hat{\omega} + \sum_{i=1}^{p} \left(\hat{\alpha}_i \left|\eta_{t+1-i} - E\left(\eta_{t+1-i}\right)\right| + \hat{\theta}_i \eta_{t+1-i}\right) + \sum_{j=1}^{q} \hat{\beta}_j \hat{\sigma}_{t+1-i}\right)$$

The 1-step forecast for APARCH(p,q) :

$$\hat{\sigma}^{\delta}_{t+1|t} = \omega + \sum_{i=1}^{p} \hat{\alpha}_i \left(\left|\varepsilon_{t+1-i}\right| - \hat{\gamma}_i \varepsilon_{t+1-i}\right)^{\delta} + \sum_{j=1}^{q} \hat{\beta}_j \hat{\sigma}^{\delta}_{t+1-j}$$

3.3.2. Quality Criteria

As was already mentioned all models will be used to forecast volatility based on historical data, thus a benchmark of results should reveal their true potential in a comparison to real values and also should state, which of the models is the most suitable for further analysis intended in chapters about structural breaks and volatility spillover effects. The quality will be tested using several forecast evaluation measures, namely a mean square error (MSE), the Theil inequality coefficient (TIC) and the Mincer-Zarnowitz regression[20].

3.3.2.1. Mean Square Error

The mean square error is a classical measure, which quantify a difference between an estimator, in this case represented as a forecast, and a true value, which is described in data set. The formula of MSE:

$$MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right] ,$$

where $\hat{\theta}$ represents a forecast and θ a true value. In another form MSE can be written as a sum of a variance and a squared bias of the forecast.

20 Mentioned forecast evaluation measures are computed through G@RCH 4.2 package implemented in OxEdit 5.10.

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + \left(Bias(\hat{\theta}, \theta)\right)^2$$

Thus MSE reveals a quality of a forecast in terms of its variance and unbiasedness. The measure can be easily compared between models estimating the same time series and also same type of estimators, because the values of mean and variance among all models should reach as low bias as possible. This means that a model with lower MSE should be regarded as more precise.

3.3.2.2. Theil Inequality Coefficient

The measure is also known as Theil's U and provides a ratio of how precise a time series of estimated values compares to a corresponding time series of real observed values. The statistic proposed in THEIL (1961) computes the degree to which one time series $\left(\{X_t\}, t=1,2,3,\ldots n\right)$ differs from another $\left(\{Y_t\}, t=1,2,3,\ldots n\right)$. Theil's U is calculated as:

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{T}\left(X_t - Y_t\right)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{T} X_t^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{T} Y_t^2}}$$

U statistic varies from 0 to 1. A value around 0 means a full harmony or a compliance of true data series with estimated values and on contrary a value near 1 means that estimated model has no significance for an estimation of true realized values. TIC in comparison with MSE also decomposes a forecast error into a bias, variance and covariance as mentioned in BALDER, KOERTS (1992), which makes TIC even a more reliable measure of a forecast performance.

3.3.2.3. Mincer Zarnowitz Regression

A method proposed in MINCER AND ZARNOWITZ (1969) is testing an unbiasedness and efficiency though a simple regression model. The main idea is a regression based on both information from forecasts and realized values. Mincer-Zarnowitz regression is defined as follows

$$y_{t+h}=\alpha+\beta\hat{y}_{t+h,t}+\varepsilon_t ,$$

imposing conditions that $\alpha=0$ and $\beta=1$, which states that forecasts should differ from realized values only by an unforecastable error described as ε_t . If mean values of predictions and realizations are equal, which is fulfilled when $\alpha=0$, a forecast can be regarded as unbiased. An efficiency of the forecast is reached, when a slope of the regression $\beta=1$, so predictions are uncorrelated with errors.

This method can be also used in a case of forecasted volatility based on GARCH class models. This would lead to redesign of the Mincer-Zarnowitz regression into a following form:

$$\sigma_{t+h}=\alpha+\beta\hat{\sigma}_{t+h,t}+\varepsilon_t ,$$

where σ_{t+h} means a realized volatility and $\hat{\sigma}_{t+h,t}$ stands for a forecasted volatility based on information available at time *t*. Thus real values of parameters α, β can be compared with their assumed conditions, which will indicate, whether estimates are unbiased or efficient. A helpful statistics, which can reveal a bias and an inefficiency of forecasts, are standard deviations and p-values[21] of estimated parameters α, β , because they can state, whether α, β parameters differ from imposed conditions on a set level of confidence. And finally also the R-squared statistic of the Mincer-Zarnowitz will show how precise fit estimated forecasts into real values.

3.4. Model Estimations

Proposed models were estimated in their (1,1)[22] form by QMLE using BFGS[23] algorithm in OxEdit 5.10 with G@RCH 4.2 library[24]. Estimations used all 3679 observations available from data series for PX index - 5th April 1994 to 31st March 2009. Estimated coefficients for GARCH (1,1) are in Table 2, volatility was represented by the squared daily returns approximation.

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21 A value of probability at which level the null hypothesis can be rejected in favour of alternative, i.e. $H_0: \alpha=0, A: \alpha\neq 0 ; H_0: \beta=1, A: \beta\neq 1$

22 p=1, q=1

23 BFGS - Broyden–Fletcher–Goldfarb–Shanno method

24 The sample mean of squared residuals was used to start a recursion.

Table 2: GARCH(1,1) Model Estimation

GARCH(1,1)	Coefficient	Std. Dev.	P-value
ω	0.040	0.007	0.000
α	0.154	0.014	0.000
β	0.833	0.014	0.000

The positivity constraint for the GARCH (1,1) was observed $\alpha/(1-\beta)\geq 0$ and also a stationarity condition was fulfilled. The unconditional variance was 3.12439. The condition for existence of the fourth moment assumes that $(\alpha+\beta)^2+2\alpha^2<1$ [25]. The constraint calculated from results of Table 2 equalled 1.02189 and it should be less than unity and thus the condition for existence of the fourth moment of the GARCH (1,1) was not observed in the data set, however this result needs an assumption about normality of residual distribution. In addition there is possibility of error in the estimation of coefficients, which would affect value of the constraint near unity and thus existence of the fourth moment cannot be clearly denied.

All necessary conditions were fulfilled in order to estimate the models. Following tables show estimates of IGARCH(1,1), EGARCH(1,1) and APARCH(1,1) in respective tables:

Table 3: IGARCH(1,1) Model Estimation

GARCH(1,1)	Coefficient	Std. Dev.	P-value
ω	0.033	0.005	0.000
α	0.166	0.014	0.000
β	0.834		

Table 4: EGARCH(1,1) Model Estimation

EGARCH(1,1)	Coefficient	Std. Dev.	P-value
ω	0.580	0.126	0.000
α	-0.056	0.198	0.777
β	0.962	0.008	0.000
θ_1	0.055	0.014	0.000
θ_2	0.274	0.054	0.000

25 Ling, McAleer (2002)

TABLE 5: APARCH(1,1) MODEL ESTIMATION

APARCH(1,1)	Coefficient	Std. Dev.	P-value
ω	0.047	0.012	0.000
α	0.146	0.016	0.000
β	0.850	0.016	0.000
γ	-0.220	0.060	0.000
δ	1.097	0.222	0.000

Estimated results in Table 3 show that model IGARCH (1,1) produced similar outcomes to the GARCH (1,1), which indicates a long persistence of volatility during the time. Estimations of all models show that all coefficients are significantly different from zero[26] and thus it indicates that the coefficients should be used in further forecast estimations. The result of EGARCH (1,1) in Table 4 shows that positive shocks cause more volatility than negative shocks, because both θ parameters are significantly greater than zero.

This is analogous to APARCH (1,1) model, which resulted in all significant parameters, described in Table 5, indicating that they are necessary to further forecasts. The parameters are also different from definitions, which would cause the APARCH model to behave same like ARCH or GARCH models, and it indicates that APARCH model should be used instead of its nested models.

A computed mean of the data series was positive (0.00770), which means positive daily returns on average[27]. An estimated skewness was positive too (0.51206) meaning that it is right-skewed, which implies more positive than negative values. Finally also kurtosis was above zero (15.81541), which indicates that the distribution of the data set is leptokurtic.

3.5. Forecasting Results

The estimations were made with OxEdit 5.10 software including G@RCH 4.2 library using OPG[28] matrices. The estimations were made as was previously defined[29]. The data series was split in ratio 4 to 1, which means that approximately first 12 years i.e. data from 5th April 1994 to 31st March 2006; were used to estimate coefficients of models, which were used in following forecast estimations, while remaining data were used as a benchmark. Estimations were made for 1-step

26 There is only one exception - coefficient alpha in EGARCH(1,1) model.
27 Variance was 2.08186.
28 OPG - outer product of gradients
29 A constant term in the mean equation is included in G@RACH 4.2 at default setting.

(one day), 5-step (one week), 10-step (two weeks) and 20-step (four weeks[30]) forecasts to compare a pace of degradation assumed from computations making forecasts into further future[31]. For a realized volatility was used an approximation based on squared daily returns:

$$\sigma_t^2 = R_t^2 \quad ,$$

$$R_t = \log(P_t / P_{t-1}) \quad ,$$

where P_t denotes a value of PX index at time *t*.

Table 6: Estimations of Forecasts Based on GARCH (1,1) Process

	MSE	TIC	M-Z α	Std. Dev.	P-Value	M-Z β	Std. Dev.	P-Value	R-squared
1-Step Estimation	205.700	0.541	0.595	0.543	0.273	0.925	0.190	0.346	0.236
5-Step Estimation	229.700	0.583	1.208	0.644	0.061	0.812	0.212	0.187	0.158
10-Step Estimation	235.300	0.604	1.277	0.623	0.041	0.836	0.210	0.218	0.141
20-Step Estimation	277.800	0.683	2.807	0.513	0.000	0.488	0.107	0.000	0.033

Table 7: Estimations of Forecasts Based on IGARCH (1,1) Process

	MSE	TIC	M-Z α	Std. Dev.	P-Value	M-Z β	Std. Dev.	P-Value	R-squared
1-Step Estimation	208.200	0.522	0.659	0.533	0.216	0.824	0.169	0.150	0.236
5-Step Estimation	237.200	0.555	1.301	0.624	0.037	0.675	0.176	0.032	0.158
10-Step Estimation	246.200	0.562	1.433	0.594	0.016	0.636	0.160	0.011	0.140
20-Step Estimation	312.600	0.627	2.961	0.511	0.000	0.306	0.068	0.000	0.033

Table 8: Estimations of Forecasts Based on EGARCH (1,1) Process

	MSE	TIC	M-Z α	Std. Dev.	P-Value	M-Z β	Std. Dev.	P-Value	R-squared
1-Step Estimation	213.700	0.639	-0.544	0.680	0.424	1.615	0.304	0.022	0.247
5-Step Estimation	241.200	0.741	-1.122	1.042	0.282	2.232	0.534	0.011	0.173
10-Step Estimation	257.100	0.803	-2.160	1.247	0.083	3.115	0.736	0.002	0.137
20-Step Estimation	276.700	0.866	-2.218	1.083	0.041	3.786	0.805	0.000	0.048

30 This is approximately one month period of time.

31 Estimated forecasts with higher "h" in a h-step estimation term will perform worse forecasts, because the input lag between real and forecasted values increase and thus a larger amount of unpredictable error terms have to estimated.

Table 9: Estimations of Forecasts Based on APARCH (1,1) Process

	MSE	TIC	M-Z α	Std. Dev.	P-Value	M-Z β	Std. Dev.	P-Value	R-squared
1-Step Estimation	217.400	0.628	0.055	0.695	0.937	1.336	0.296	0.128	0.209
5-Step Estimation	235.100	0.692	0.095	0.857	0.911	1.516	0.381	0.088	0.156
10-Step Estimation	253.300	0.755	0.314	0.765	0.682	1.652	0.399	0.051	0.097
20-Step Estimation	273.700	0.831	0.810	0.642	0.207	1.763	0.380	0.022	0.033

The outcomes of forecasting quality criteria estimated for defined estimations are described in Table 6, Table 7, Table 8 and Table 9 for each researched process. Based on a definition of Mincer-Zarnowitz regression[32], estimated forecasts remained unbiased for most of the models until a 5-step estimation at 5% level of confidence[33] according to computed p-values. The GARCH(1,1) and APARCH (1,1) forecasts kept its efficiency till 5-step estimations, while IGARCH(1,1) forecast succeeded only in 1-step estimation and APARCH(1,1) failed at all. The only strong feature of APARCH(1,1) forecasts can be perceived in its unbiasedness, which remained even in case of 20-step estimation.

A minimal difference between GARCH(1,1) and IGARCH(1,1) forecasts indicates that volatility shocks affected a long periods of time, this means that a long term volatility memory effect can be assumed, which was indicated in previous chapter. The best outcome in a term of Mincer-Zarnowitz regression's R-squared has been achieved with EGARCH(1,1) model, which indicates an existence of a asymmetric effects. However the model was not clearly superior to a simple GARCH(1,1) model, which does not take into account a leverage effect at all. This can be proved by worse MSE or TIC values, in addition the forecasts of EGARCH(1,1) were not efficient even for 1-step estimation. So although more complex models performed slightly better in some criteria, the outcomes were not unambiguous and it could not be stated that GARCH(1,1) is inferior to other processes and its results were neither biased nor inefficient.

Thus for simplicity's sake a GARCH(1,1) can be regarded as the best model for further estimations, because its outcomes were fully comparable with other models. This is consistent with findings of study Lunde and Hansen (2005), which stated that the GARCH(1,1) model does not need to be replaced by other more complicated models and it is a sufficient model for forecasting estimations[34].

32 While an assumption of unbiasedness or efficiency cannot be rejected in favour of alternative, which assumes that forecasts are biased or inefficient, I state that models kept proposed features on a particular level of confidence.

33 For any following statements a 5% level of confidence is used as default measure, until other percentage level is explicitly mentioned.

34 Better models were identified only in fractionally integrated models, which complexity is beyond this analysis.

IV. Structural Change Models

Structural change models are used in order to analyse the inner structure of a researched data series and reveal breakpoints, when a structure of perceived real values changes in a substantive manner, so that models have to be estimated independently in sub-periods of a whole data series. In this chapters the original Inclan-Tiao (1994) test, including its successors proposed by Sansó et al. (2003), will be employed to find possible breakpoints detecting changes in unconditional variance, which would indicate a change in index PX inner structure defined by GARCH(1,1) process.

The ICSS algorithm can be used for a detection of influential events as used in Wang (2007), which described a time period including data from the Asian crisis. The results revealed a significant breakpoint, which occurred during the crisis and thus confirmed a structural change arising from an important financial event. Also Morales and Andreosso-O'Callaghan (2008) employed ICSS algorithm in order to reveal significant breakpoints of various indices coinciding with important global events.

The performance of the proposed models will be then tested through the forecasting abilities, which should differ during periods of time when breakpoints occur, because a substantial change in the structure would disallow any possibility of precise forecasts[35]. A rising number of breakpoints should lower the precision of forecasts and thus it can be reversely tested, which model revealed real breakpoints or which breakpoints were spuriously estimated and also whether some of them lack certain breakpoints.

4.1. The Inclan-Tiao Test

The purpose of the Inclan-Tiao test is to analyse, whether there are one or more structural breakpoints, which would divide a researched time series into different periods in terms of different unconditional variance. This test is based on ICSS[36] method, where is initially estimated intended process, which should describe a time series. The resulted residuals are a basis to count sum of squares, which are cumulated and iterated through a next step of the estimation process in order to test the null hypothesis of constant unconditional variance. The concrete description of the process follows.

Inclan and Tiao (1994) proposed to use the statistic given by

35 This is consistent with a definition of structural breakpoints, which should find important changes during estimated period of time in terms of defined processes.

36 Iterated Cumulated Sum of Squares

$$IT = \sup_k |\sqrt{T/2} D_k|$$

where

$$D_k = \frac{C_k}{C_t} - \frac{k}{T}$$

, $C_k = \sum_{t=1}^{k} \varepsilon_t^2, k = 1, \ldots, T$ is the cumulative sum of squares of ε_t . Under the assumption that ε_t are a zero-mean, normally, identically and independently distributed random variables, $\varepsilon_t \sim iidN(0, \sigma^2)$, the asymptotic distribution of the test is given by:

$$IT \Rightarrow \sup_r |W^*(r)|$$

where is $W^*(r) \equiv W(r) - rW(1)$ a Brownian Bridge, $W(r)$ is a standard Brownian motion and $\Rightarrow$ stands for weak convergence of the associated probability measures.

There is a drawback of the IT test is that its asymptotic distribution free of nuisance parameters critically depends on the assumption of normally, independently and identically distributed random variables ε_t . Hence SANSÓ ET AL. (2003) proposed new types of test called Kappa 1 and Kappa 2.

4.2. Kappa Tests

The original Inclan Tiao test is based on the assumption that the disturbances are independent and Gaussian distributed, which means that conditions could be considered as too strong for financial time series. The financial series show empirical distributions with fat tails (leptokurtic) and persistence in the unconditional variance. Thus the successors of the original Inclan-Tiao test are able to cope with possible problems arising from a nature of financial data series.

The first type of the Kappa tests resolves a possible problem with fourth moment of a researched data set. This problem with fourth moment is very common for a real financial stock market data as in ALFARANO ET AL. (2008). It was also shown that this existence of the fourth moment cause that Inclan-Tiao test is not effective and it overestimates number of structural break points and thus adjusted models should be used as in ANDREOU AND GHYSELS (2001). The Kappa 1 test is tackling a theme of the forth moments in financial data series, while The Kappa 2 test is trying to solve a problem arising from a usage on conditionally heteroskedastic variance processes.

Kappa tests are based on a modified technique of ICSS test algorithm and its critical values are computed via Monte Carlo method using 50,000 estimations for various numbers of observations, which ensures that estimated results will be precise enough for a general usage. These adjustments are being made to prevent estimations of spurious breakpoints, which would invalidate results of the analysis and could lead to a misinterpretations.

4.2.1. Kappa 1 Test

The existence of the fourth moment in real financial data is almost natural, because they tend to have fat tails, which is a result of investors' behaviour on financial markets[37]. Kappa 1 test of Sansó et al. (2003) is based on a further generalization of Inclan-Tiao test. It assumes that if $\varepsilon_t \sim iid$ and there exists finite fourth moment $E\left(s_t^4\right) \equiv \eta_4 < \infty$, then the result of the Inclan Tiao statistic should be modified as follows:

$$IT \Rightarrow \sqrt{\frac{\eta_4 - \sigma^4}{2\sigma^4}} \sup_r \left|W^*(r)\right|$$

And thus the distribution includes nuisance parameters, which can bias estimated results. Important distortions should be expected when the critical values of the supremum of a Brownian Bridge are calculated. For classical Gaussian processes, where $\eta_4 = 3\sigma^4$ the value of Inclan Tiao test statistic remain unchanged, which namely means $IT \Rightarrow \sup_r \left|W^*(r)\right|$. In case of $\eta_4 > 3\sigma^4$, the distribution can be described as leptokurtic and thus more rejections of the null hypothesis of constant variance should be expected, with an effective size greater than the nominal one. On the other hand, when $\eta_4 < 3\sigma^4$ the test will be simply too prudent. Proposed consequences suggest that following correction to the original Inclan-Tiao test should be incorporated in order to remove mentioned nuisance parameters for identical and independent zero-mean random variables as specified in following Kappa 1 test:

37 In addition an existence of the fourth moment was not rejected in estimated index PX data series.

$$\kappa_1 = \sup_k \left| T^{-1/2} B_k \right| \quad \text{where}$$

$$B_k = \frac{C_k - \frac{k}{T} C_T}{\sqrt{\hat{\eta}_4 - \hat{\sigma}^4}}$$

$$\hat{\eta}_4 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^4 \quad \text{and} \quad \hat{\sigma}^2 = T^{-1} C_T \quad .$$

Asymptotic distribution of the test is set as follows:

If $\varepsilon_t \sim iid$, and $E\left(s_t^4\right) \equiv \eta_4 < \infty$, then $\kappa_1 \Rightarrow \sup_r \left| W^*(r) \right|$

Calculated sample critical values have been included in ICSS library developed by Rapach and Strauss (2008) and reprogrammed by Sansó et al. (2003) in GAUSS language, which calculations were conducted in OxGauss 5.10 extension in OxEdit 5.10.

4.2.2. Kappa 2 Test

Although the Kappa 1 test brought generalization to Inclan Tiao test including an assumption of non-constant fourth moment, which is a typical case of financial market data, it is still dependent on an assumption of random variables independence. This is a very strict condition for financial data, because there is evidence of conditional heteroskedasticity in this kind of data samples as proved in Bollerslev et al. (1992, 1994). This fact requires to take into account the essence of heteroskedasticity in order to correct the cumulative sum of squares algorithm.

Sansó et al. (2003) assumed that the data sample can be described as a sequence of random variables $\{\varepsilon_t\}_{t=1}^{\infty}$ and that it is consistent with following conditions:

1) $E(\varepsilon_t) = 0, E(\varepsilon_t^2) = \sigma^2 < \infty, \forall t \geq 1;$

2) $\sup_t E\left(|\varepsilon_t|^{\psi+\epsilon}\right) < \infty, \psi \geq 4, \epsilon > 0$

3) $\exists \omega_4 = \lim_{T\to\infty} E\left(T^{-1}\left(\sum_{t=1}^{T}\left(\varepsilon_t^2 - \sigma^2\right)\right)\right) < \infty$

4) $\{\varepsilon_t\}$ $is\ \ \alpha - mixing\ \ with\ \ coefficient\ \ \alpha_j, when \sum_{j=1}^{\infty} \alpha_j^{1-2/\psi} < \infty$

The condition 1) is describing zero expected value of ε_t and also its finite variance. Conditions 2) and 3) state that ε_t in the data sequence cannot be independent and identically distributed as a t-Student with three degrees of freedom. ω_4 in a condition 4) is describing long-run fourth moment of ε_t or a long-run variance of the zero mean variable $\xi_t \equiv \varepsilon_t^2 - \sigma_t^2$. The last condition is handling "degree of independence" of data sequence and display a trade-off relation of the serial dependence and the "high order moments" existence. Imposed finiteness of the fourth moments however does not exclude serial dependence of higher degrees.

Those stated conditions led Sansó et al. (2003) to establish a following statistic:

$$\kappa_2 = \sup_k \left| T^{-1/2} G_k \right| \quad ,$$

where

$$G_k = \hat{\omega}_4^{-1/2} \left(C_k - \frac{k}{T} C_T \right) \quad .$$

$\hat{\omega}_4$ has to be a consistent estimator of ω_4 , while Sansó et al.. (2003) decided to compute following non-parametric estimator of ω_4

$$\hat{\omega}_4 = \frac{1}{T} \sum_{t=1}^{T} \left(\varepsilon_t^2 - \hat{\sigma}^2 \right)^2 + \frac{2}{T} \sum_{t=1}^{m} \omega(\text{l,m}) \sum_{t=l+1}^{T} \left(\varepsilon_t^2 - \hat{\sigma}^2 \right) \left(\varepsilon_{t-1}^2 - \hat{\sigma}^2 \right) \quad ,$$

where $\omega(\text{l,m})$ is a lag window defined as $\omega(\text{l,m}) = 1 - \frac{l}{(m+1)}$. It should be added that if $\xi_t = \varepsilon_t^2 - \hat{\sigma}^2$ then $\hat{\omega}_4 \rightarrow E\left(\xi_t^2\right) = \eta_4 - \sigma^4$. Described assumptions 1) to 4) cause that Inclan Tiao, Kappa 1 and Kappa 2 tests will have following:

$$IT \Rightarrow \sqrt{\frac{\omega_4}{2\sigma^4}} \sup_r \left| W^*(r) \right|$$

$$\kappa_1 \Rightarrow \sqrt{\frac{\omega_4}{\eta_4 - \sigma^4}} \sup_r \left| W^*(r) \right|$$

$$\kappa_2 \Rightarrow \sup_r \left| W^*(r) \right|$$

These particular equations were used by Sansó et al. (2003) to compute critical values for mentioned tests.

4.3. Results Analysis

Breakpoints were calculated using OxEdit 5.10 and OxGauss 5.10 extension with ICSS library developed by Rapach and Strauss (2008) and reprogrammed by Sansó et al.. (2003) in GAUSS language. The data series included 3678 samples i.e. from 5th April 1994 to 31st March 2009. The programmed algorithm identified as breakpoints also starting and ending dates, which can be omitted, but their inclusion help to better perceive periods limited by real breakpoints[38]. Table 10 shows particular breakpoints with corresponding dates of observations for all three tests.

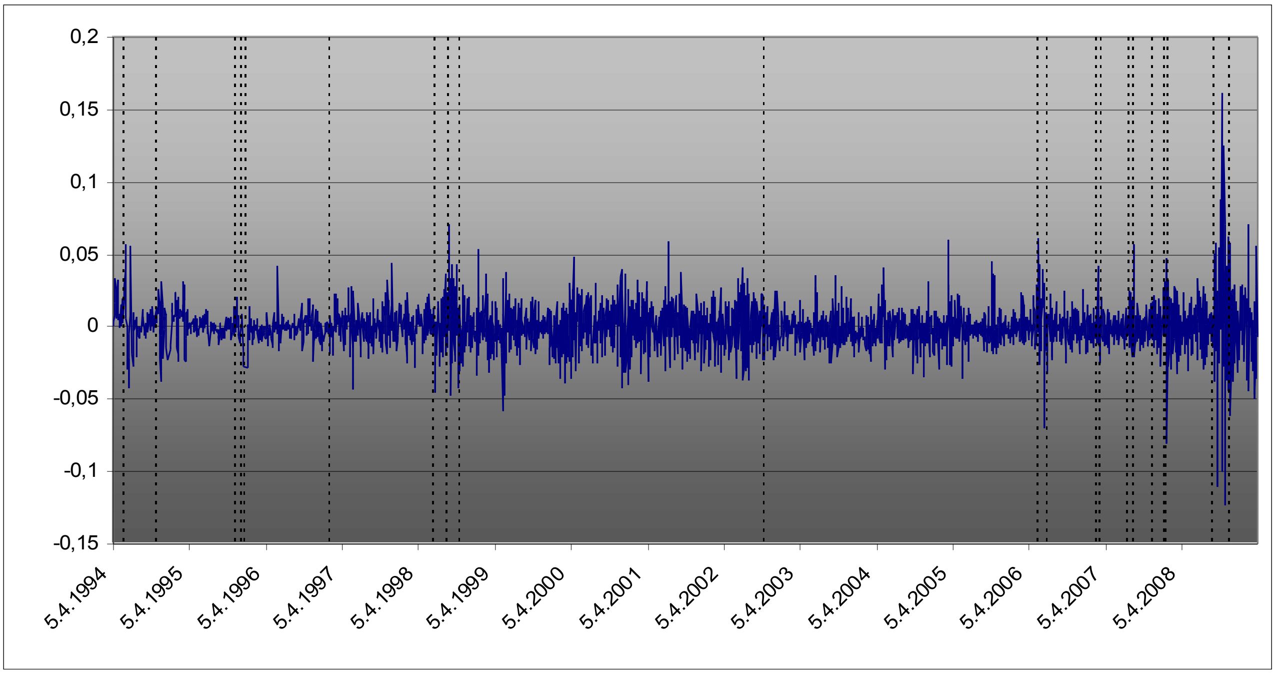
Table 10: List of Breakpoints with Dates of Observations

Inclan Tiao		Kappa 1		Kappa 2	
Breakpoint	Date	Breakpoint	Date	Breakpoint	Date
1	05/04/1994	1	05/04/1994	1	05/04/1994
22	24/05/1994	22	24/05/1994	3360	19/12/2007
100	01/11/1994	39	07/07/1994	3678	31/03/2009
334	10/11/1995	100	01/11/1994		
355	13/12/1995	176	15/03/1995		
358	08/01/1996	625	03/02/1997		
625	03/02/1997	970	18/06/1998		
970	18/06/1998	1015	24/08/1998		
1015	24/08/1998	1055	19/10/1998		
1055	19/10/1998	2058	16/10/2002		
2058	16/10/2002	3254	23/07/2007		
2958	17/05/2006	3536	02/09/2008		
2988	28/06/2006	3591	20/11/2008		
3152	22/02/2007	3678	31/03/2009		
3164	12/03/2007				
3254	23/07/2007				
3274	20/08/2007				
3337	16/11/2007				
3375	16/01/2008				
3381	24/01/2008				
3536	02/09/2008				
3591	20/11/2008				
3678	31/03/2009				

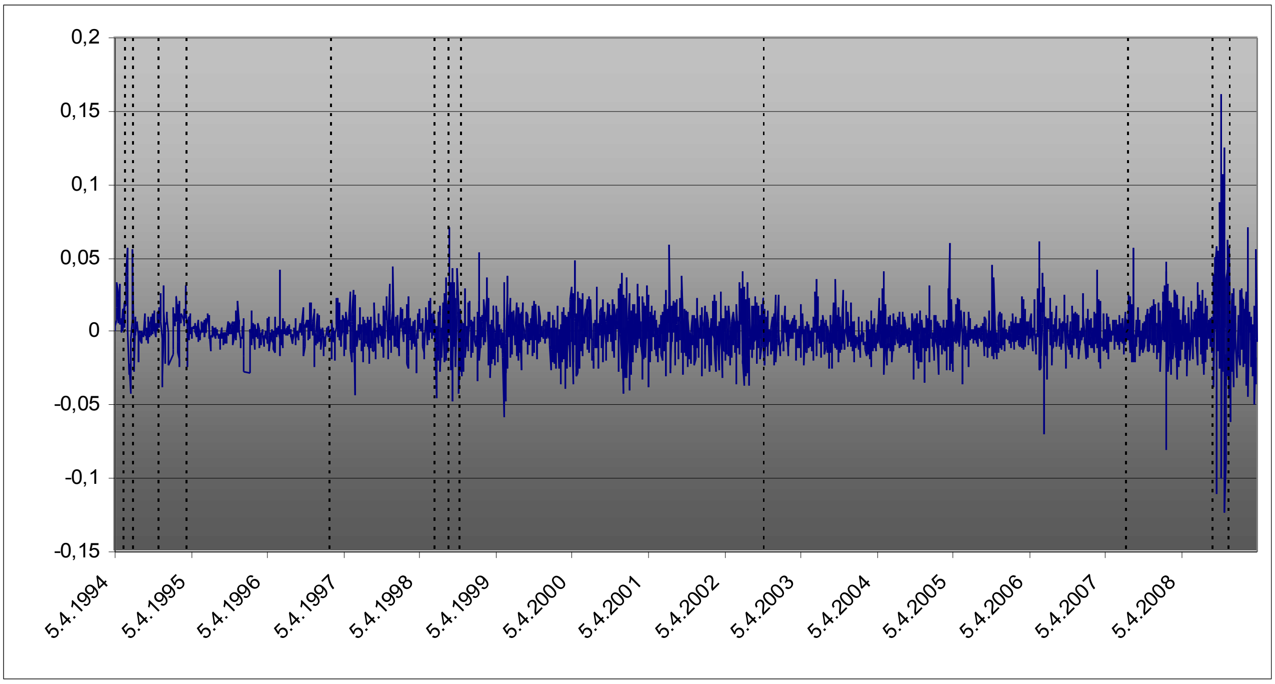
38 Number of real breakpoints is lower by two than in table, number of periods is lower by one.

Following graphs depict structural breaks with a comparison to the researched data series, it shows that Inclan-Tiao and Kappa 1 tests detected more short-term shocks such an sudden increase in volatility during end of year 2008, while Kappa 2 test divided the whole data sets just into two parts.

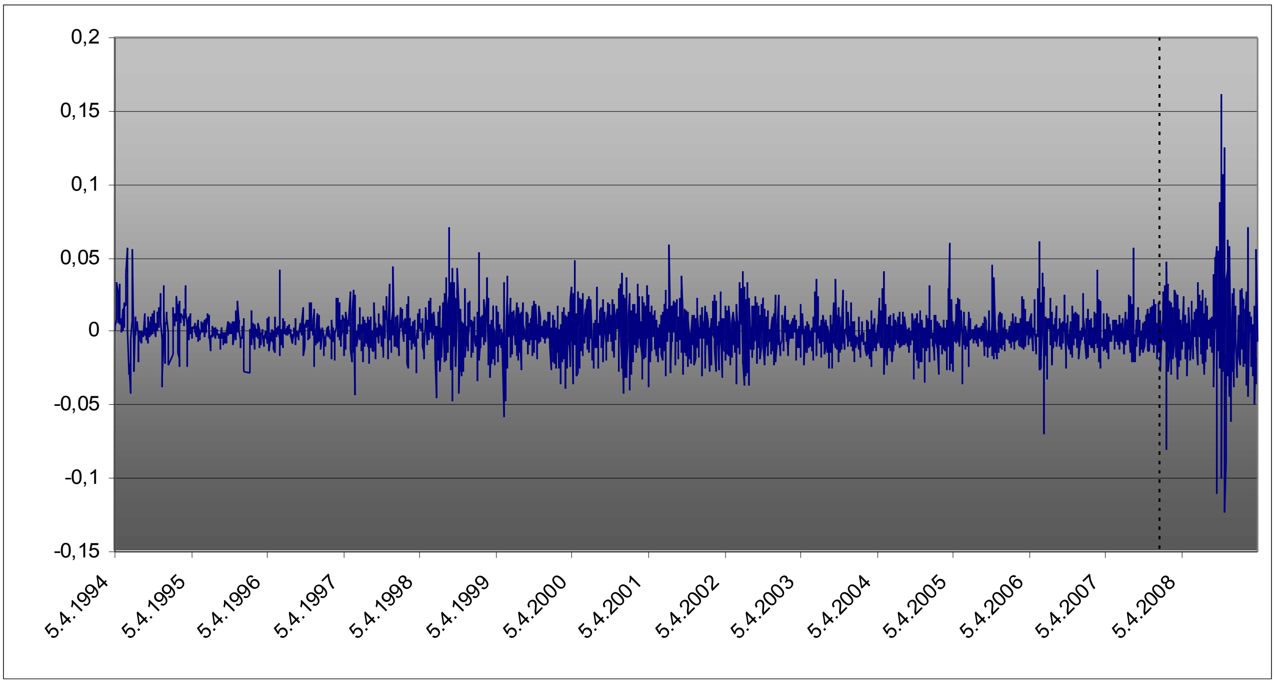
Graph 4: PX Index Daily Returns and Detected Breakpoints - Inclan Tiao Test



Graph 5: PX Index Daily Returns and Detected Breakpoints - Kappa 1 Test



Graph 6: PX Index Daily Returns and Detected Breakpoints - Kappa 2 Test



In order to test a hypothesis that a higher number of breakpoints implies worse forecasting abilities and on the contrary a lower number of breakpoints allows better forecasts, the whole observed data series was divided into five periods containing three-years of observations i.e. 1st period: 5th April 1994 - 31st March 1997; 2nd period: 1st April 1997 - 31st March 2000; 3rd period: 1st April 2000 - 31st March 2003; 4th period: 1st April 2003 - 31st March 2006; 5th period: 1st April 2006 - 31st March 2009; Table 11 indicates number of breakpoints computed by particular tests in each period.

Table 11: Number of Breakpoints in Defined Periods

	Inclan-Tiao	Kappa 1	Kappa 2
Period 1	6	5	0
Period 2	3	3	0
Period 3	1	1	0
Period 4	0	0	0
Period 5	11	3	1

Then a GARCH (1,1) process was used for a testing of the hypothesis, when one period was used as a basis for an estimation of parameters, which were used in computations of forecasts using similar techniques as in previous chapters, and a following period was used as a benchmark for estimated 1-step or 5-step forecasts. Following tables show quality of results using defined quality criteria.

TABLE 12: 1-STEP FORECASTS USING GARCH(1,1) PROCESS IN DEFINED PERIODS

1-Step Estimation

	MSE	TIC	MZ α	Std. Dev.	P-Value	MZ β	Std. Dev.	P-Value	R-squared
Period 1-2	2.598	0.655	0.347	0.084	0.000	0.316	0.133	0.000	0.011
Period 2-3	2.603	0.684	0.271	0.144	0.061	0.536	0.265	0.040	0.009
Period 3-4	2.673	0.621	0.149	0.198	0.452	0.488	0.241	0.017	0.009
Period 4-5	2.790	0.605	-0.246	0.418	0.556	0.796	0.421	0.314	0.008

TABLE 13: 5-STEP FORECASTS USING GARCH(1,1) PROCESS IN DEFINED PERIODS

5-Step Estimation

	MSE	TIC	MZ α	Std. Dev.	P-Value	MZ β	Std. Dev.	P-Value	R-squared
Period 1-2	2.567	0.656	0.320	0.127	0.012	0.336	0.212	0.001	0.009
Period 2-3	2.601	0.691	0.266	0.214	0.214	0.543	0.397	0.125	0.006
Period 3-4	2.735	0.613	0.161	0.282	0.567	0.433	0.313	0.035	0.006
Period 4-5	2.874	0.601	-0.148	0.552	0.788	0.669	0.523	0.264	0.004

According to quality of forecasts evaluated by R-squared of Mincer-Zarnowitz regression, which reveal a goodness of fit of forecasts to real values, forecasts made with a usage of new setting of period shows much worse fitness with a comparison to previously forecasted values. However this does not mean that new forecasts are bad, because a proper measure of realized volatility has to be chosen according to ANDERSEN AND BOLLERSLEV (1998) and in this case the realized volatility was replaced by an approximation based on squared daily returns. Thus a data with higher frequency should be used to fully utilise the power of autoregressive conditional heteroskedasticity models.

When the quality of forecasts should be compared with a proposed hypothesis based on number of breakpoints, the worst forecasts should be perceived in period 4-5 according to Inclan-Tiao and Kappa 2 tests, however Inclan-Tiao also suggests that the best forecasts can be computed in period 3-4, which is not consistent with the results. On the other hand the Kappa 1 test is not consistent neither with the best forecasts nor with the worst forecasts. Thus only Kappa 2 test is consistent with both statements, because it suggests only that during periods 1-2, 2-3 and 3-4 the forecasts should be better that in period 4-5.

A next comparison between forecasting abilities and the proposed existence of breakpoints by particular test statistics was made in the way that breakpoints indicated periods within the forecasts would be computed. It namely means that the data between each two breakpoints were split into two parts in ratio 2 to 1, when a basis for the estimation of forecasts used two thirds of the subsample and one third served as a benchmark for estimated forecasts. The forecasts were made only for 1-step estimation and the only quality criterion was R-squared obtained from the Mincer-Zarnowitz regression[39]. The following table shows resulted R-squared with matching period.

Table 14: Mincer-Zarnowitz R-squared for Defined Subsamples[40]

Inclan Tiao		Kappa 1		Kappa 2	
Period	R-squared	Period	R-squared	Period	R-squared
1	0.0438	1	0.0438	1	0.0405
2	0.0402	2	0.1114	2	0.0222
3	0.1626	3	0.0341		
4	0.0097	4	0.0004		
5	N/A	5	0.1593		
6	0.0788	6	0.0651		
7	0.0651	7	0.0018		
8	0.0018	8	0.2580		
9	0.2580	9	0.0003		
10	0.0003	10	0.0283		
11	0.0004	11	0.0670		
12	0.0486	12	0.0718		
13	0.0104	13	0.0038		
14	N/A				
15	0.0003				
16	0.1513				
17	0.0632				
18	0.0005				
19	N/A				
20	0.0070				
21	0.0718				
22	0.0038				

The results obtained from the Table 14 suggest that Inclan-Tiao test marks too many breakpoints, because some subsamples contained even less that 15 observations, which cannot be enough to reveal a real structure or even to make forecasts based on the data series. However when this drawback is omitted, the results suggest that Inclan-Tiao test sorted the data sample into three main groups: the first group can be characterised by mediocre/good forecasting abilities inside of the subsample using GARCH(1,1) process, which means R-squared above 10% level; second group shows subsamples, in which GARCH(1,1) achieved worse results, R-squared exceeded 1%, but were below 10% level; third group contains subsamples, which contains nearly unforecastable data,

39 According to previous findings additional criteria would be redundant.
40 N/A in the table means that there was not enough observations to compute forecasts.

R-squared was even lower than 1%.

When a Kappa 1 test results are analysed, the same sorting can be perceived and even the Kappa 1 test did not make any breakpoints, which would prevent an estimations of forecasts. Thus in this task Kappa 1 test performed better than the original Inclan-Tiao test. On the other hand the Kappa 2 test did not sort the data sample into different "classes" of subsamples, it only divided the original data series into two parts, where GARCH(1,1) performed roughly same. Alas as it was already stated, this outcome does not reveal whether GARCH (1,1) was not appropriate for the estimations[41].

Although models researching structural changes stated interesting results, which would suggest a precise dates to structural breaks, their results are not very consistent, when their abilities were deeply analysed, and thus the results should be cautiously interpreted. Inclan-Tiao failed during both tests, Kappa 1 and Kappa 2 performed better, but the results were not unambiguous. When results from the Kappa 1 test would be regarded as the most precise, it would suggest that most of the structural breaks occurred before 1998, which could be regarded as an early stage of development of PSE, according to events listed in the Czech market overview chapter, characterised by frequent structural changes. And then there is a period of time coincidental with world financial crisis starting in 2008, when also structural changes occurred in higher amount. Kappa 2 test identified only one structural break, which occurred after a merger of minor and main markets and prior a world financial crisis in 2008.

41 Andersen and Bollerslev (1998) suggest data series with higher frequencies or an appropriate evaluation of volatility.

V. Volatility Spillover Effect Models

There was a vast amount of significant events, which changed the structure of domestic and world equity markets and indicated that world economics should be reconsidered in a new context. This means a reconsideration based on a new understanding of information highways, which became a standard piece of our lives. This outlook is important for investors realizing investments in all markets and also raised a lot of questions about volatility spillovers between related markets, when some markets experienced even simultaneous incidents.

Reasons for increased market dependencies and an occurrence of a similar behaviour could be various. International spillovers may be associated in cases of cross-listed securities in various markets, which is analogous for an increasing number of abroad listed depositary receipts representing domestic securities. The international trade can affect the correlations of consumption and business cycles across countries. This will enhance the level of economic and financial integration process as was described in Ng (2000), which suggested stronger links in regional markets and also described a significance in volatility transmission in case of local developed and emerging markets.

An increase in the degree of market integration into international structures can be a significant event, which can change a correlation among interconnected markets as was shown in Cappiello et al. (2006). Furthermore also periods of crises tightened interlinks between equity markets as showed Saleem (2008) or Ng (2000), which described a precise turbulent events resulting from a contagion of equity markets. All these information and relations can be powerful tools, which can be useful in case of a search for different stages of development especially in the Czech Republic.

Useful aspects for a country's stage of a liberalization process and a common evolution of the equity market can be described in point of view of the volatility spillovers. This can be related to a situation of PSE, which dramatically changed from its beginning to the status in the 3rd millennium. I will investigate whether a development and a strong integration processes have affected forces guiding volatility and cross-market correlations at PSE in comparison with other developed markets. Namely the models offer to trace back an intensity of transmission mechanisms. This research opens a possibility of perception of interlink between PSE and other developed capital markets, which can answer whether or when PSE became a part of global markets and also determine at what extent it occurred. On a field of volatility spillover effects there are two main classes of models, it namely means univariate models and multivariate models.

5.1. Univariate Models

Although univariate models are only capable to capture a single data series, there are options, which enable to quantify a volatility spillovers between surveyed markets. BAELE(2003) suggested to compute a complex system of estimations with various conditions as shows Appendix IV. The system employs residuals of primary univariate models into cross-sectional estimations, where are residuals denoted as independent variables. There is way to estimate conditional correlation on a basis of forecasts made from the final model, which was composed from all univariate residuals, all cross-sectional residuals and also auxiliary models[42].

However there are weaknesses, which are embedded into this method. At first the system of equations is rigid and it can not be flexibly used for greater amount of variables, which limits the outcomes of a research, because when there are more variables involved in the analysis, it is necessary to impose additional conditions on can result in incompatibility. Secondly the essence of estimations is based on forecasts, which have shown insufficient results during some time periods as proved in previous chapters of the thesis.

5.2. Multivariate Models

A multivariate approach to volatility spillover analysis is much more flexible, because it treats all variables equally and it does not require manipulations with input data series in a case of more estimated variables. One of the most popular multivariate GARCH models is constant conditionally correlation multivariate GARCH model proposed in BOLLERSLEV (1990), which can be defined in following way[43]:

$$H_t = D_t R D_t \quad ,$$

where $D_t = diag\left\{\sqrt{h_{i,j}}\right\}$

$$E_{t-1}\left(\varepsilon_t \varepsilon_t{'}\right) = D_t^{-1} H_t D_t^{-1}$$

$$\varepsilon_t = D_t^{-1} r_t$$

$$r_t | \Psi_{t-1} \sim N(0, H_t)$$

R denotes a correlation matrix, which contains conditional correlations, *r* stands for random

42 For further information see Ng(2000) or Baele(2003).

43 This is proposed form of the CCC MVGARCH model used in Engle (2002) for further generalisation into DCC MV GARCH.

variables, which are assumed to be normally distributed, and *h* are standalone univariate GARCH models. This model brings significant advantages over previously mentioned univariate approach, because it has less number of parameters and is relatively simple to estimate[44]. However there are also drawbacks included in the model, it means an assumption of a conditional correlations, which can be only extended by a band of confidence, and it disallows to perceive changes of conditional correlations during estimated time period. Thus a generalization of CCC MVGARCH was proposed in order to eliminate these flaws, which enabled a dynamization of the conditional correlations and resulted in the dynamic conditional correlation MVGARCH model.

5.2.1. Dynamic Conditional Correlation Multivariate GARCH model

One of the sophisticated econometric models, which is able to show volatility spillover effects across different countries in selected data sample, is DCC MVGARCH model described by ENGLE (2002).

The model is defined as follows see also ENGLE (2002):

$$r_t|\Psi_{t-1} \sim N(0, D_t R_t D_t) \quad (1)$$
$$D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} r_{t-1} r'_{t-1} + diag\{\gamma_i\} D_{t-1}^2 \quad (2)$$
$$\varepsilon_t = D_t^{-1} r_t \quad (3)$$
$$Q_t = S(\iota\iota' - A - B) + A\varepsilon_{t-1}\varepsilon'_{t-1} + BQ_{t-1} \quad (4)$$
$$R_t = diag\{Q_t\}^{-1} Q_t diag Q_t^{-1} \quad (5)$$

A relation (1) describes an assumption of normality. An equation (2) expresses the assumption that each subset follow an univariate GARCH process. (3) describes behaviour of residual terms and finally (4) and (5) describe matrix composition necessary for the estimation and iteration processes. Without the assumption of normality in (1), the estimator would be only QME. The log likelihood for the estimator is following:

$$\log(L) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t'\varepsilon_t + \log|R_t| + \varepsilon_t' R_t^{-1}\varepsilon_t\right)$$

which is being maximised through estimated parameters. The log-likelihood can be further divided into two parts

44 As proposed in Nakatani and Teräsvirta (2006).

$$\log(L)(\theta,\phi)=\log(L_V)(\theta)+\log(L_C)(\theta,\phi)$$

$$\log(L_V)(\theta)=-\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log(2\pi)+\log(h_{i,t})+\frac{r_{i,t}^2}{h_{i,t}}\right) \quad ,$$

which shows that this part reflecting volatility is a sum of individual univariate GARCH log-likelihoods, which can be maximized separately. This emphasize a need of prior estimations of all involved univariate GARCH models. While a second term describing conditional correlation parameters is maximized individually meaning a two stage estimation.

$$\hat{\theta}=arg\,max\left\{L_V(\theta)\right\}$$

$$max\,\underline{\phi}\left\{L_C(\hat{\theta},\phi)\right\}$$ [45]

These definitions can be adjusted to fit into elliptical distribution, which includes other nested distributions i.e. normal, Student, LaPlace and exponential power distributions; as used in Pelagatti and Rondena (2004), who incorporated this in their MultiGARCH library[46]. The elliptical distribution has following likelihood function[47]:

$$l(\theta)=\sum_{t=1}^{T}\{\log c_m-\frac{1}{2}\log|\Sigma_t|+\log g(r_t\Sigma_t^{-1}r'_t)\} \quad (6)$$

Because their results stated that normal distribution performed very well, in a comparison to other distributions, I used it in estimations for a simplicity's sake. A final estimation of the model consists of three steps. In the first step univariate GARCH models are estimated for each data set and the resulting coefficients ω,α,β of equation (2) are used for next step as starting values. Next step begins recursion and following estimation of (3) and also residuals estimated in step 1 are used as estimate of matrix S in equation (4). Finally a third step evaluating dynamical conditional correlation is made fully automatically through MultiGARCH library.[48]

45 The particular algorithm maximizing log-likelihood of the conditional correlation part is described in Appendix II.
46 MultiGARCH library is a package used for DCC MVGARCH estimation.
47 The estimation process is divided into original code and redesigned routines of MultiGARCH package.
48 The particular algorithm used in the library is described in Appendix III.

5.3. Data Description

The main goal of the analysis is to describe stages of PSE development and its relations to other advanced markets, which could indicate whether PSE became also a part of developed market. It can be assumed that the Czech Republic is mostly dependent on European markets and thus also European indices mostly occur in the data series, which is enriched by two other important stock exchanges represented by USA and Japan. Because different indices listed in one country tend to act simultaneously and thus it would not improve the outcome, only one representative is chosen from each country i.e. ATX in Austria, BEL 20 in Belgium, CAC 40 in France, FTSE 100 in Great Britain, DAX 30 in Germany, NIKKEI 225 in Japan, AEX in Netherlands, IGBM in Spain, OMX SPI in Sweden, SMI in Switzerland, NYSE 100 in USA and finally PX index traded in the Czech Republic on Prague Stock Exchange, which is clearly irreplaceable in the analysis. This means that a whole data sample includes 12 national indices dating from 5th April 1994[49] until 30th March 2009 and thus an analyses of many important events of a recent economical history are available. For the purpose of clarity the names of variables are described by abbreviations of names of states instead of indices.

Data estimated in the routine were calculated in following form:

$$R_t = \log(P_t / P_{t-1}) \times 100 \quad ,$$

where P_t stands for closing value of computed index. This means that input values of national stock indices were transformed into daily net returns R_t computed as Close-to-Close value in percentages. When a expression net daily return is mentioned it is important also to clarify from which point of view they are computed to be net, because there are two basic choices. The first one take into account only daily returns of local investor, who invests into national stocks and thus in my case into a particular national index. On the other hand there is another option, which takes into account real net returns adjusted by exchange rate effects, which are important for investors investing on global markets, who utilise benefits from international diversification. This means that they are interested in strategies incorporating also a currency risk, which is significantly affecting a success of their strategies.

It is common to use daily returns denominated in local-national currencies as in Diebold (2007) or Cappiello et al. (2006). However it is possible also to test dynamic conditional correlation

49 Initial date was set as a beginning day of Czech national index PX, which is the latest stock exchange index in the sample. At 5th April 1994 the value of PX 50 was set to initial value 1000.

among currencies as in KITAMURA (2007) was presented, which implies that a synthesis of these analyses would result into a point of view of fully informed investor, who is able to modify his strategy according to all available data. Thus a following analysis is conducted for both types of data i.e. net index returns[50] and also adjusted net index returns[51].

Because of a lack of data sample synchronization[52] an original samples obtained from data servers[53] sorting routine in OxEdit 5.10 was used, which approved only opening dates, which were common for all countries, in order to minimize possible problems during DCC MVGARCH model estimation, which could occur when matrices are being inverted. This is a common problem of DCC MVGARCH studies, which use rather weekly or averaged weekly data free of 'holiday-gaps'. But the data sample based on weekly data would offer only 780 samples for 15 years, which is approximately 4 times less than was achieved with a sorting procedure, which resulted in 3174 samples. This implies that the precision of the output should be higher than e.g. in CAPPIELLO ET AL.. (2006) or DIEBOLD (2007). All values of net returns and adjusted net returns, which were used in the analysis, are depicted in Appendix V.

5.4. Result analysis

Using an programmed procedures and the econometric software OxEdit 5.10 [54] univariate GARCH(1,1) processes were computed for each particular national index using both data sets, which is depicted in Table 15 and Table 16[55]. As was mentioned this is a basis for a next step of a DCC MVGARCH analysis. At this stage results in both tables confirmed that all estimated models fulfilled necessary conditions for both data sets of net returns and adjusted net returns - parameters were positive $\omega, \alpha_i \geq 0, \beta_i \geq 0$ and also all processes were stationary $\alpha + \beta < 1$. From this point the result analysis is divided into two parts i.e. the analysis of net returns and the analysis of adjusted net returns.

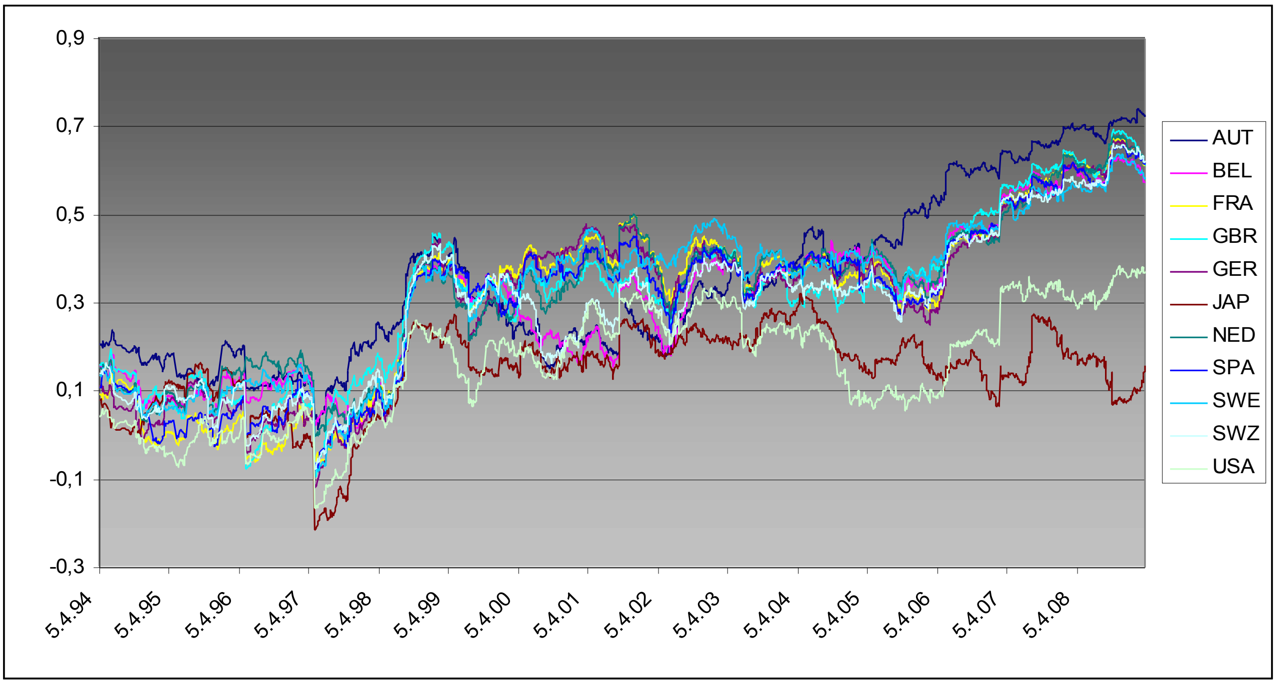
5.4.1. Net Returns

The DCC MVGARCH model was successfully estimated and thus its all necessary

50 Net index returns denote net index returns without exchange rate effects.
51 Adjusted net index returns include exchange rate effects and thus can be qualified as real net returns.
52 i.e. that it is common that some exchanges close on holidays, which are unique in their countries and thus list of dates, when are stock exchanges open, is specific for a particular country.
53 Data have been gathered from yahoo.finance.com, PSE and also CNB through www.kurzy.cz database.
54 including package G@rch 4.2 and package MultiGarch 0.3
55 See on pages 69 to 70.

conditions were fulfilled, otherwise the convergence of the model would not be achieved, because the model is very sensitive to input data. Conditional correlations estimated by DCC MVGARCH in Graph 7 model shows a gradual increasing trend of interdependencies of Czech capital market among nearly all perceived data sets. This can be interpreted as a gradually increasing interdependence of Czech stock market to developed markets. A very interesting consequence of the output shows that this gradual integration of Czech stock exchange is common for all remaining data sets including relatively far Sweden, which is not even a part of EMU similarly to Switzerland and Great Britain. This proves that capital market interrelations are deepening without regards to membership in EMU. However there are two exceptions. Japan and USA indices behave differently and stay in a -0.15 to 0.4 band of correlation for all the time, this can be perceived in individual graphs of conditional correlation in Appendix I also with individual conditional covariances.

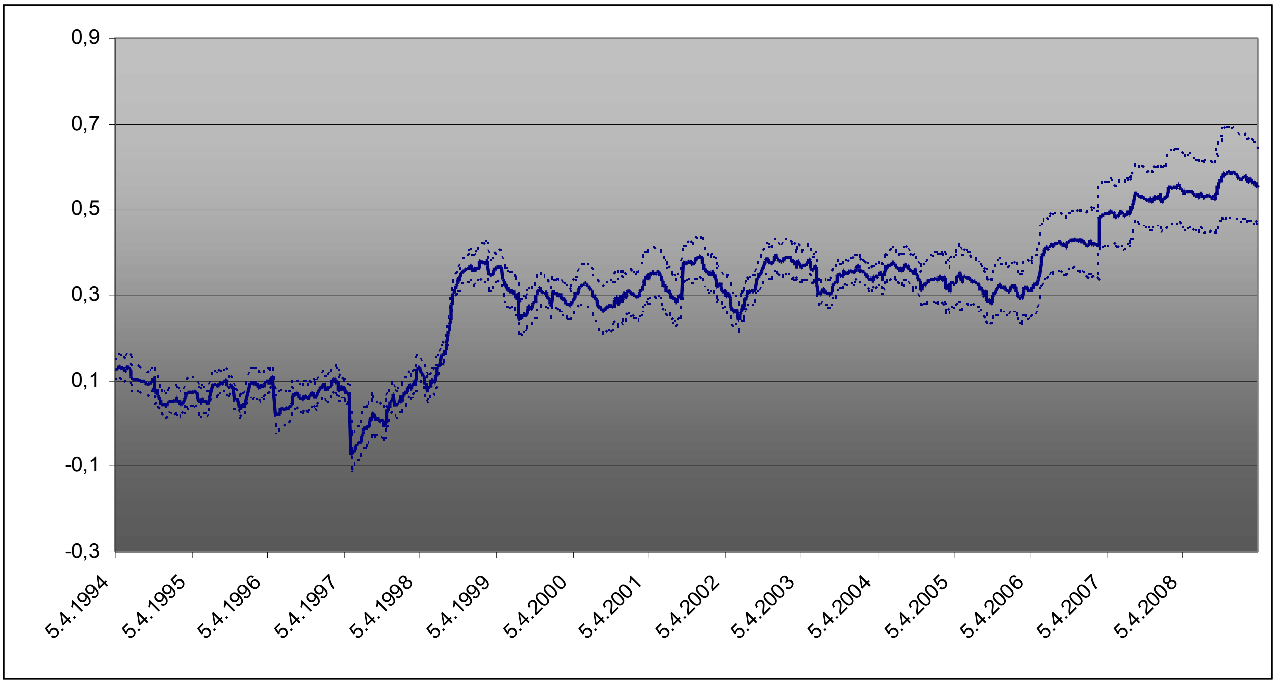
Graph 7: Aggregated Conditional Correlations - Net Returns



The most illustrative picture of a typical behaviour of the correlation can be achieved through a computation of the expected value based on values of all estimated correlations. This approach is similar to Cappiello et al. (2006), where average correlations are computed for particular regions. Thus if the average of all estimated correlations is computed, the result is an average correlation to world markets from point of view of the Czech Republic. This computed measure will be named in the rest of the document as the 'average world correlation' for a simplicity's sake. The final outcome of the average correlation is in Graph 8, which is even amended with its band of confidence calculated for 95% level of confidence and based on the

Student distribution[56].

Graph 8: Average of Conditional Correlations with Band of Confidence Using Net Returns



When the band of confidence was computed, it is also possible to compare, which national indices get off the band at most. The Austrian ATX correlation over excesses the band most of the time and thus can be referred as the market with the highest correlation. On the other hand indices of USA and Japan under excess the band and it implicates that markets out of the Europe have lower interconnections with the PSE. A comparison of last values of USA and Japanese indices finally reveals that recently the USA equity market is more interlinked to PSE than the Japan market.

Although the average correlation behaviour can be smoothed with a rising linear trend, it is not perfectly linear and several important leaps can be perceived in the estimation. The average correlation can be divided into three different periods of time. The first period lasts from an establishment of the PX[57] index until a half of the year 1998, when the average correlation stayed in a band from -0.1 to 0.15. It indicates very low or even zero correlation between PSE and other markets, which implies that PSE was in a position typical for unintegrated emerging markets as described Khalid and Rajaguru (2007) or Hyde et al. (2008). A second period is characterised by a

56 The band of confidence requires an assumption of a normal distribution of individual conditional correlations and was computed with 10 degrees of freedom.

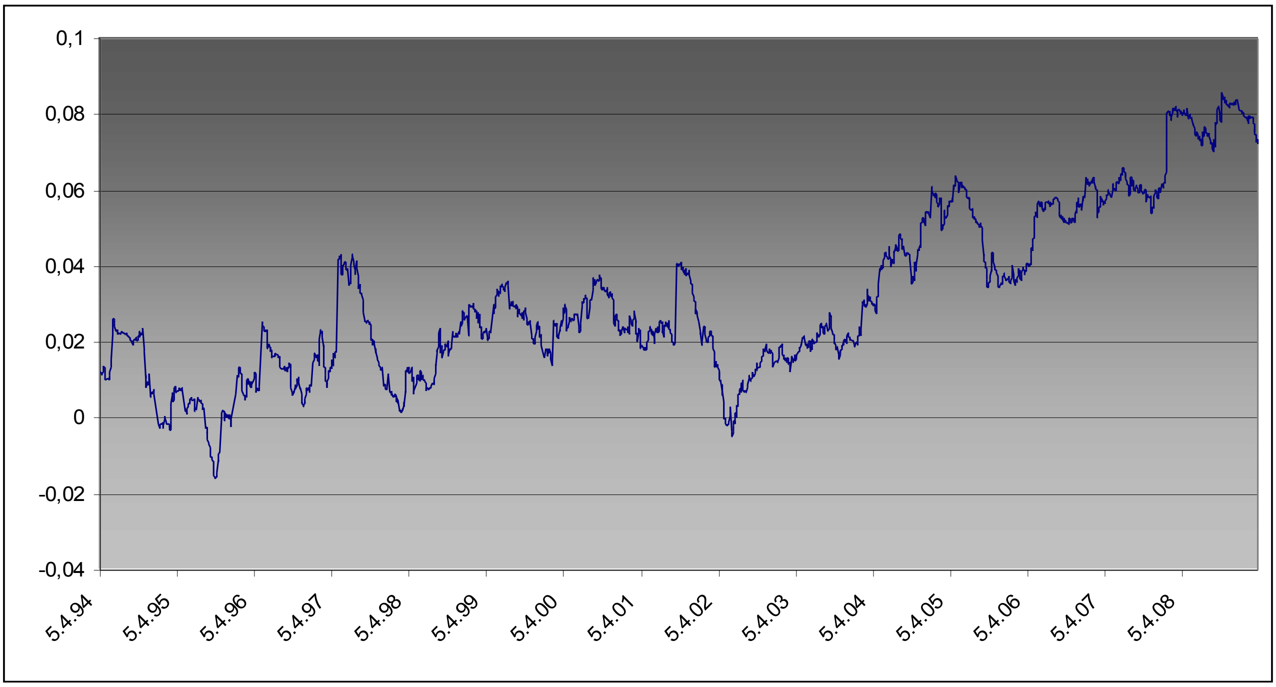
57 formerly PX 50 index

significant increase in a correlation, which lies between 0.2 and 0.45, lasting until 2006. This means that the correlation is significantly positive and it fills the gap between periods of low and high correlations, which occurred in the last period. The final period starts in 2006 and remains until nowadays. The main characteristic is a continual increase in correlation up to values around 0.6, which is typical to developed and integrated states of EU according to CAPPIELLO ET AL. (2006).

When the analysis is enriched by important economical events it can reveal the spirit of a development of PSE. This means that Czech stock market was rather "stand-alone" than integrated into Europe in the first period, which is typical for emerging markets. When a following development is researched year 1998 shows very important change, which can be associated with various economic events. According to SALEEM(2008) this change could be related to Russian crisis, which occurred during the same period of time, however there is possible also another explanation.

CAPPIELLO ET AL. (2006) suggests that during 1998 Euro had already effects on financial markets. This implicates that the correlation with EMU should be increased from 1998 or 1999, when compared to the average world correlation. Thus a Graph 9 was made, which compare correlation of EMU states represented in the sample[58] with the average world correlation.

GRAPH 9: DIFFERENCE BETWEEN EMU AVERAGE AND WORLD AVERAGE CORRELATIONS



The Graph 9 shows that the difference between suggested average correlations was often

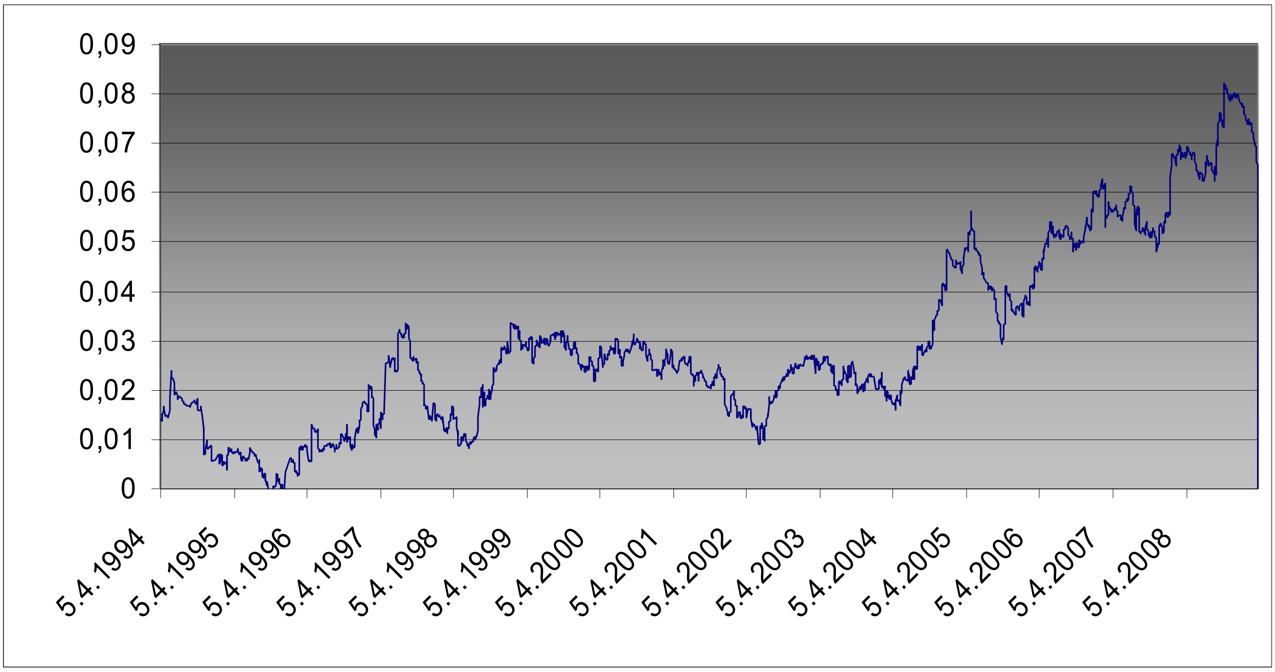
58 It means ATX, AEX, BEL 20, CAC 40, DAX 30 and IGBM indices.

positive, which indicates stronger interlinks with EMU countries, but there was no significant increase during 1998 or 1999, which would confirm a hypothesis of an importance of Euro adoption in context to the Czech stock market. This concludes that during 1998 correlation with all market indices stood up steeply, because the Russian crisis contagion, but lasted for longer period of time, which is consistent with SALEEM (2008). This sudden difference in a volatility transmission is typical for emerging markets in a case of period of Russian crisis as was researched in CAPORALE ET AL. (2006).

A next important event, which affected the Czech market was an accession to EU in May 2004. A flow of the average world correlation suggests that integration of PSE strengthened later, but it is possible to analyse correlations similarly as in a case of Euro adoption. Thus a Graph 10 was made, which compares the average world correlation to the 'average correlation to EU countries'. The Graph 10 shows that before 2004, the difference between world and EU was positive in terms of correlations, but from 2004 the difference increased significantly and exceeded a band of previous values[59]. The result suggest that the EU enlargement was an important event, which increased a degree of PSE interlinks to world markets and allowed PSE to become a developed market with a full-fledged integration. The particular date of a new stage of a development can be perceived in year 2004, in a case of analysis of differences among markets, or in year 2006, when the average correlation amongst world markets increased., but in both cases the date is after the accession, which suggests that the EU enlargement was rather a reason for a change than an anticipated event.

59 The difference between the average world and average EU correlations did not exceeded a bordering value 0.035.

Graph 10: Difference between EU Average and World Average Correlations



Finally it is possible to interpret an impact of a global financial crisis in 2008 on PSE in terms of volatility spillovers. The outcomes indicate that a financial crisis in 2008 did not affect a steady trend, which started during 2006 and lasted until the end of a data sample in March 2009. There is no sudden change in a correlation development, which means that although correlations increased in 2008 on PSE a trend remained the same.[60] This offers a conclusion that the global financial crisis did not affected a degree of integration of PSE into developed markets, but it was an inevitable event, which is a cost united with a 'membership in developed markets club'.

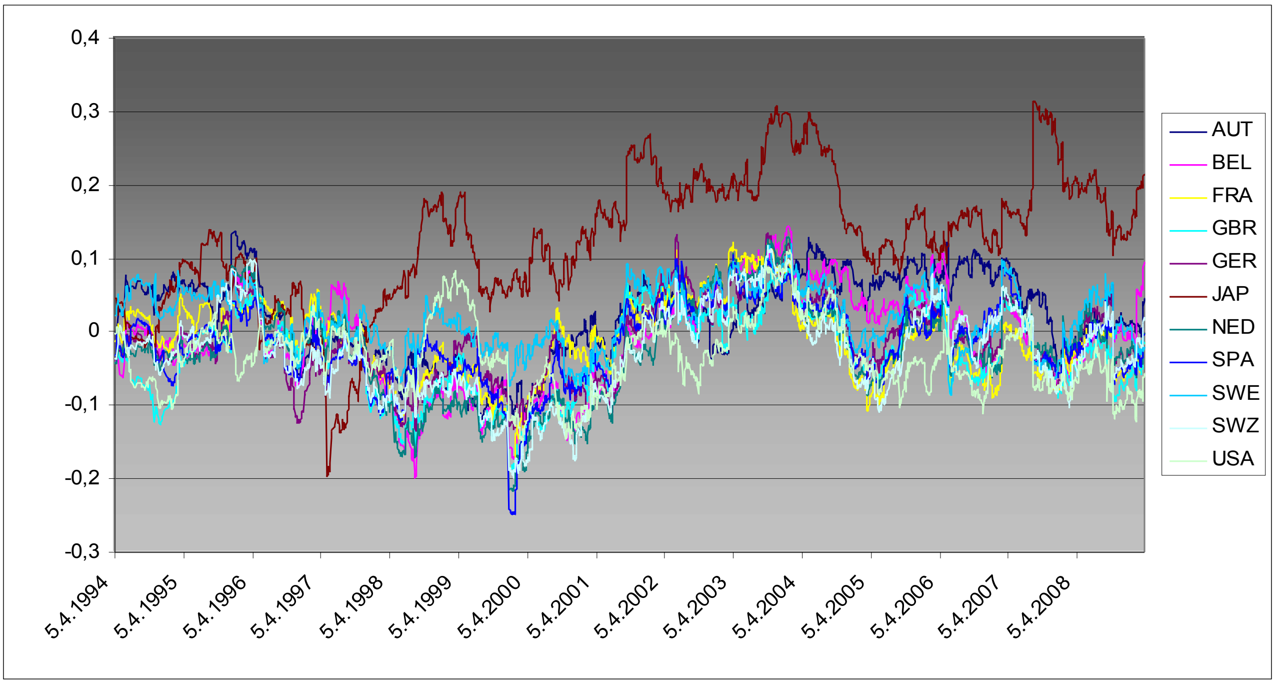
5.4.2. Adjusted Net Returns

As in a previous analysis of daily net returns the estimation of DCC MVGARCH was computed using adjusted daily net returns, which incorporate an exchange rate effects. All returns were weighted by CZK, which was chosen as a basis for a comparison. The result of the model is depicted in a Graph 11, which shows volatility spillovers were not significant during the whole period of time, all values remained in a band from -0.25 to 0.3. This indicates that although volatility spillovers occurred in case of net daily returns, which analyse a situation from point of view of a local investor or a global investor interested only in returns in a same currency as is

60 This statement can be supported by a fact that correlation over 50% can be perceived from year 2007, which is not regarded as a time of a global financial crisis.

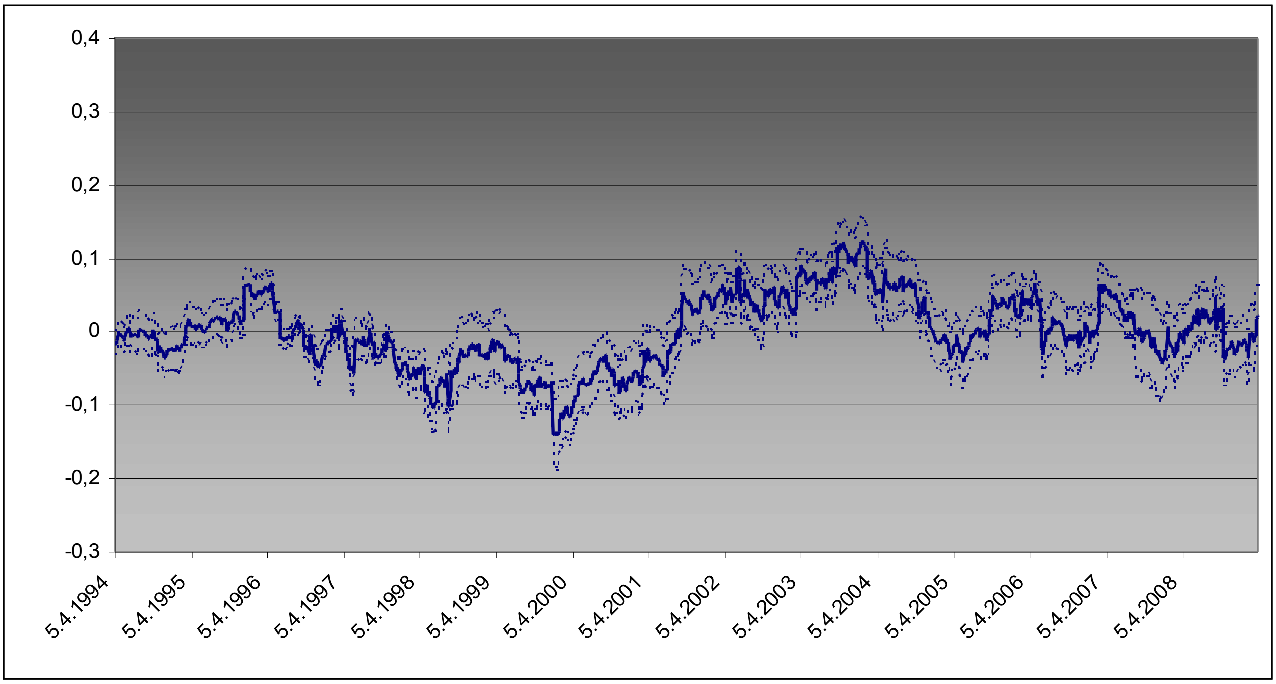
denominated the index, the volatility of adjusted daily net returns remained almost the same. A good signal for a global investor, who is interested in investments with low correlations, which would offer a maximum diversification effect[61]. A volatility of the investment on PSE remained unchanged in a comparison to investments on other markets, when weighted in CZK.

Graph 11: Aggregated Conditional Correlations - Adjusted Net Returns



61 Amount of the diversification effect arises from a degree of co-movement and thus also correlations, higher correlations imply lower diversification effect and on contrary lower correlations mean higher diversification effect.

Graph 12: Average of Conditional Correlations with Band of Confidence Using Adjusted Net Returns



The Graph 12 shows that an average correlation of PSE among the world sample remained even in band bordered by values -0.15 and 0.15, which is more typical for CCC MVGARCH model, because a correlation stayed almost constant. This outcome shows that adjusted net returns would be only little affected by excessive volatility and thus volatility spillovers or market contagions have low effects.

5.5. Granger Causality Test

Although previous chapters clarified changes in volatility spillovers, the directions of spillovers remained unsolved. The theme was researched in Mathur and Subrahmanyam (1990), where Granger causality was suggested as a toll, which can determine directions of interdependencies.

Causality test employed by Granger (1969) is relatively easy test using standard Fisher test to find whether a zero hypothesis can or cannot be rejected. Granger causality test uses lag variables to find interconnections between researched data series. Due to lower complexity it is possible to do cross tests between all markets, however this is not a purpose of the work and thus only relations between Czech Republic and other markets are deeply analysed. The Granger causality is researched using a two-variable interdependence model described as follows:

$$x(t) = \sum_{i=1}^{n} \alpha_{1,i} x_{t-i} + \sum_{i=1}^{n} \beta_{1,i} y_{t-i} + \varepsilon_{1,t},$$
$$y(t) = \sum_{i=1}^{n} \alpha_{2,i} x_{t-i} + \sum_{i=1}^{n} \beta_{2,i} y_{t-i} + \varepsilon_{2,t},$$

A relation assuming $x \rightarrow y$ can be computed through testing zero hypothesis $H_0: \alpha_{2,i} = 0,$ *for i = 1,...,n*, which rejection indicates that *y* is caused by *x* in terms of Granger causality, while opposing relation assuming $y \rightarrow x$ involves testing hypothesis $H_0: \beta_{1,i} = 0$ *for i = 1,...,n*.

As it was shown in previous equations, if the Granger causality e.g. in case of $x(t)$ variable is intended to be computed, it necessary to use its own lagged variables $x_{t-i},$ *for i = 1,...,n* in the model in order to compare a benefit of new data series $y_{t-i},$ *for i = 1,...,n*, which is regarded as a 'Granger origin'.

5.5.1. Akaike Information Criterion

A need for a proper definition of the Granger causality test brings a question "How many lagged variables should be used in the estimation?", which can be answered with a usage of Akaike information criterion (AIC). AIC can determine the optimal number of independent variables in the Granger causality model. AIC was proposed in Akaike (1974), it is a relative measure of the information lost when a given model is used for a purpose to describe a reality. The basic idea is to determine the relation between a precision and a complexity of the model. Akaike's test suggest to choose a model with the lowest possible AIC value. It compares benefits of additional variables with their total amount, the definition is as follows:

$$AIC = 2\mathrm{k} - 2\ln(L)$$

where *k* is a number of parameters in the model and $\ln(L)$ is the value of maximized log-likelihood function for the estimated model. In my case I used another option how to compute AIC.

Under an assumption that errors of a model are normally, independently and identically distributed I computed sum of squared residuals:

$$SSR=\sum_{i=1}^{n}\hat{\varepsilon}_i^2$$

which can lead into another form of AIC test statistic:

$$AIC=2\mathrm{k}+n\left[\ln\left(SSR/n\right)\right]$$

This equation can be interpreted as a preference of lower sum of squared residuals, because also lower AIC means better outcome. While higher number of parameters *k* imposes penalty to estimated model in terms of AIC.

The AIC values were computed for all models characterized by previous hypothesis $H_0:\beta_{1,i}=0$ and thus a number of lagged variables in tested alternative were set to same amount. The maximum number of lags checked through AIC sorting algorithm were 10 lagged variables. This was conducted in order to achieve the best restricted model so resulting p-values reveal the Granger causality with a substantial elimination of possible spurious outcomes, which would resulted from an inappropriate model definition.

5.5.2. Estimations of Tests

For a purpose of more precise calculations, the whole data series, which starts on 5th April 1994 and ends on 31st March 2009, was divided on a basis of whole years into 15 periods as is depicted in following tables[62]. The reason for the division was an assumption, that Granger causality could differ during long term. And finally because of a dual analysis of volatility spillover effects based on both net returns and adjusted net returns, also all results involving Granger causality and AIC comparison have to be conducted two times.

Table 17 shows advised number of lagged variables according to the lowest AIC based on net returns for each country, while Table shows advised number of lags based on values including exchange rate effects for each country.[63]

62 Period 1994 starts on 5th April 1994 and ends on 31st December 1994, all periods from 1995 to 2007 starts on 1st January and ends on 31st December of depicted years, finally period 2008 starts on 1st January 2008 and ends on 31st March 2009.

63 Number of advised lagged variables is the same for the Czech republic in both estimations. This is caused, because values of indices are weighted by real returns in CZK.

Table 17: Number of Lagged Variables Suggested by AIC - Net Returns

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA	CZE
1994	8	4	2	1	1	1	2	9	4	2	1	10
1995	1	1	1	1	1	1	1	1	1	4	1	1
1996	4	1	2	1	8	1	4	4	1	1	3	1
1997	1	1	3	1	1	2	1	2	1	9	10	7
1998	1	1	2	2	2	2	2	1	2	1	1	1
1999	1	9	4	4	4	1	10	1	1	5	3	4
2000	1	1	1	3	1	4	1	1	1	1	1	1
2001	1	8	5	3	5	1	5	1	2	1	1	1
2002	1	1	7	7	7	1	7	7	1	6	1	1
2003	2	1	8	2	1	1	5	1	1	1	1	1
2004	5	2	1	1	2	1	2	1	2	1	1	6
2005	1	1	2	1	2	1	1	2	1	1	2	1
2006	9	6	6	1	6	6	1	6	7	1	6	1
2007	1	1	1	1	1	1	1	1	1	1	1	1
2008	1	5	5	5	5	2	9	5	1	5	2	1

Table 18: Number of Lagged Variables Suggested by AIC - Adjusted Net Returns

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA	CZE
1994	10	4	9	1	1	7	1	10	4	10	1	10
1995	1	1	3	4	1	1	1	1	2	5	3	1
1996	4	1	1	1	4	1	1	4	1	1	1	1
1997	1	1	3	1	1	2	1	5	1	10	1	7
1998	1	1	4	1	1	7	1	1	1	1	1	1
1999	3	1	1	5	4	1	6	1	1	6	2	4
2000	1	2	1	3	8	3	8	1	1	1	1	1
2001	1	8	5	3	8	1	5	1	1	2	1	1
2002	1	1	7	7	7	2	7	7	1	1	1	1
2003	2	1	8	3	1	1	1	1	1	9	1	1
2004	5	2	2	1	2	1	2	2	2	1	1	6
2005	1	1	1	7	1	1	1	1	1	1	1	1
2006	9	6	6	1	6	4	1	6	6	1	2	1
2007	1	1	1	1	1	8	1	1	1	4	1	1
2008	1	7	6	7	8	2	9	5	2	1	2	1

In Tables 19, 20, 21 and 22[64] the computed p-values of F-tests testing depicted zero hypothesis are shown. The names of tables indicate, which direction of Granger causality is tested. Resulting p-values describe at which level of confidence a hypothesis of a non-existence of Granger causality can be rejected.

64 See on pages 71 to 72.

5.5.3. Results Analysis

Lower p-values indicate that the market is affected by Granger causality, while high p-values reject the causality relation. When results of net returns are analysed and level of confidence is set to 5%, it can be stated that the Czech market is dependent on other countries in Granger sense[65] since 2004, when the occurrence of lower p-values is more often, but this relationship was only unidirectional. A bidirectional relation can be dated only in year 2008, when 70% of the countries was dependent on the PSE. Before year 2004 the dependences are only sporadic, which is consistent with results of DCC MVGARCH, which revealed that from year 2004 PSE can be marked as developed market. That also confirms that year 2004 was important for the Czech market and thus an accession of the Czech Republic improved an integration of PSE to other markets. The process of integration seems to be still in progress, because results from the latest year 2008 show that the interlinks are bidirectional.

Results of adjusted net returns implies, that Granger causality occurred even earlier, but it could not be perceived through net returns, because since 1998 p-values are near zero for most of the indices in the sample. This also confirms that relations between PSE and other markets were almost always unidirectional and in addition the dependences can be perceived through data including exchange rate effects. However it cannot be clearly answered whether the Granger causality is connected with exchange rates only and thus the impact of equity market could be marginal. A comparison of the Granger causality with DCC MVGARCH estimates can conclude, that in case of adjusted net returns the dependence occurred only in terms of returns, but volatility spillovers were not observed.

Alas the results of the Granger causality cannot give unambiguous answers, but they offer a useful outlook to interdependencies of PSE to other markets and it supports findings that years 1998 and 2004 were important milestones in history of the Czech equity market. The outcomes also show that the Granger causality was only unidirectional in a history of PSE, but it can be assumed that this will change in a near future, because year 2008 already recorded bidirectional relations.

65Further mentioned dependencies are assumed to be in sense of Granger causality.

VI. Conclusion

Findings of the thesis are various. At first it was proved that GARCH (1,1) process is suitable for an analysis of the Czech stock market, while it also performed very well in a comparison to more sophisticated models. The existence of a conditional heteroskedasticity was confirmed. A test of forecasting abilities showed mediocre performance and quickly deteriorating outcomes, when more that 5-step estimations were computed. Alas true forecasting abilities could not be tested, because only an approximation of a daily volatilities was used. This recommends that also higher frequencies should be included in a further research. A part involving structural change models indicated that a period before year 1998 is different than later era, which means that the evolution of procedures and rules affected a development of PSE and a behaviour perceived on the market. Structural models also provided a guide through less or more predictable periods, when GARCH (1,1) showed different quality of performance in forecasting abilities.

The DCC MV GARCH model demonstrated that the best outcomes can be received after a comparison of PSE with different markets, while a research of a solely national data series provided only a limited descriptive power. Dynamic model marked two important events in the history of the Czech equity market, i.e. year 1998 and the Asian/Russian crisis and also year 2004 and the accession of the Czech Republic into European Union. Before year 1998 PSE had all signs of a emerging market. In 1998 the awareness about the Czech market was spread out and an intermediate period began. It meant that an integration of PSE into developed markets stood up to higher level. The intermediate period is typical with a mediocre interlinks to developed markets. Finally year 2004 was a very important event for PSE, a reason is not only the accession into EU, but also a full membership in the Federation of European Stock Exchanges and a granted status 'designated offshore securities market' to PSE from U.S. Securities and Exchange Commission. This 'invitation' to a club of developed markets was 'accepted' by PSE and furthermore proved during the analysis. The outcomes showed that from year 2004 PSE reached a new stage, which is typical for other developed exchanges. This indicates that the accession was not anticipated by market agents and rather was a reason for changes.

Alas a membership in a 'advanced club' also brought costs, which counted during the global financial crisis in 2008. Because of a high degree of the integration the crisis was a cause of a development on other markets, which was inevitable to PSE. Finally an effect of the Czech crown showed that although the volatility spillovers are a serious issue for the Czech market, net outcome of contagions is minimized through exchange rates.

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List of Abbreviations:

AIC - Akaike information criterion

ARCH - autoregressive conditional heteroskedasticity

APARCH - asymmetric power autoregressive conditional heteroskedasticity

AUT - Austria

BEL - Belgium

CNB - Czech National Bank

CZE - Czech Republic

CZK - Czech Crown

EGARCH - exponential generalised autoregressive conditional heteroskedasticity

EU - European Union

EMU - European Monetary Union

FRA - France

GARCH - generalised autoregressive conditional heteroskedasticity

GER - Germany

GBR - Great Britain

ICSS - Iterated cumulated sum of squares

JAP - Japan

NED - Netherlands

OPG - Outer Product of Gradient

QME - Quasi-Maximum Estimator

SPA - Spain

SWE - Sweden

SWZ - Switzerland

TIC - Theil inequality coefficient

USA - United States of America

Data Sources:

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List of Graphs:

List of Tables:

TABLE 15: PARAMETERS OF UNIVARIATE GARCH ESTIMATIONS - NET RETURNS

0.060	ω	AUT
0.133	α	AUT
0.844	β	AUT
0.066	ω	BEL
0.183	α	BEL
0.793	β	BEL
0.033	ω	FRA
0.101	α	FRA
0.888	β	FRA
0.018	ω	GBR
0.112	α	GBR
0.882	β	GBR
0.037	ω	GER
0.107	α	GER
0.884	β	GER
0.039	ω	JAP
0.105	α	JAP
0.888	β	JAP
0.031	ω	NED
0.130	α	NED
0.866	β	NED
0.033	ω	SPA
0.094	α	SPA
0.894	β	SPA
0.032	ω	SWE
0.080	α	SWE
0.911	β	SWE
0.046	ω	SWZ
0.124	α	SWZ
0.855	β	SWZ
0.018	ω	USA
0.082	α	USA
0.909	β	USA
0.114	ω	CZE
0.145	α	CZE
0.822	β	CZE

TABLE 16: PARAMETERS OF UNIVARIATE GARCH ESTIMATIONS - ADJUSTED NET RETURNS.

0.042	ω	AUT
0.078	α	AUT
0.899	β	AUT
0.022	ω	BEL
0.117	α	BEL
0.877	β	BEL
0.015	ω	FRA
0.068	α	FRA
0.928	β	FRA
0.012	ω	GBR
0.070	α	GBR
0.926	β	GBR
0.023	ω	GER
0.076	α	GER
0.917	β	GER
0.046	ω	JAP
0.072	α	JAP
0.917	β	JAP
0.022	ω	NED
0.090	α	NED
0.905	β	NED
0.013	ω	SPA
0.052	α	SPA
0.944	β	SPA
0.017	ω	SWE
0.057	α	SWE
0.939	β	SWE
0.022	ω	SWZ
0.080	α	SWZ
0.910	β	SWZ
0.013	ω	USA
0.042	α	USA
0.952	β	USA
0.064	ω	CZE
0.144	α	CZE
0.837	β	CZE

TABLE 19: GRANGER CAUSALITY P-VALUES FOR NET RETURNS - DIRECTION OF CAUSALITY FROM FOREIGN COUNTRIES TO THE CZECH REPUBLIC:

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.26	0.85	0.88	0.54	0.45	0.46	0.35	0.7	0.43	0.07	0.76
1995	0.66	0.07	0.19	0.47	0.07	0.79	0.01	0.04	0.02	0.02	0.75
1996	0.12	0.13	0.09	0.06	0.57	0.58	0.14	0.06	0.04	0.1	0.24
1997	0.11	0.35	0.09	0.03	0.09	0.29	0.19	0.13	0.29	0.33	0.01
1998	0.6	0.86	0.46	0.43	0.74	0.03	0.23	0.52	0.03	0.41	0.04
1999	0.06	0.56	0.84	0.37	0.11	0.17	0.55	0.4	0.78	0.37	0.57
2000	0.04	0.07	0.04	0.69	0.24	0.01	0.01	0.38	0.24	0.04	0.01
2001	0.39	0.72	0.43	0.29	0.19	0.44	0.74	0.35	0.35	0.35	0.07
2002	0.19	0.31	0.17	0.15	0.9	0.66	0.08	0.02	0.25	0.21	0.01
2003	0.52	0.49	0.65	0.6	0.97	0.33	0.97	0.45	0.78	0.81	0.05
2004	0.51	0.76	0.56	0.76	0.58	0.02	0.82	0.86	0	0.99	0
2005	0.01	0.2	0.02	0	0.03	0	0.01	0.03	0.11	0.17	0
2006	0.83	0.45	0.35	0.6	0.31	0	0.26	0.6	0.23	0.85	0
2007	0.33	0.01	0	0.01	0	0	0	0.15	0	0	0
2008	0.12	0	0.27	0.21	0	0	0.13	0.2	0	0.23	0

TABLE 20: GRANGER CAUSALITY P-VALUES FOR NET RETURNS- DIRECTION OF CAUSALITY FROM THE CZECH REPUBLIC TO FOREIGN COUNTRIES

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.53	0.99	0.95	0.15	0.64	0.16	0.76	0.88	0.83	0.58	0.1
1995	0.03	0.32	0.14	0.21	0.17	0.98	0.29	0.68	0.35	0.48	0.48
1996	0.52	0.5	0.81	0.68	0.65	0.07	0.68	0.98	0.27	0.33	0.57
1997	0.18	0.51	0.08	0.26	0.74	0.66	0.22	0.1	0.98	0.51	0.58
1998	0.71	0.9	0.91	0.44	0.29	0.13	0.9	0.26	0.05	0.82	0
1999	0.19	0.49	0.17	0.05	0.12	0.9	0.23	0.03	0.12	0.4	0.98
2000	0.87	0.15	0.29	0.23	0.74	0.19	0.45	0.57	0.92	0.04	0.02
2001	0.96	0.53	0.24	0.02	0.23	0.17	0.44	0.69	0.47	0.71	0.41
2002	0.7	0.98	0.24	0.36	0.4	0.02	0.27	0.58	0.54	0.69	0.91
2003	0.28	0.22	0.27	0.44	0.09	0.39	0.57	0.34	0.63	0.19	0.31
2004	0.15	0.64	0.9	0.45	0.96	0.75	0.48	0.98	0.28	0.86	0.6
2005	0.7	0.25	0.65	0.78	0.66	0.08	0.79	0.66	0.41	0.62	0.75
2006	0.28	0.3	0.16	0.8	0.02	0.62	0.16	0.03	0.07	0.73	0.03
2007	0.44	0.14	0.25	0.19	0.2	0.47	0.17	0.35	0.74	0.07	0.75
2008	0.78	0	0	0	0	0.04	0.02	0	0.08	0	0.08

TABLE 21: GRANGER CAUSALITY P-VALUES FOR ADJUSTED NET RETURNS - DIRECTION OF CAUSALITY FROM FOREIGN COUNTRIES TO THE CZECH REPUBLIC

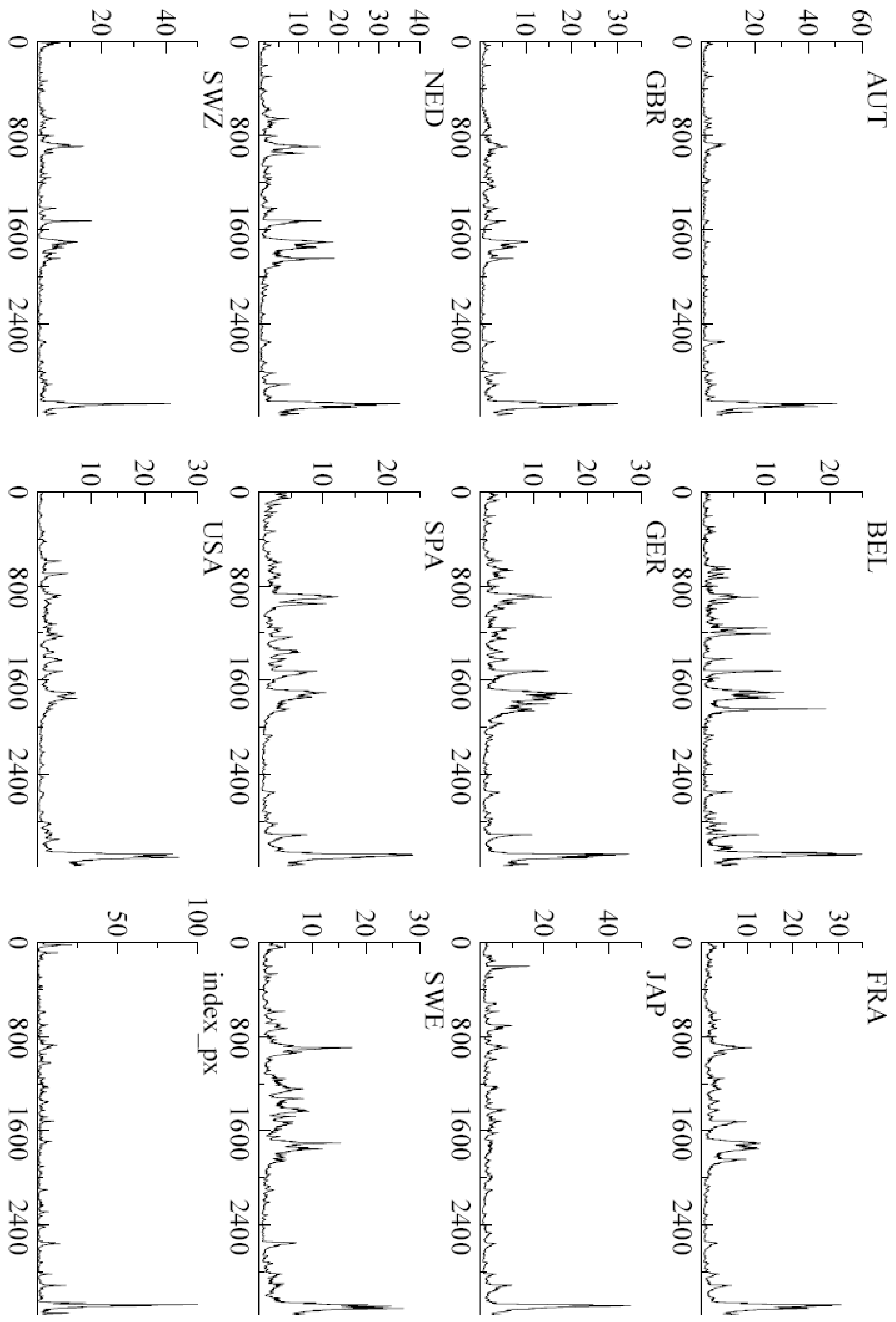
	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.47	0.5	0.83	0.66	0.55	0.72	0.19	0.6	0.29	0.03	0.46
1995	0.36	0.12	0.71	0.14	0.5	0.46	0.2	0.56	0.68	0.74	0.99
1996	0.93	0.64	0.06	0.91	0.54	0.31	0.31	0.65	0.58	0.43	0.64
1997	0.01	0.29	0.25	0.36	0.15	0.18	0.3	0.1	0.65	0.12	0.42
1998	0	0	0	0	0	0.2	0	0	0	0	0
1999	0	0	0	0	0	0.09	0	0	0	0	0.15
2000	0.07	0.16	0	0	0	0.02	0	0	0	0.05	0
2001	0.01	0	0	0	0	0.3	0	0	0	0	0
2002	0	0	0	0	0	0.97	0	0	0	0	0
2003	0	0	0	0	0	0.1	0	0	0	0	0
2004	0	0	0	0	0	0.01	0	0	0	0	0
2005	0	0	0	0	0.01	0	0	0.01	0	0	0.89
2006	0	0	0	0	0	0	0	0	0	0	0.01
2007	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	0	0	0	0	0	0	0	0	0

TABLE 22: GRANGER CAUSALITY P-VALUES FOR ADJUSTED NET RETURNS - DIRECTION OF CAUSALITY FROM THE CZECH REPUBLIC TO FOREIGN COUNTRIES:

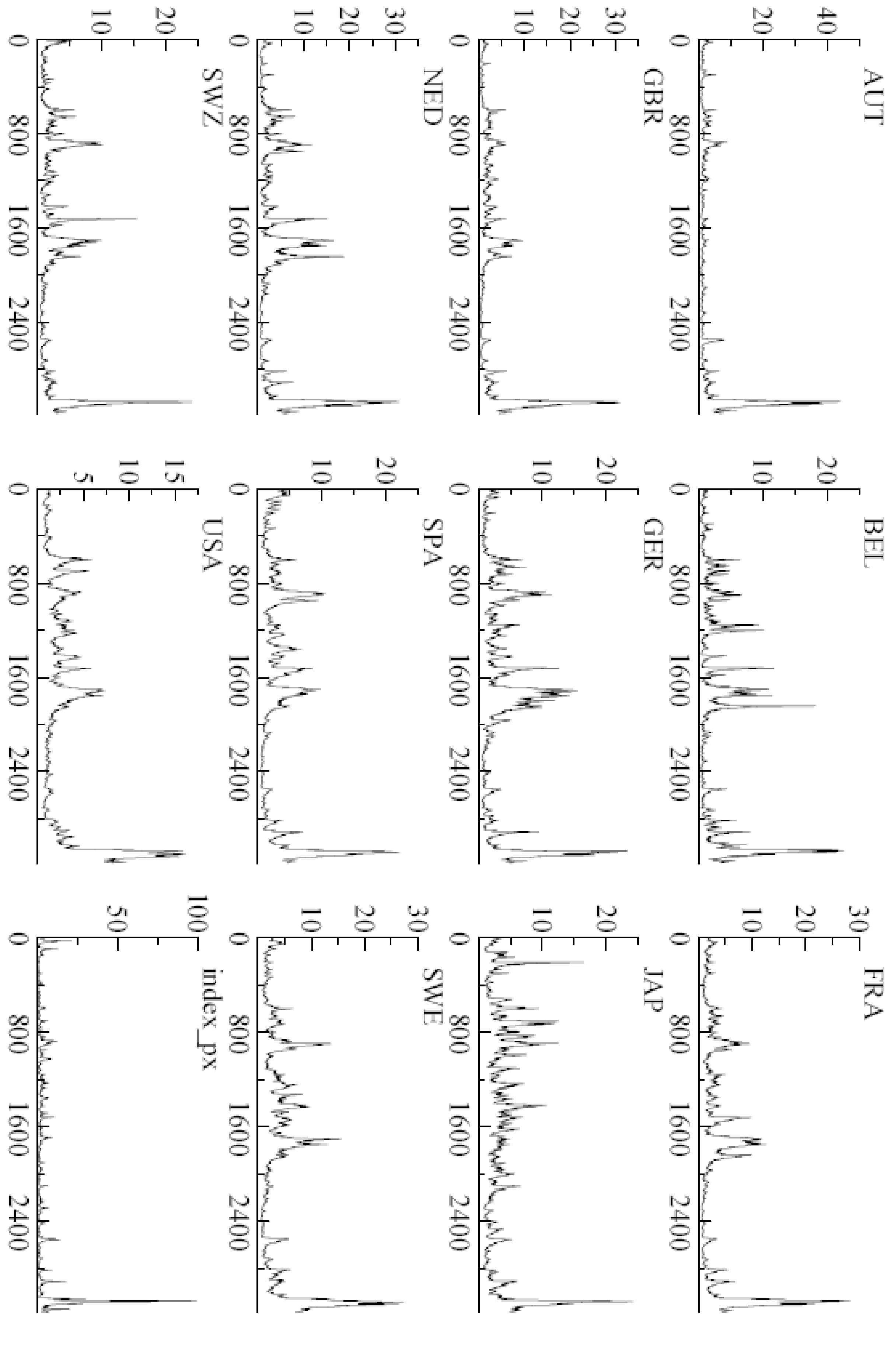
	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.86	0.98	0.64	0.47	0.8	0.22	0.93	0.93	0.84	0.96	0.14
1995	0.82	0.42	0.74	0.77	0.31	0.76	0.98	0.34	0.18	0.86	0.32
1996	0.64	0.97	0.88	0.65	0.64	0.25	0.87	0.96	0.83	0.59	0.98
1997	0.1	0.41	0.01	0.05	0.3	0.57	0.13	0	0.29	0.44	0.07
1998	0.7	0.96	0.32	0.74	0.88	0.21	0.95	0.85	0.17	0.74	0.56
1999	0.17	0.67	0.84	0.33	0.26	0.91	0.22	0.93	0.55	0.69	0.82
2000	0.79	0.56	0.81	0.59	0.72	0.67	0.54	0.69	0.9	0.87	0.32
2001	0.63	0.56	0.36	0.01	0.17	0.25	0.41	0.62	0.78	0.15	0.98
2002	0.19	0.15	0.27	0.25	0.63	0.02	0.33	0.79	0.23	0.26	0.49
2003	0.4	0.96	0.18	0.99	0.71	0.85	0.53	0.34	0.76	0.76	0.62
2004	0.04	0.86	0.73	0.75	0.76	0.93	0.66	0.46	0.69	0.37	0.73
2005	0.5	0.94	0.55	0.43	0.56	0.12	0.67	0.25	0.84	0.87	0.89
2006	0.01	0.32	0.14	0.3	0.03	0.27	0.06	0.03	0.13	0.12	0.68
2007	0.24	0.82	0.37	0.74	0.61	0.42	0.62	0.37	0.55	0.59	0.92
2008	0.79	0.03	0	0.02	0	0.15	0	0	0.34	0.22	0.05

Appendix I

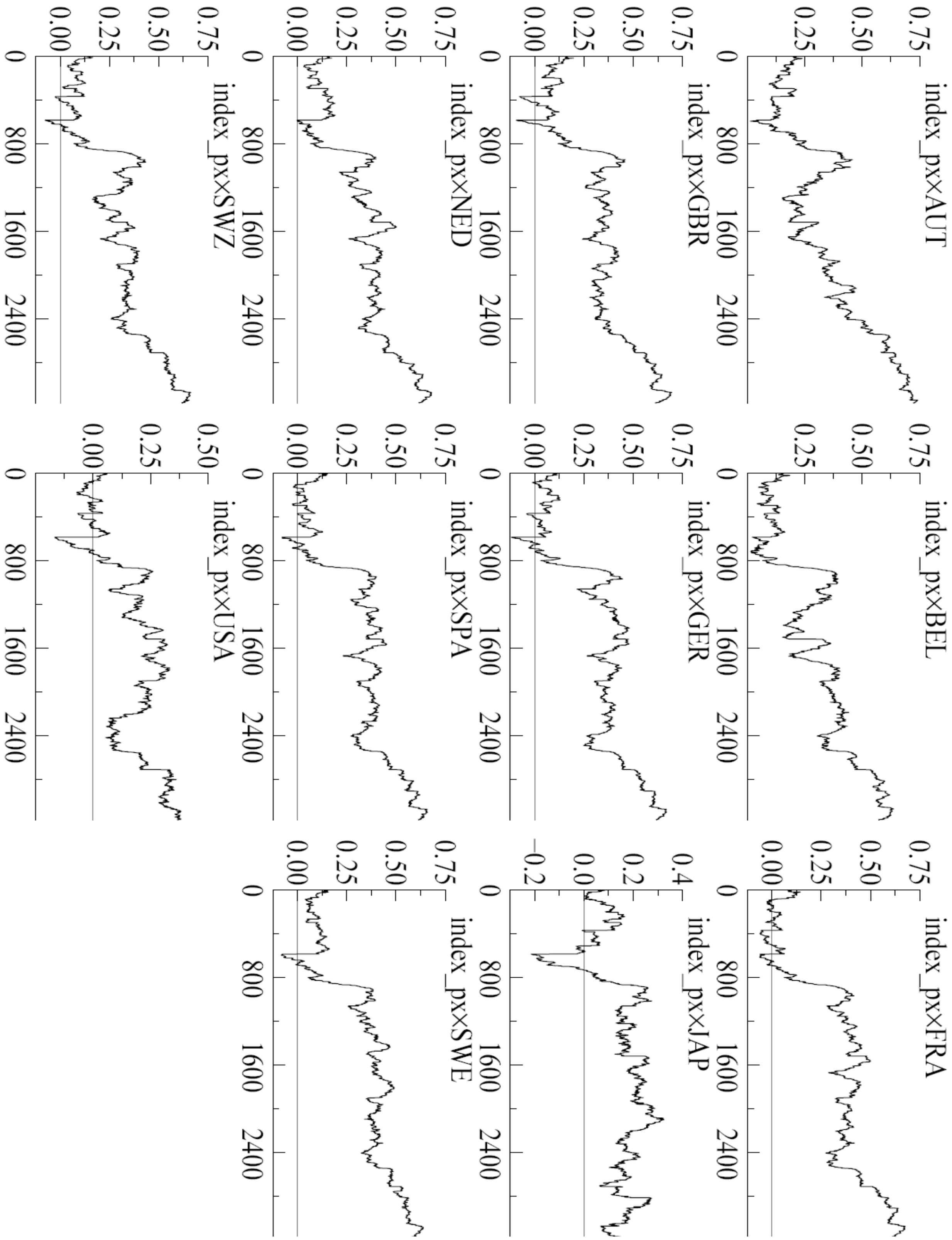
Variance Graphs - Net Returns



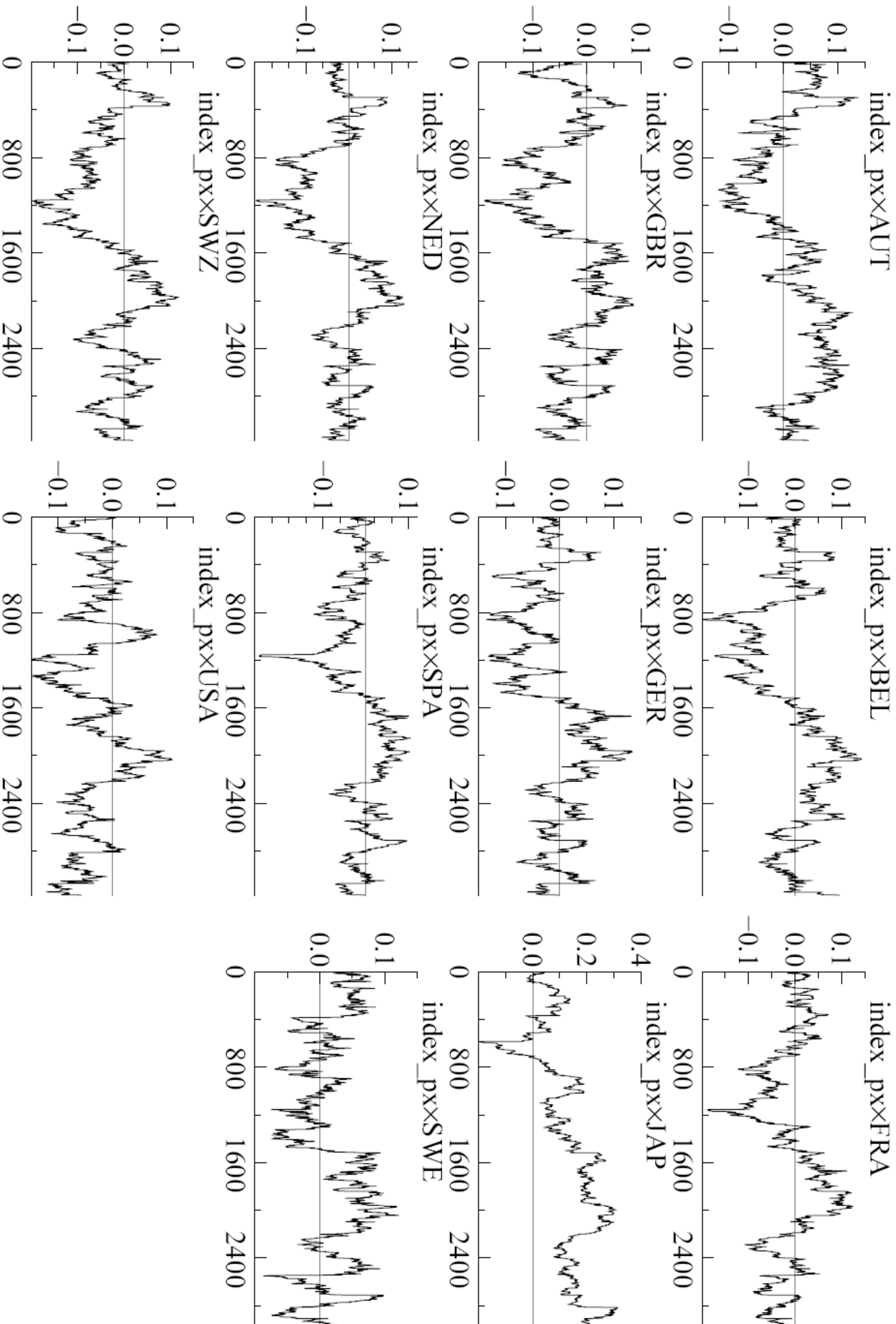
Variance Graphs - Adjusted Net Returns



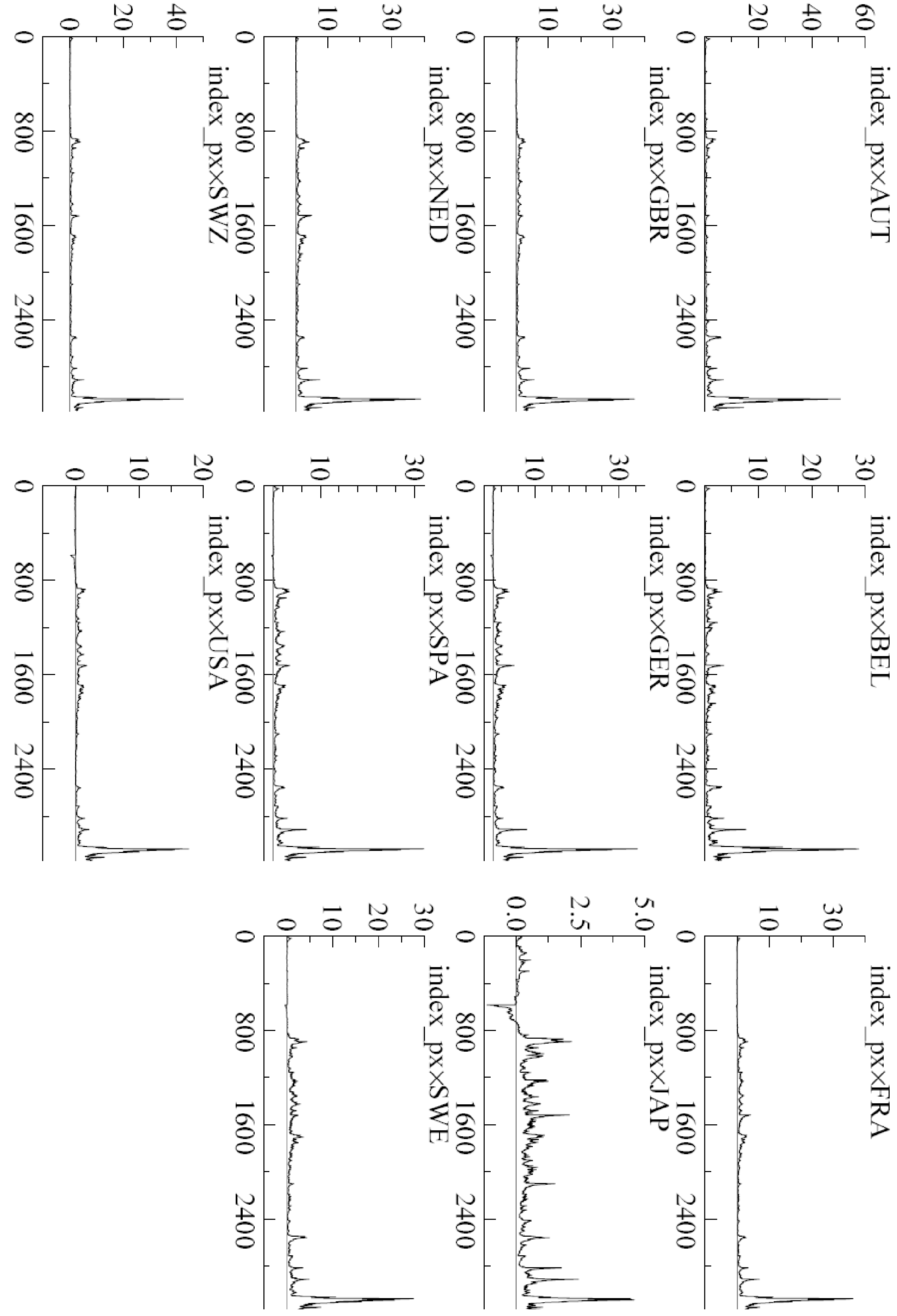
Conditional Correlation Graphs - Net Returns



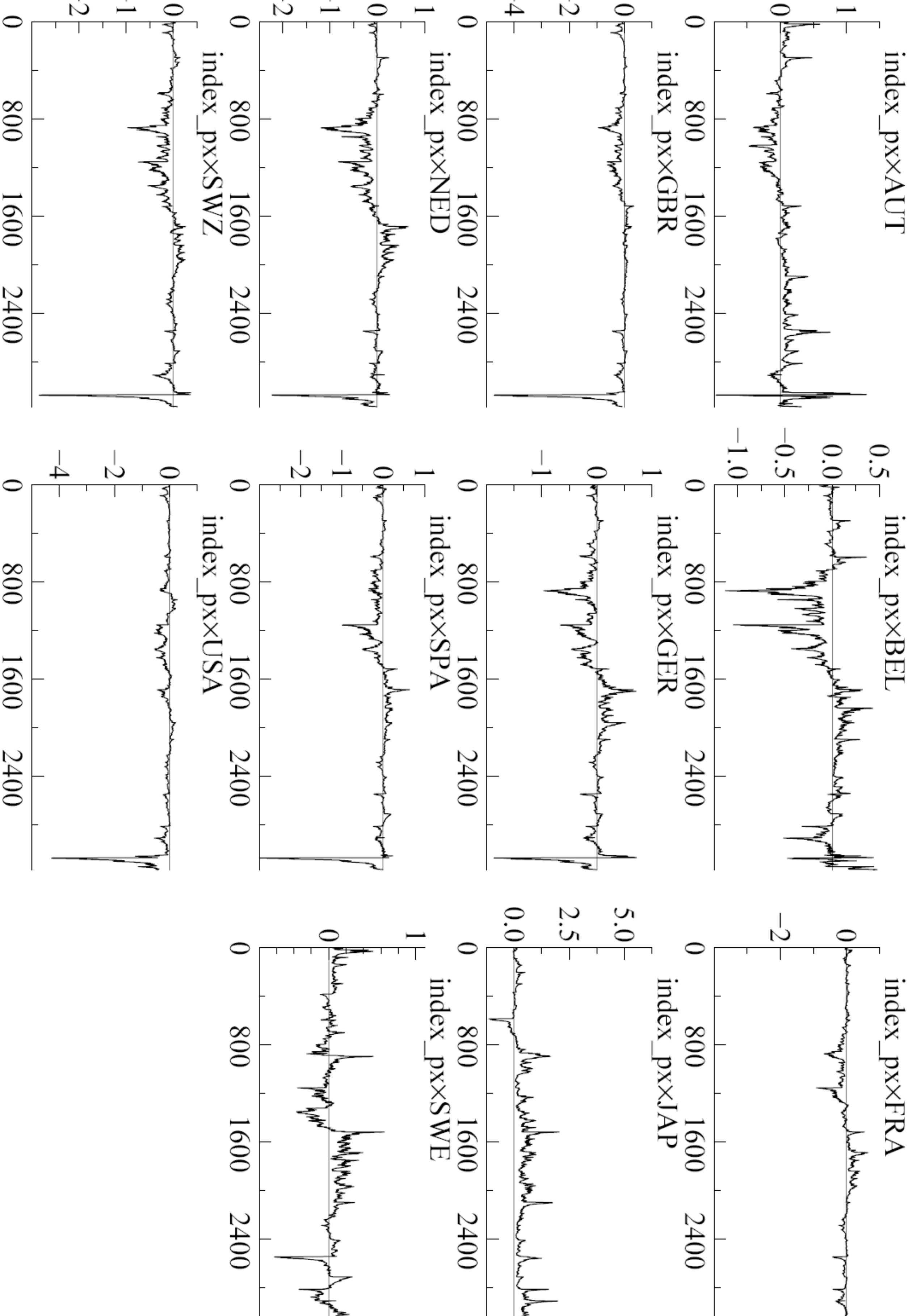
Conditional Correlation Graphs - Adjusted Net Returns Data



Conditional Covariance Graphs - Net Returns



__Conditional Covariance Graphs - Adjusted Net Returns__



Appendix II

Elliptical distribution used in MultiGARCH library

The m-dimensional random vector is X said to be distributed elliptically[66], symbolically $X \sim EC_m(\mu, \Sigma, \phi)$, if its characteristic function may be expressed in the form

$$E[\exp(it'X)] = \exp(it'\mu)\phi(t'\Sigma t) \quad ,$$

with μ m-dimensional vector, definite positive $m \times m$ matrix, and $\phi(.)$ scalar function, referred to as *characteristic generator*. They stated following principal properties of elliptical distributions:

P1. if $X \sim EC_m(\mu, \Sigma, \phi)$ has a density, this has the form

$$f(x) = c|\Sigma|^{-1/2} g[(x-\mu)'\Sigma^{-1}(x-\mu)]$$

with $g(.)$ a scalar function, referred to as *density generator* and the notation $X \sim EC_m(\mu, \Sigma, g)$ may also be used;

P2. suppose that $X \sim EC_m(\mu, \Sigma, \phi)$ possess k moments, if , $k \geq 1$ then , $E(X) = \mu$ and if $k \geq 2$, then $Cov(X) = \gamma\Sigma$, with $\gamma = -2\psi'(0)$;

P3. if $X \sim EC_m(\mu, \Sigma, \phi)$, for any given $p \times m$ matrix A with rank $p \leq m$ and any p-dimensional vector b

$$AX + b \sim EC_p(A\mu + b, A\Sigma A', \phi)$$

P4. if

$$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim EC\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$$

then

$$X_1 / X_2 \sim EC\left(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21,} \phi_{q(x_2)}\right)$$

where $\phi_{q(x_2)}$ depends on the value assumed by X_2 through the function

$$q(X_2) = (X_2 - \mu_2)'\Sigma_{22}^{-1}(X_2 - \mu_2);$$

P5. using previous equations we get X_1
$X \sim EC_m(\mu, \Sigma, \phi)$

For further details refer to Fang et al. (1990) or Pelagatti and Rondena (2004)

66An alternative name for elliptical distributions is *elliptically contoured distributions.*

Appendix III

The elliptical DCC model as used in MultiGARCH library

Pelagatti and Rondena (2004) shows in their OxMetrics library following relations, which lead to final estimation of the model:

Let r_t be *k*-dimensional a vector process defined by

$$r_t / \Omega_{t-1} \sim EC_k(O, \Sigma_t, g) \quad (1)$$

where Ω_t is the filtration on which r_t is adapted and Σ_t is a positive definite Ω_{t-1} measurable dispersion matrix defined by

$$\Sigma_t = D_t R_t D_t \quad (2)$$

with D_t diagonal matrix defined by the recursion

$$D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} r_{t-1} r'_{t-1} + diag\{\lambda_i\} D_{t-1}^2 \quad (3)$$

$\circ$ representing element by element multiplication , and with R_t , conditional correlation matrix defined by the set of equations

$$\xi_t = D_t^{-1} r_t$$
$$Q_t = S(11' - A - B) + A \xi_{t-1} \xi'_{t-1} + B Q_{t-1} \quad (4)$$
$$R_t = diag\{Q_t\}^{-1/2} Q_t \, diag\{Q_t\}^{-1/2}$$

Equation (3) is a set of univariate GARCH models with parameters ω_i, κ_i *and* $\lambda_i, (i=1,\ldots,n)$, applied to every element of the vector r_t . Equation (4) controls the dynamics of the conditional correlation matrix R_t through the square symmetric matrices of parameters S, A and B . Ding and Engle (2001) show that if A, B and $(11' - A - B)$ are positive semi-definite and S is positive definite, then Q_t is also positive definite. In order to keep small the number of parameters to be simultaneously estimated, A and B are usually taken as scalars or set equal to $A = \alpha\alpha'$ and $B = \beta\beta'$, with α and β *k*-dimensional vectors of parameters. For the same reason, S , which can be shown to be the unconditional correlation matrix, is estimated using the sample correlation of the standardized residuals ξ_t .

If in equation (1) we take an elliptical distribution with density, then it is easy to build the log-likelihood function

$$l(\theta)=\sum_{t=1}^{T}\{\log c_m-\frac{1}{2}\log|\Sigma_t|+\log g(r_t\Sigma_t^{-1}r_t')\} \quad (5)$$

which, for a moderate number k of assets, may be maximized by numerical methods. When the number of assets, and with it, the number of parameters is too large, then a three steps estimation procedure may be exploited to obtain consistent, asymptotically normal, although inefficient, estimates of the parameters.

1st step

Since the marginals of an elliptical distribution are elliptical distributions of the same family (property P2.), the parameters ω_i, κ_i *and* λ_i of the sequence of univariate GARCH models in equation (3) may be estimated by maximizing the k univariate likelihoods $EC(0,\sigma_{ii}, g)$, for $i=1,\ldots,k$. Through the recursion (3) the matrices D_t and the standardized residuals, $\xi_t=D_t^{-1}r_t$ may be estimated.

2nd step

The sample correlation matrix of the standardized residuals estimated in the first step is then used as estimate of the matrix S in equation (4).

3rd step

Using the estimated D_t and S , the likelihood

$$l(A,B)=\sum_{t=1}^{T}\{\log c_m-\frac{1}{2}\log|R_t|-\log|\hat{D}_t|+\log g(\hat{\xi}_t R_t^{-1}\hat{\xi}'_t)\}$$

is maximized with respect to the parameters in A and B (usually the two scalars α and β).

Consistency and asymptotic normality of the 3-step estimates may be demonstrated exploiting the same results of Newey and McFadden (1994) used by Engle and Sheppard (2001) and Pelagatti and Rondena (2004).

Let $\phi=(\omega_1,\kappa_1,\lambda_1,\ldots,\omega_k,\kappa_k,\lambda_k)'$ be the parameters' vector of the first step, $\rho=(s_{1,2},\ldots,s_{1,k},\ldots,s_{k,1},\ldots,s_{k,k-1})'$ contain the unique elements of matrix S , which are the 2nd step parameters, and $\psi=(\alpha,\beta)'$ be the vector of the parameters estimated in the 3rd step. Furthermore let

$$\begin{aligned} h^{(1)}(r,\phi) &= \nabla_\psi\{l_i(r_i,\omega_i,\kappa_i,\lambda_i)\}_{i=1,\ldots,k} \\ h^{(2)}(r,\phi,\rho) &= vech(\hat{\xi}\hat{\xi}'-S) \\ h^{(3)}(r,\phi,\rho,\psi) &= \nabla_\psi l_c(r,\phi,\rho,\psi) \end{aligned}$$

where $l_i(r_i,\omega_i,\kappa_i,\lambda_i)$ for $i=1,\ldots,k$, is the t-th contribution to the log-likelihood of the i-th univariate GARCH model (1st step) and $l_c(r,\phi,\rho,\psi)$ is the t-th contribution to the log-likelihood of the 3rd step. Letting $\theta=(\psi',\rho',\phi')'$, the 3-step procedure can be cast in GMM form with sample "orthogonality" conditions

$$\bar{h}(\theta)=\frac{1}{T}\sum_{t=1}^{T} h(r_t,\theta)=0$$

where

$$h(r_t,\theta)=\begin{bmatrix} h^{(1)}(r,\psi) \\ h^{(2)}(r,\psi,\rho) \\ h^{(3)}(r,\psi,\rho,\phi) \end{bmatrix}$$

and the estimates are obtained by solving

$$\hat{\theta}_T=\{\theta:\min_{\theta}\bar{h}(\theta)'\bar{h}(\theta)\}$$

Since the system is just-identified with so many equations as parameters, the absolute minimum of the quadratic form (that is, 0) can be reached, and the orthogonality conditions relative to $h^{(i)}$ are independent of those relative to $h^{(i+j)}$ with j positive integer, the GMM estimate is equivalent to the 3-step estimate.

Now let

$$\begin{aligned}
H_{\phi}^{(1)} &= E\left[\nabla_{\phi} h^{(1)}(r,\phi_0)\right], \\
H_{\phi}^{(2)} &= E\left[\nabla_{\phi} h^{(2)}(r,\phi_{0,}\rho_0)\right], \\
H_{\rho}^{(2)} &= E\left[\nabla_{\rho} h^{(2)}(r,\phi_{0,}\rho_0)\right], \\
H_{\phi}^{(3)} &= E\left[\nabla_{\phi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right], \\
H_{\rho}^{(3)} &= E\left[\nabla_{\rho} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right], \\
H_{\psi}^{(3)} &= E\left[\nabla_{\psi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],
\end{aligned}$$

the expected Jacobian matrix is given by

$$H=\mathrm{E}\left(\frac{\partial h(r,\theta)}{\partial\theta}\right)=\begin{pmatrix} H_{\psi}^{(1)} & 0 & 0 \\ H_{\psi}^{(2)} & H_{\rho}^{(2)} & 0 \\ H_{\psi}^{(3)} & H_{\rho}^{(3)} & H_{\phi}^{(3)} \end{pmatrix} \quad (6)$$

By adapting from Newey and McFadden (1994), under regularity conditions

$$\sqrt{T}(\hat{\theta}_T-\theta_0)\xrightarrow{D}\mathrm{N}(0,H^{-1}\Omega H^{-1}), \quad (7)$$

where

$$\Omega=\mathrm{E}\left[h(r,\theta_0)h(r,\theta_0)'\right]. \quad (8)$$

Consistent estimates of H and Ω may be obtained by substituting expectations with sample means:

$$\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T} h(r,\theta_0)\, h(r,\theta_0)'$$

and

$$\hat{H}_{\phi}^{(1)} = \frac{1}{T}\sum_{t=1}^{T}\left[\nabla_{\phi} h^{(1)}(r,\phi_0)\right],$$

$$\ldots$$

$$\hat{H}_{\psi}^{(3)} = \frac{1}{T}\sum_{t=1}^{T}\left[\nabla_{\psi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],$$

as blocks of $\hat{H}$.

Appendix IV

The Univariate GARCH Approach Volatility Spillover Model Baesed on Baele (2003)

The conditional return on the US index is assumed to evolve according one of the autoregressive processes - AR(1) process:

$$R_{US,t}=c_{0,US}+c_{1,US}R_{US,t-1}+e_{US,t} \quad (1)$$

The idiosyncratic shock $e_{US,t}$ is assumed to be normally distributed with mean 0 and the conditional variance follows a symmetric GARCH(1,1) specification, according to Engle (1982), Bollerslev (1986):

$$\sigma^2_{US,t}=\omega_{US}+\alpha_{US}e^2_{US,t-1}+\beta_{US}\sigma^2_{US,t-1} \quad (2)$$

It is necessary to inspect $\omega_{US}>0$ and $\alpha_{US},\beta_{US}\geq 0$ is positive to make sure that the variance is positive and also check that and $\alpha_{US}+\beta_{US}\leq 1$, which will ensure stationarity. The assumption on European indices is as follows, it is described by the following extended AR(1) specification:

$$R_{EU,t}=c_{0,EU}+c_{1,EU}R_{EU,t-1}+\gamma_{EU,t-1}R_{US,t-1}+\phi_{EU,t-1}e_{US,t}+e_{EU,t} \quad (3)$$

Test of significance has to be employed to state an optimal lag order of the AR specification according to set significance level. The conditional mean of the European returns depend on the their own lagged returns as well as the lagged US return. The mean spillover effects are introduced by the lagged US return, $R_{US,t-1}$ (i.e. the first AR extension). The volatility spillover from the US to Europe takes place via the semifinal term, $e_{US,t}$ (i.e. the second extension of AR process). Thus, the European return depends on the US idiosyncratic shock. I will revise why this is the representation of a volatility spillover effect. In the concrete practical usage, the residual from equation (1) is used in place of $e_{US,t}$. The idiosyncratic shock $e_{EU,t}$ has mean 0 and the conditional variance evolves according to the GARCH(1,1)

$$\sigma^2_{EU,t}=\omega_{EU}+\alpha_{EU}e^2_{EU,t-1}+\beta_{EU}\sigma^2_{EU,t-1} \quad (4)$$

It is again required that $\omega_{EU}>0$, $\alpha_{EU},\beta_{EU}\geq 0$ and $\alpha_{EU}+\beta_{EU}\leq 1$ (stationarity and positive variance). The last step consists in providing a model for the individual country returns. The mean specification for the European return in equation (3) is extended even further. Specifically for the Czech Republic:

$$R_{CR,t}=c_{0,CR}+c_{1,CR}R_{CR,t-1}+\gamma_{CR,t-1}R_{US,t-1}+\delta_{CR,t-1}R_{EU,t-1}+\phi_{CR,t-1}e_{US,t}+\psi_{CR,t-1}e_{EU,t}+e_{CR,t} \quad (5)$$

The conditional mean return is depended on the lagged US, European, and own return. This specification allows mean spillover effects from both the US and European returns to the Czech Republic by the lagged returns $R_{US,t-1}$ and $R_{EU,t-1}$. Volatility spillover effects from the US and Europe to the Czech Republic are introduced by the variables $e_{US,t}$ and $e_{EU,t}$, respectively, i.e. the idiosyncratic US and European shocks. Shortly, it will become clear exactly why this corresponds to volatility spillover effects, in equation (10) below. In the estimation, the residuals from equations (1) and (3) are regarded as explanatory variables. The idiosyncratic local country shocks are subject to the same distributional assumptions as the other idiosyncratic shocks; they have mean 0 and the following conditional volatility:

$$\sigma^2_{CR,t} = \omega_{CR} + \alpha_{CR} e^2_{CR,t-1} + \beta_{CR} \sigma^2_{CR,t-1} \quad (6)$$

where $\omega_{CR} > 0$, $\alpha_{CR}, \beta_{CR} \geq 0$ and $\alpha_{CR} + \beta_{CR} \leq 1$. It is necessary to assume that idiosyncratic shocks , $e_{EU,t}$ $e_{US,t}$ and $e_{CR,t}$ are independent. However this will not apply for unexpected returns:

$$\epsilon_{US,t} = e_{US,t} \quad (7)$$

$$\epsilon_{EU,t} = \phi_{EU,t-1} e_{US,t} + e_{EU,t} \quad (8)$$

$$\epsilon_{CR,t} = \phi_{CR,t-1} e_{US,t} + \psi_{CR,t-1} e_{EU,t} + e_{CR,t} \quad (9)$$

The definitions of the unexpected returns in equations (7), (8) and (9) enable to calculate the conditional variance of the unexpected return for country i, as well as the conditional covariances and correlations between the unexpected returns. The conditional variance of the unexpected return of the Czech Republic is based on the information available at time t − 1 (I_{t-1}) is given as follows.

$$h_{CR,t} = E(\epsilon^2_{CR,t} | I_{t-1}) = \phi^2_{CR,t-1} \sigma^2_{US,t} + \psi^2_{CR,t-1} \sigma^2_{EU,t} + \sigma^2_{CR,t} \quad (10)$$

The conditional variance of the unexpected return for the Czech Republic depends on the variance of the contemporary US, European, and own idiosyncratic shocks. When e.g. the US idiosyncratic volatility is large, the volatility of the unexpected returns for the Czech Republic also tends to be large (small) if $\phi_{CR,t-1}$ is positive (negative). This is a measurement of volatility spillover effect. A sign and significance of the parameters $\phi_{CR,t-1}$ and $\psi_{CR,t-1}$ determine whether volatility spillover effects come from the US or Europe and if they are present in the Czech Republic. The conditional variance of the European unexpected return depends only on the US and its own idiosyncratic volatility. The conditional variance of the US unexpected return is equal to the variance of the US idiosyncratic shock. The conditional covariance between the unexpected return of the PSE and the US (respectively European) unexpected return depends on the US (or both US and European) idiosyncratic volatilities:

$$h_{CR,US,t} = E(\epsilon^2_{CR,t} \epsilon^2_{US,t} | I_{t-1}) = \phi_{CR,t-1} \sigma^2_{US,t} \quad (11)$$

$$h_{CR,EU,t} = E(\epsilon^2_{CR,t} \epsilon^2_{EU,t} | I_{t-1}) = \phi_{CR,t-1} \phi_{EU,t-1} \sigma^2_{US,t} + \psi_{CR,t-1} \sigma^2_{EU,t} \quad (12)$$

The conditional covariance between the US and European unexpected returns depends only on the US idiosyncratic volatility:

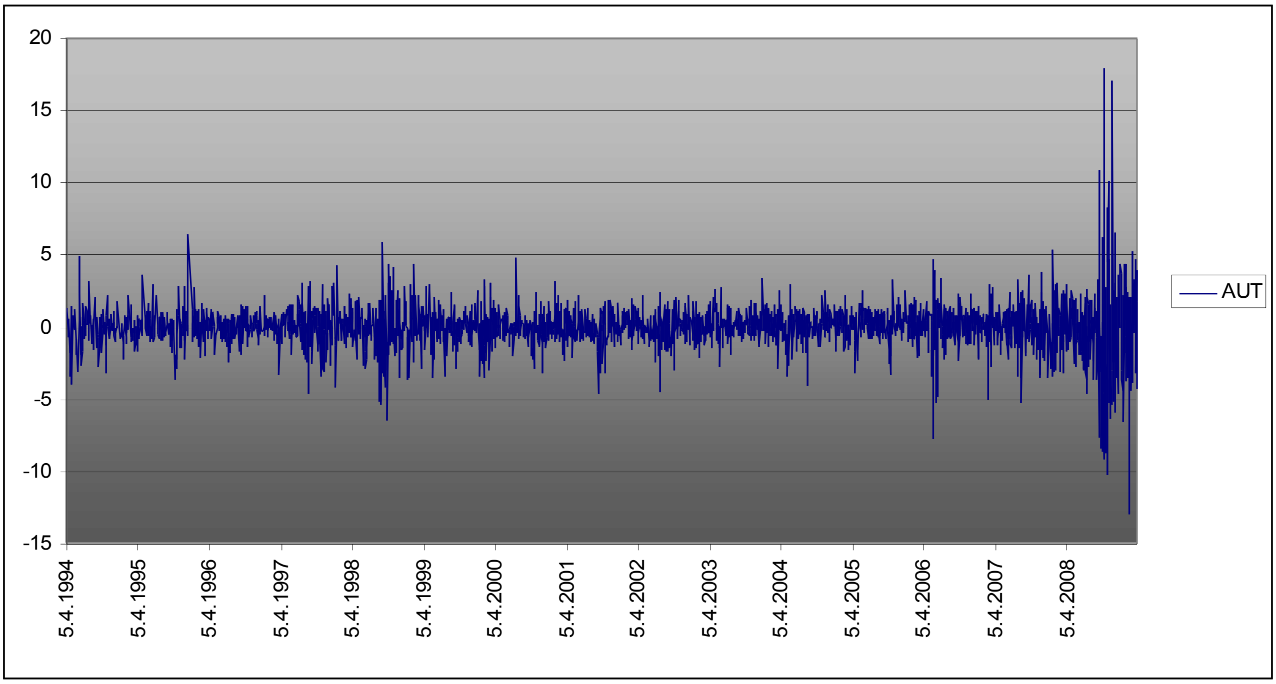
$$h_{US,EU,t} = E(\epsilon_{EU,t} \epsilon^2_{US,t} | I_{t-1}) = \phi_{EU,t-1} \sigma^2_{US,t} \quad \textbf{(13)}$$

Appendix V

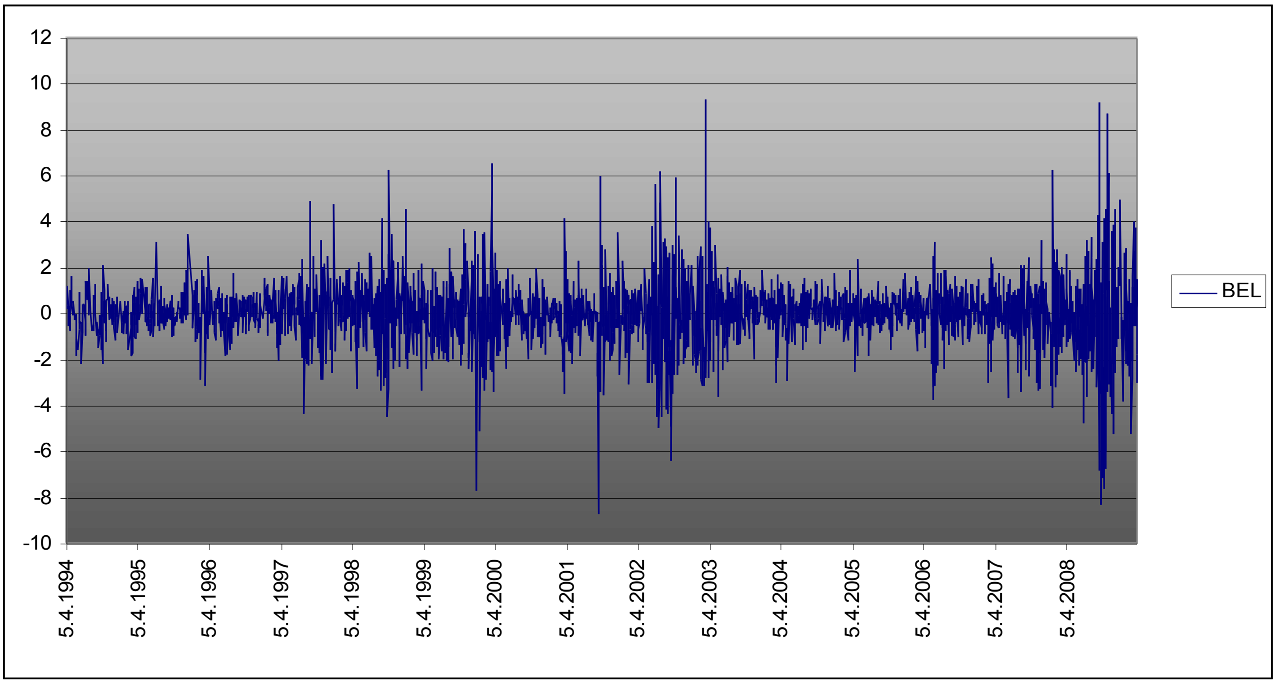
Net Daily Returns of Particular Indices (Percentage Changes)

Data source: yahoo.finance.com

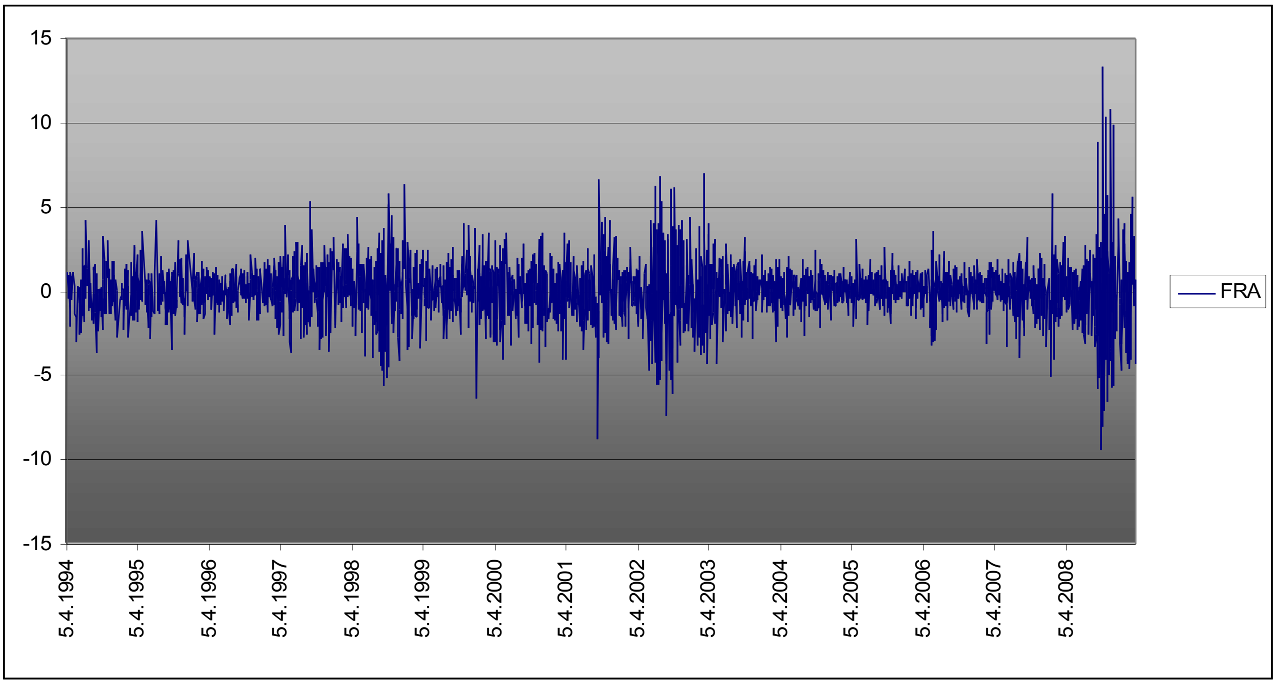
Austria - ATX



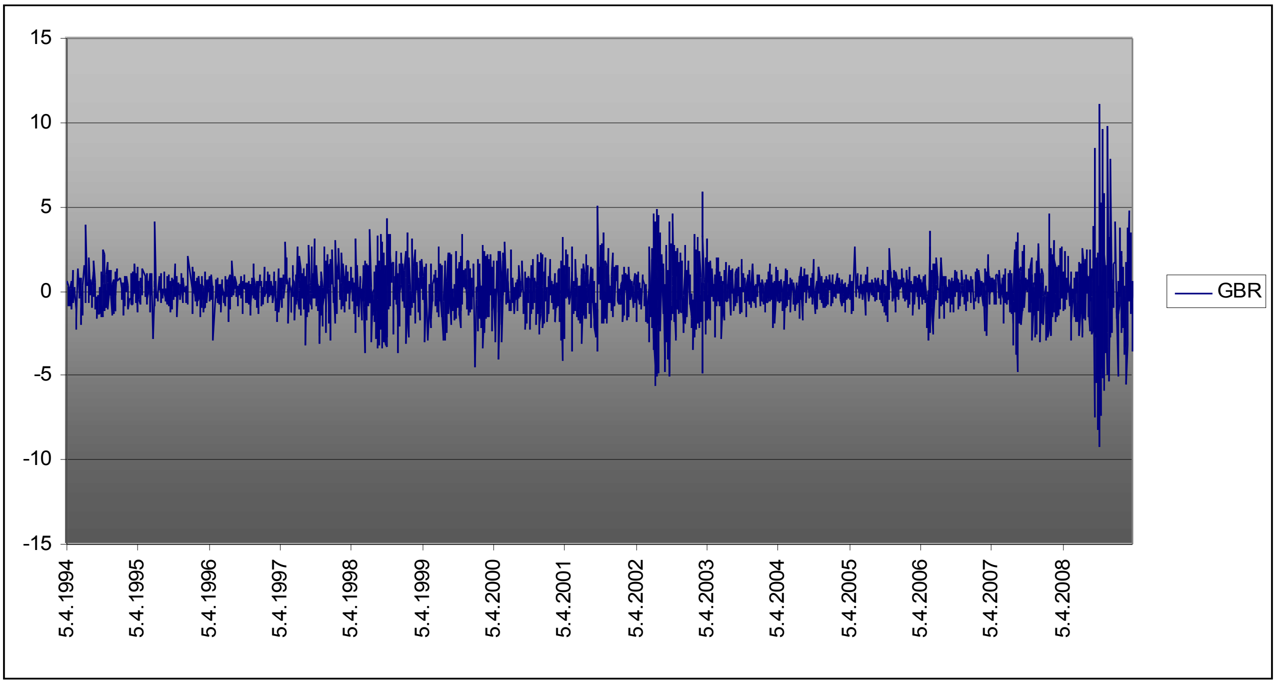
Belgium - BEL 20



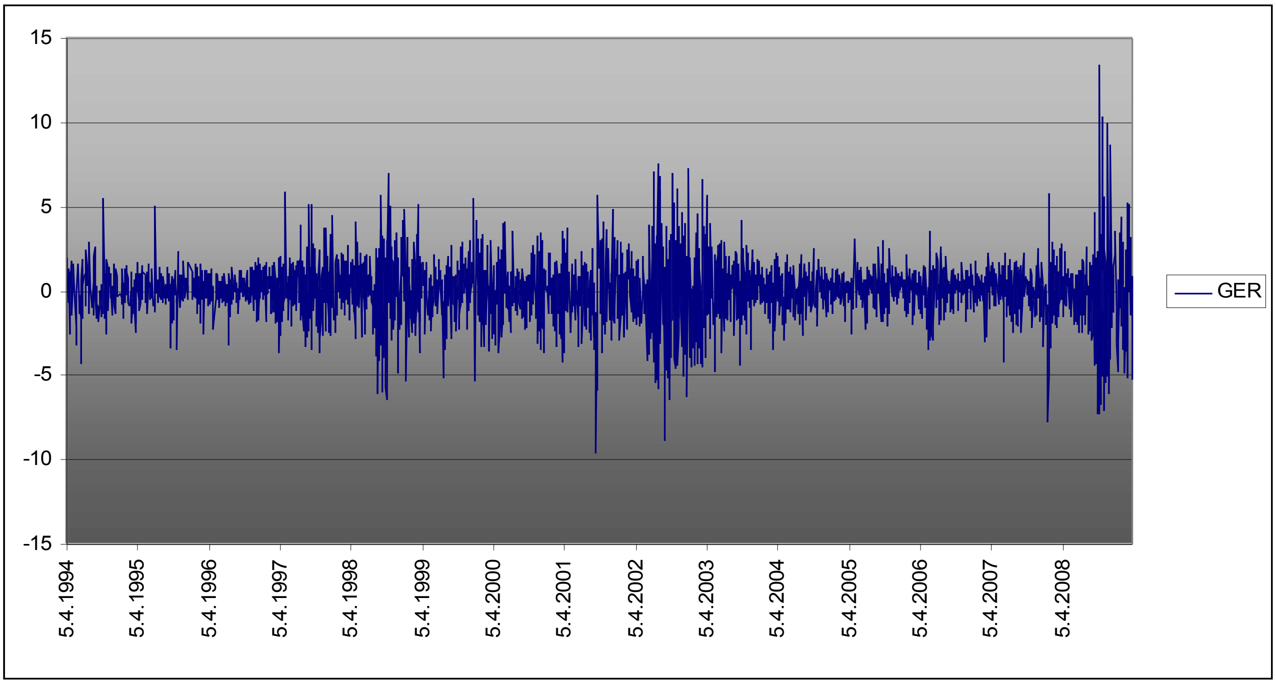
France - CAC 40



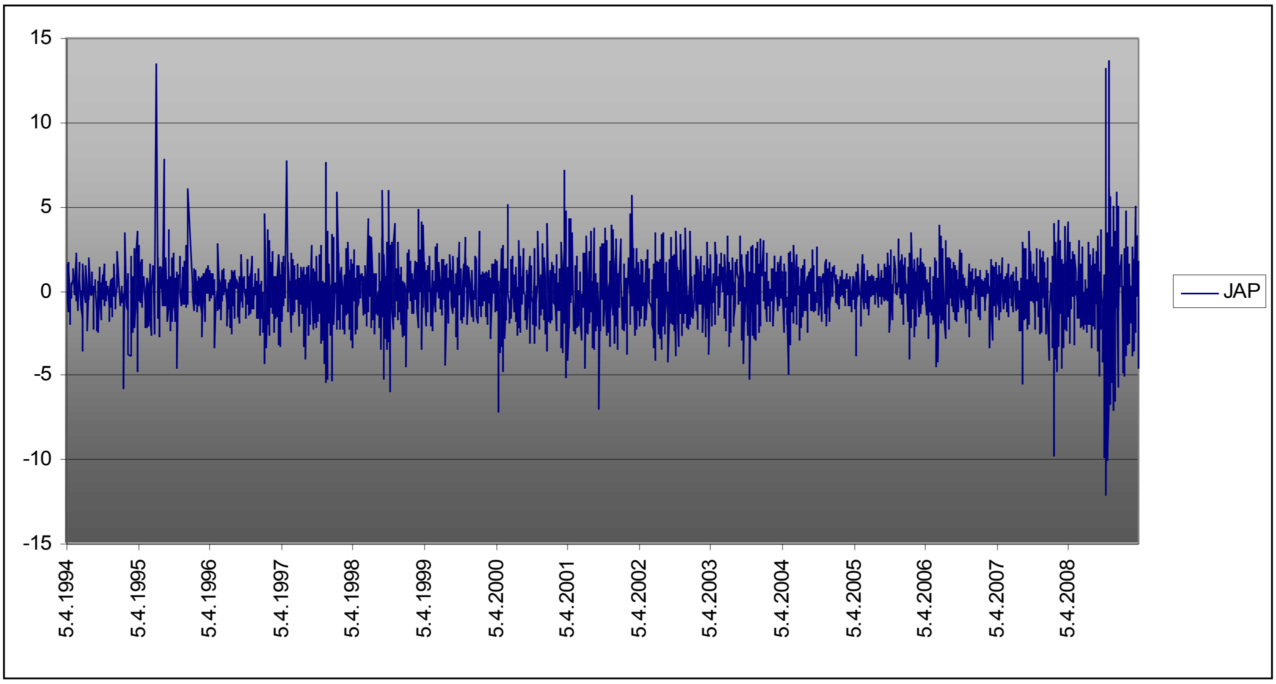
Great Britain - FTSE 100



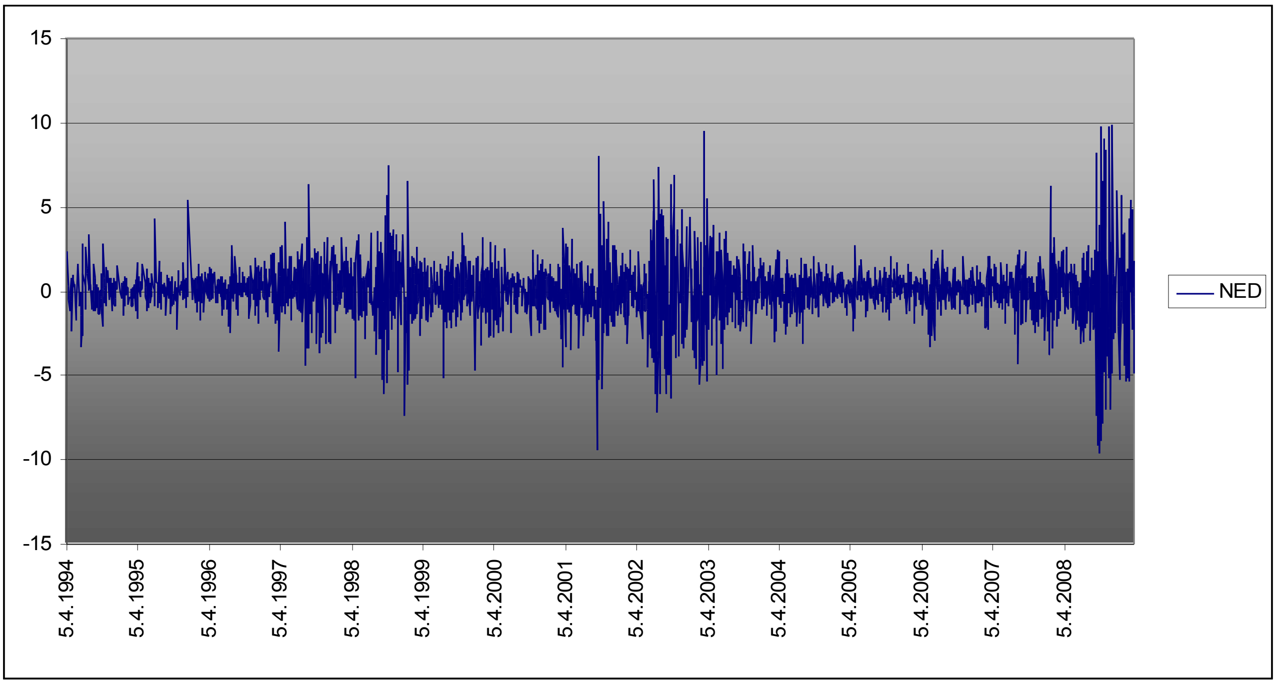
Germany - DAX 30



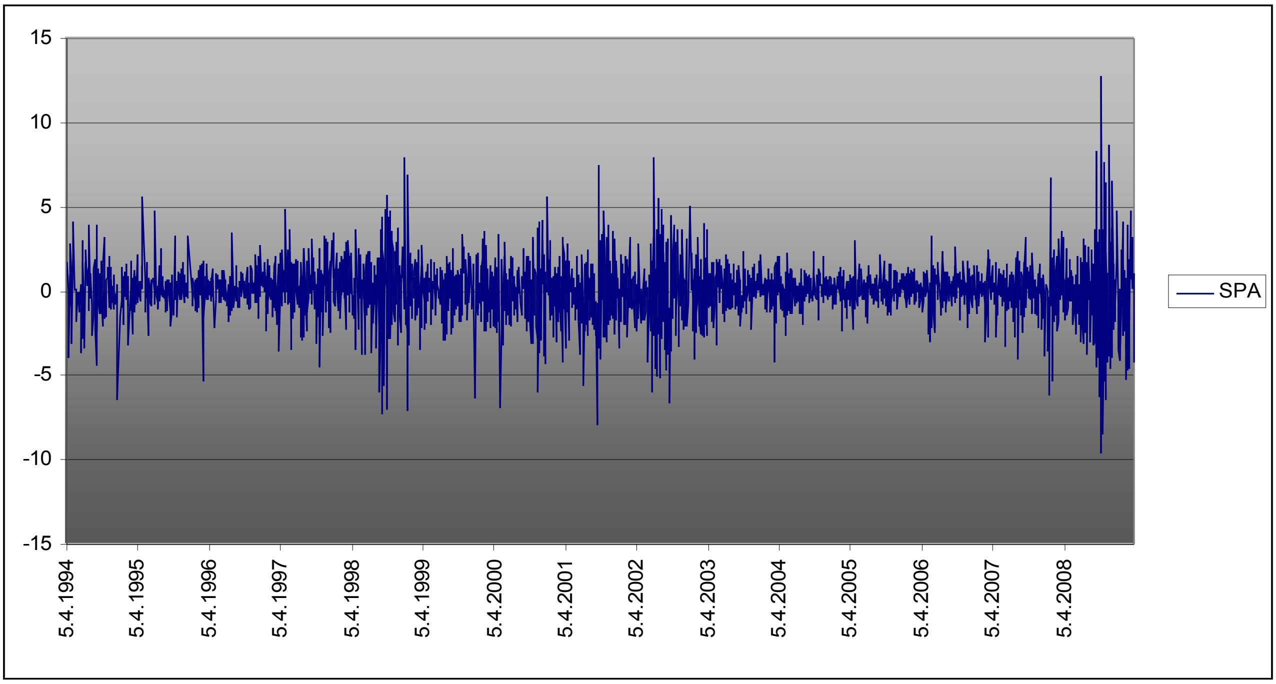
Japan - NIKKEI 225



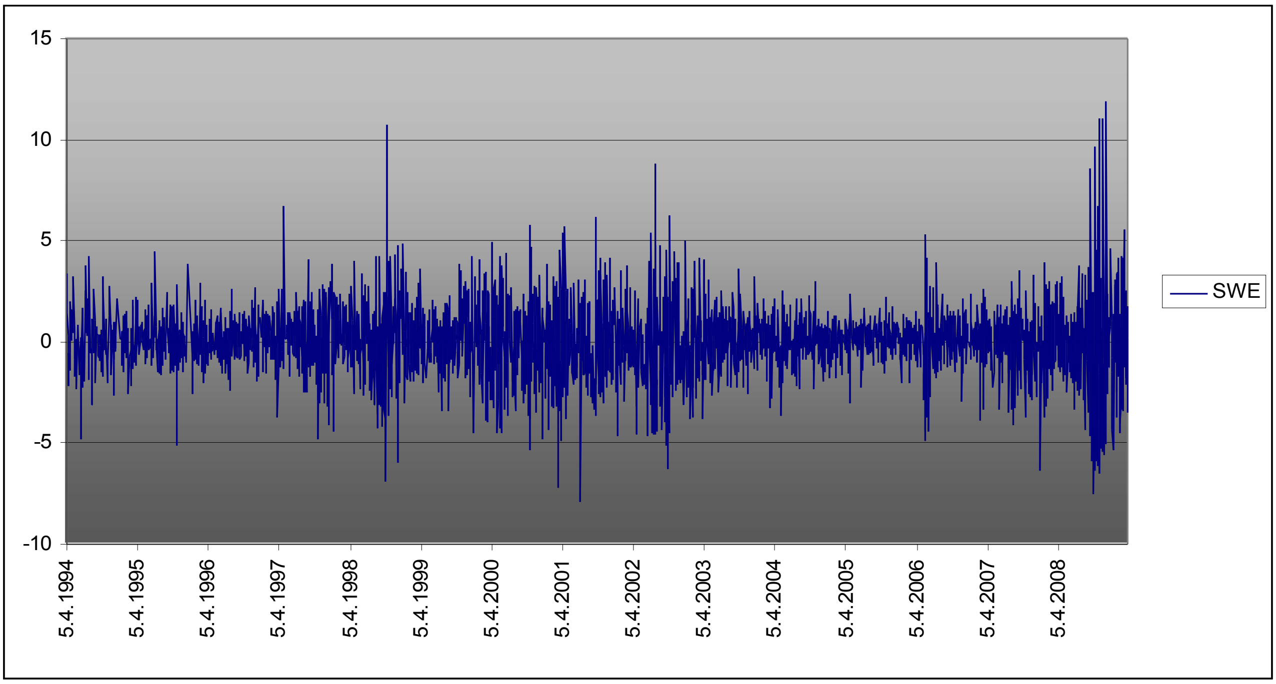
Netherlands - AEX



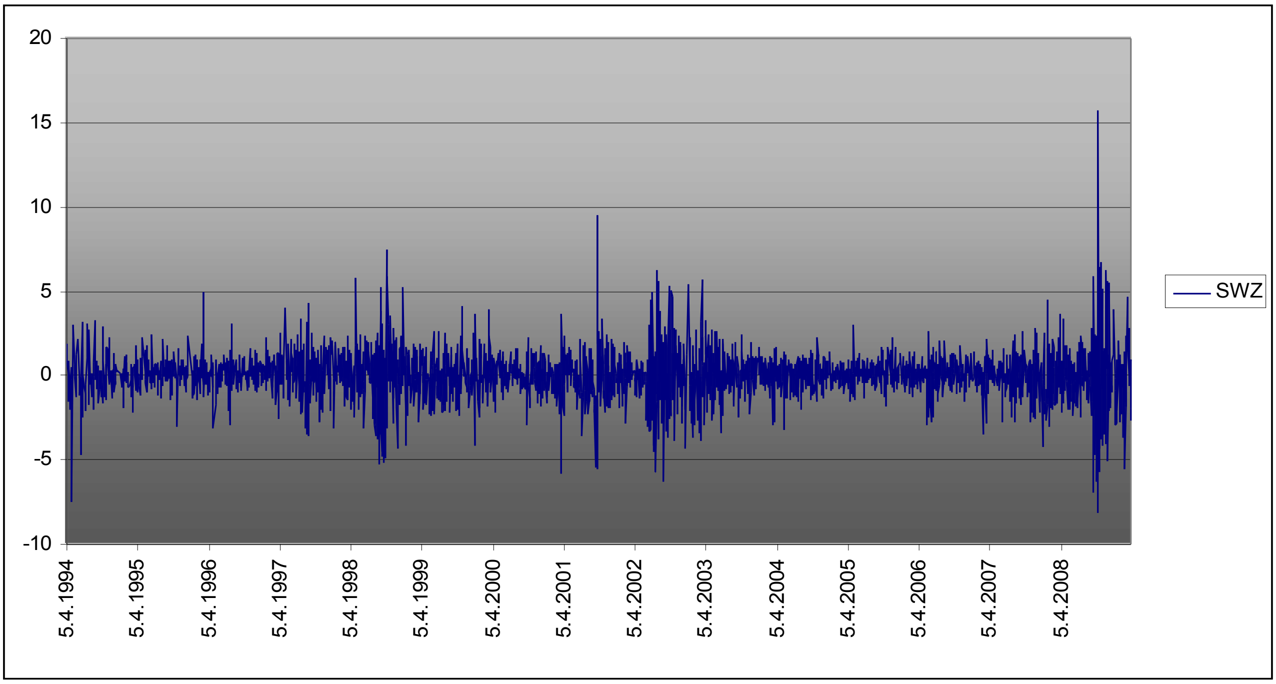
Spain - IGBM



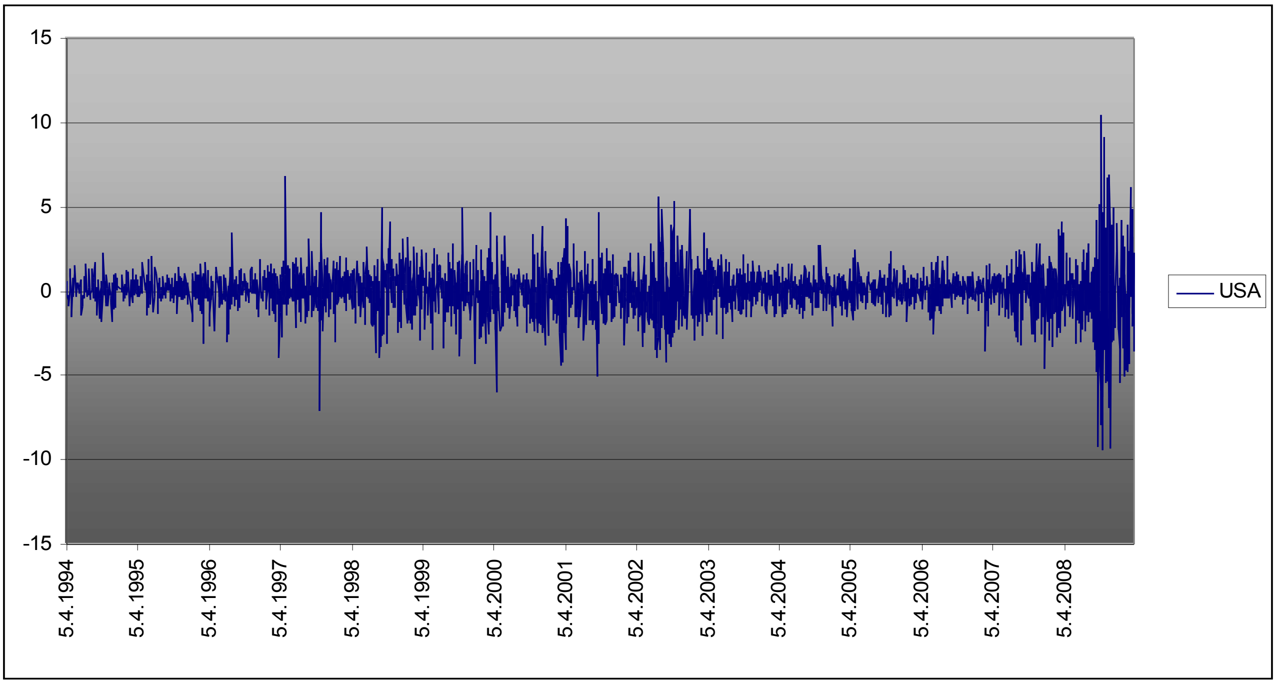
Sweden - OMX SPI



Switzerland - SMI



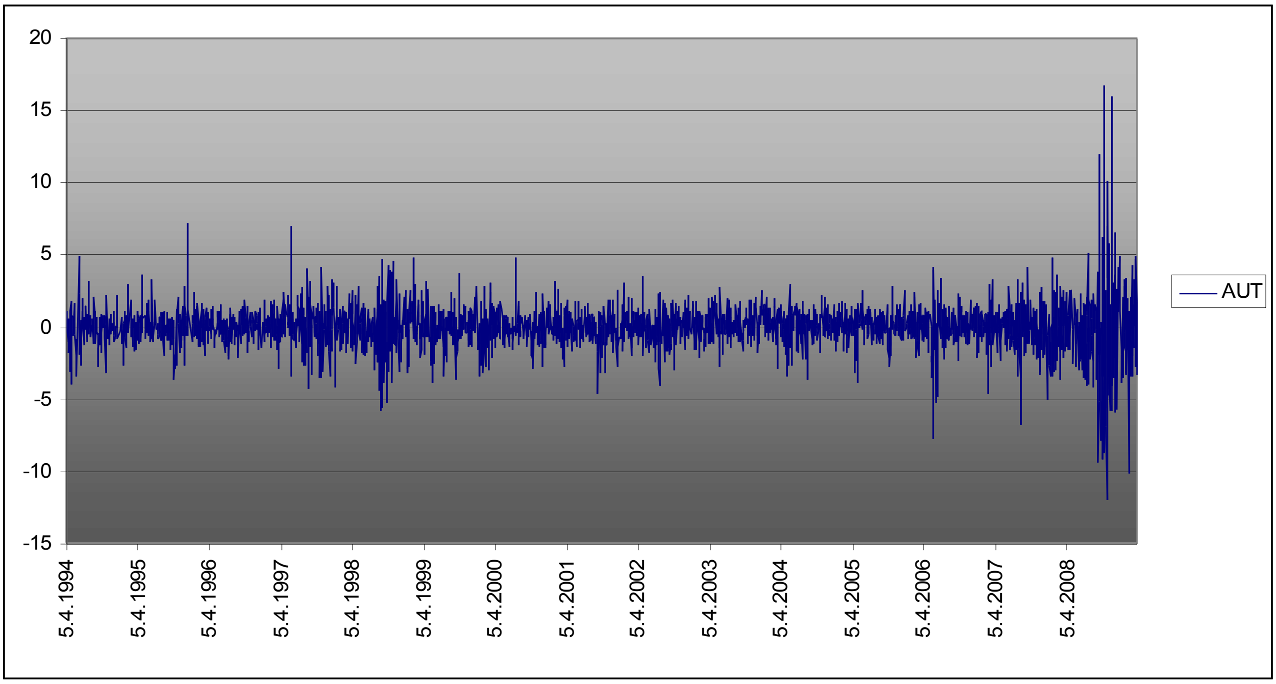
United States of America - NYSE 100



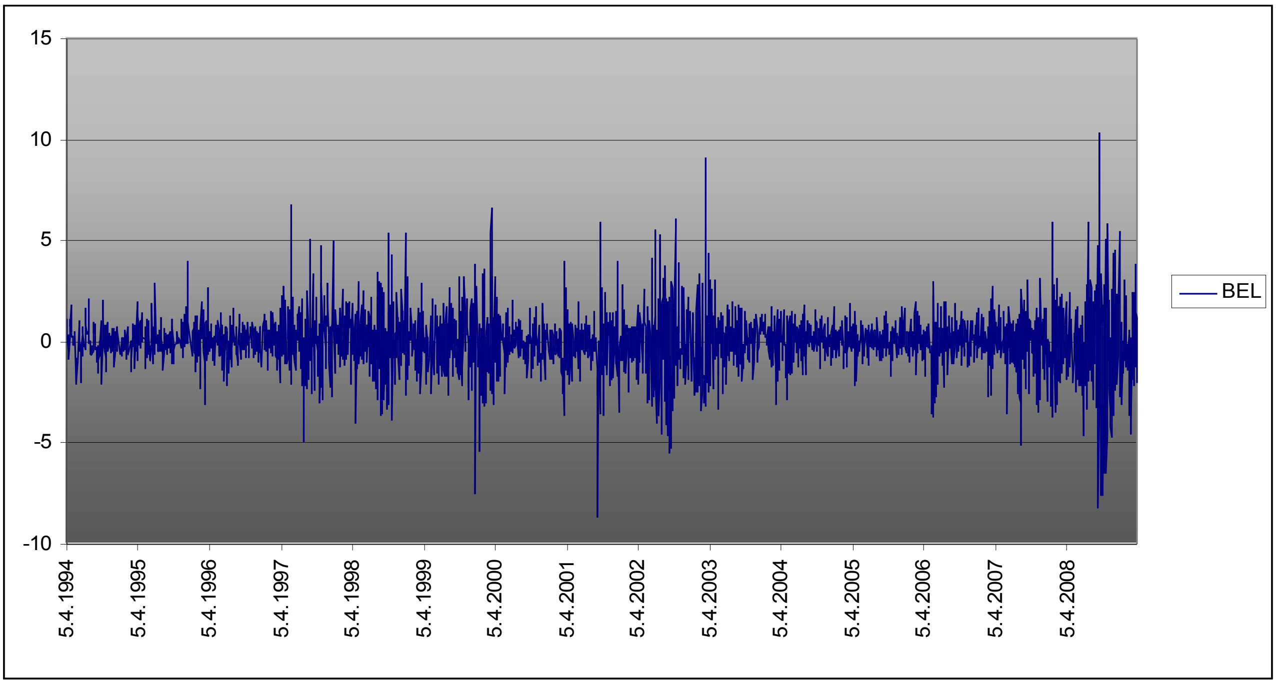
Adjusted Net Daily Returns of Particular Indices (Percentage Changes)

Data source: yahoo.finance.com + Czech National Bank database (www.kurzy.cz)

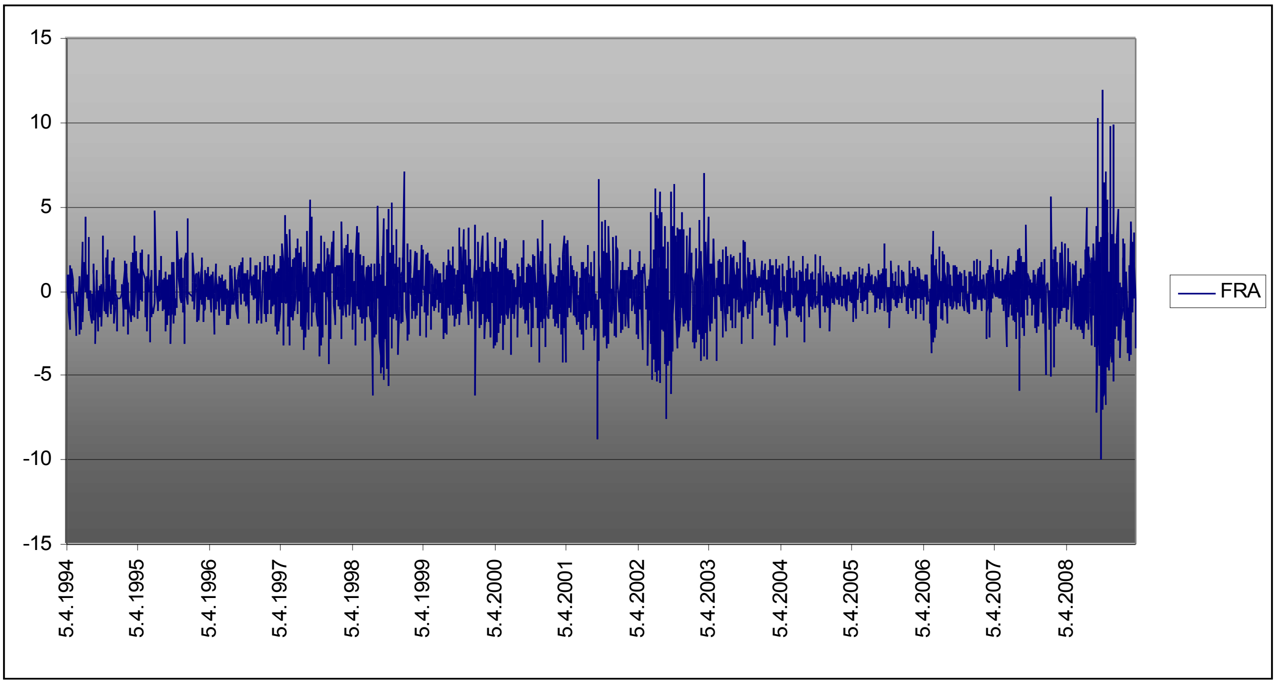
Austria - ATX



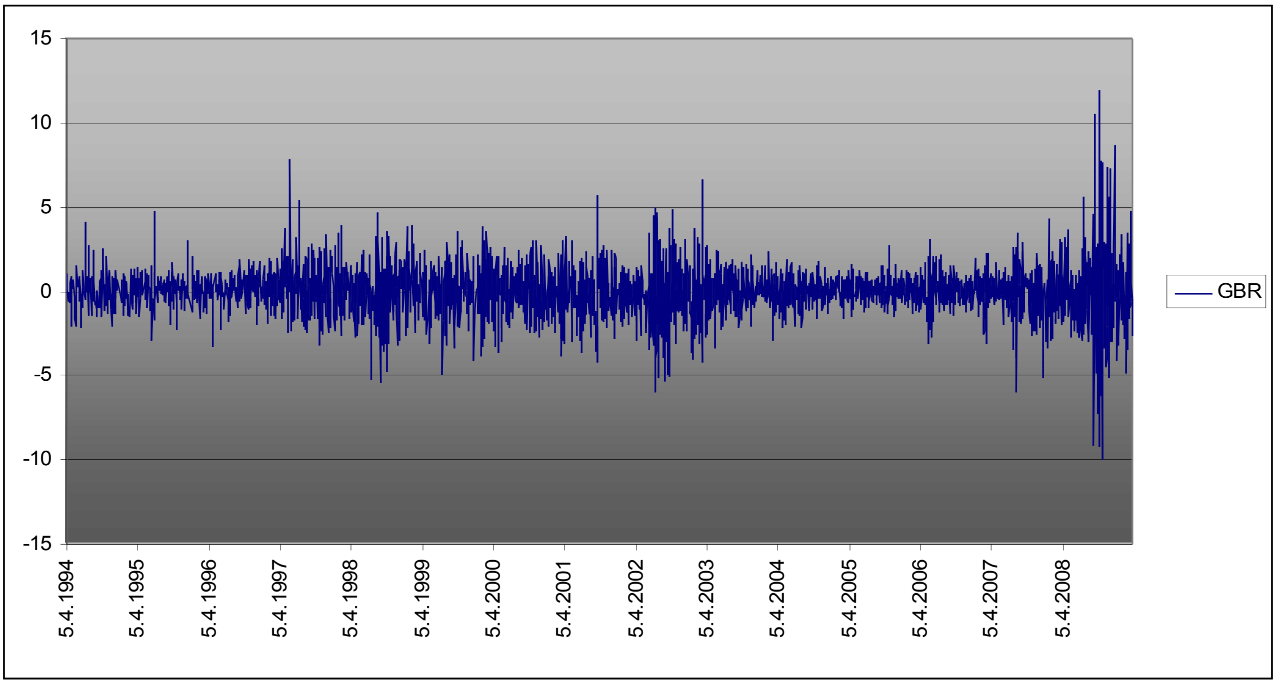
Belgium - BEL 20



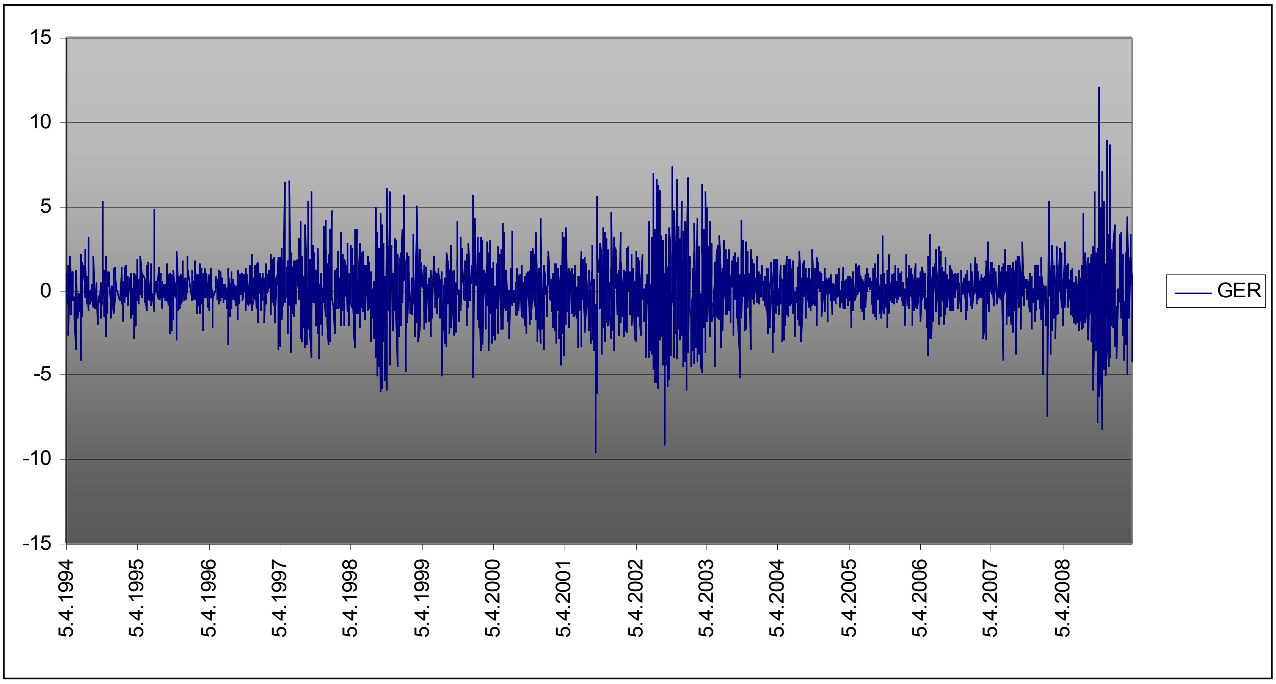
France - CAC 40



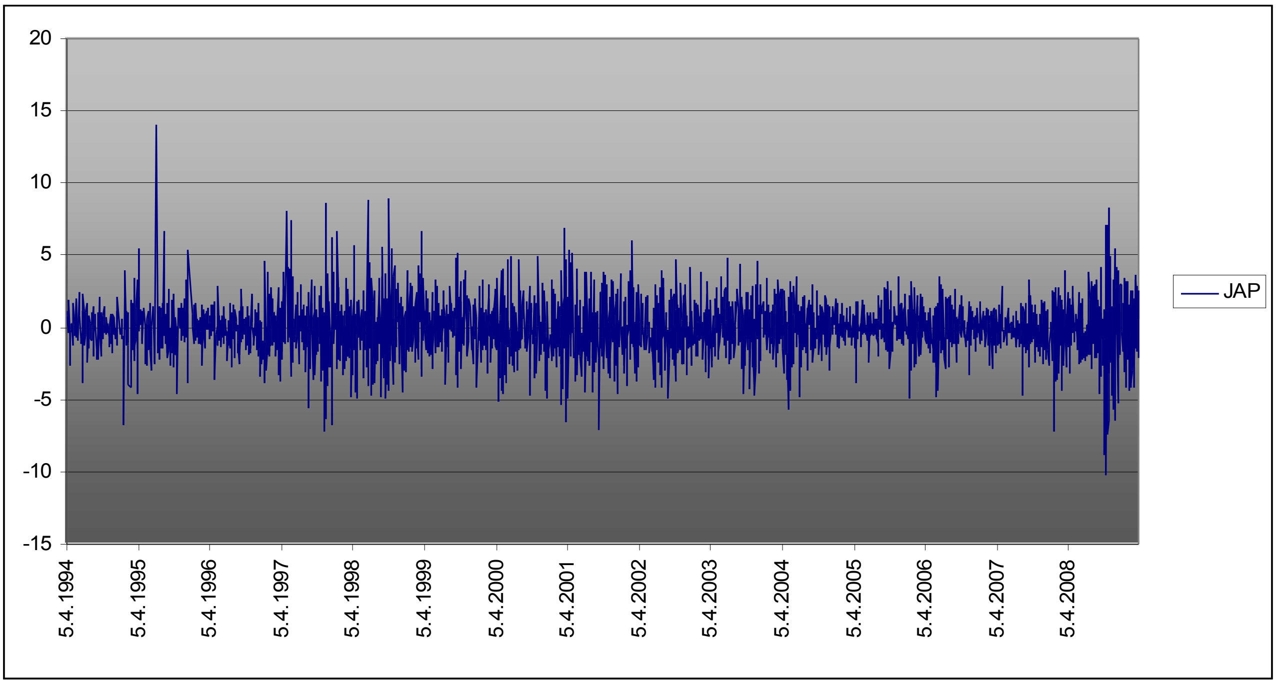
Great Britain - FTSE 100



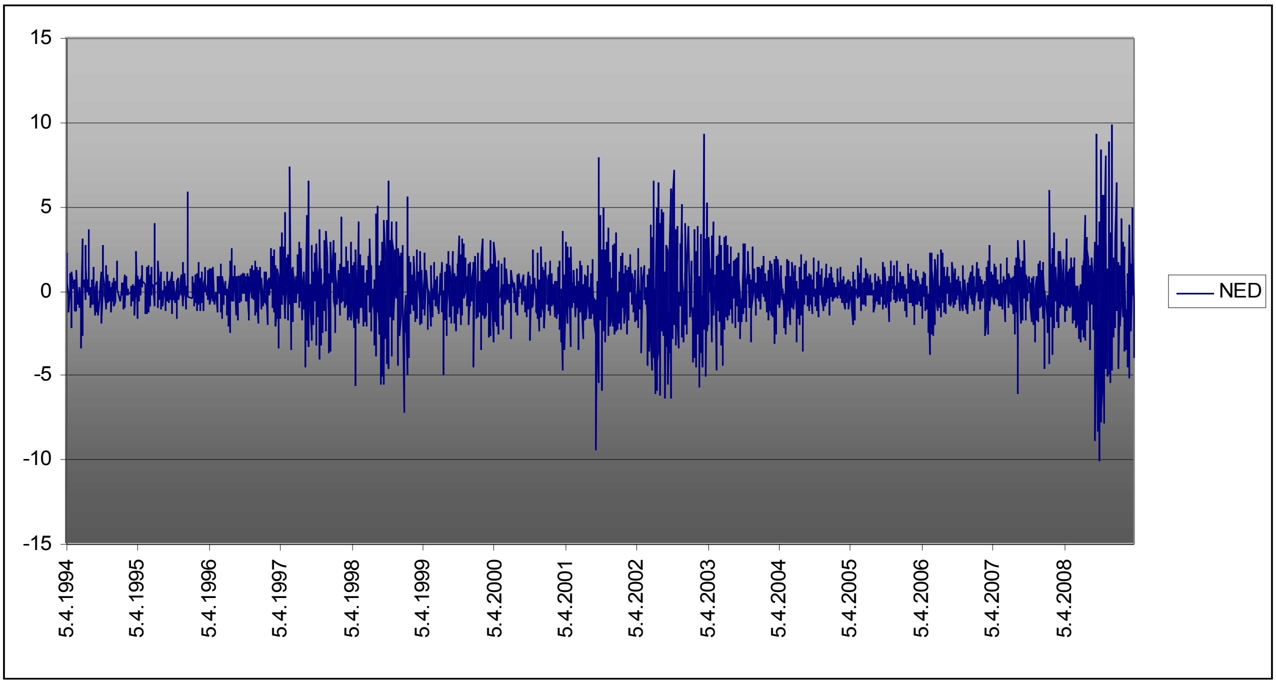
Germany - DAX 30



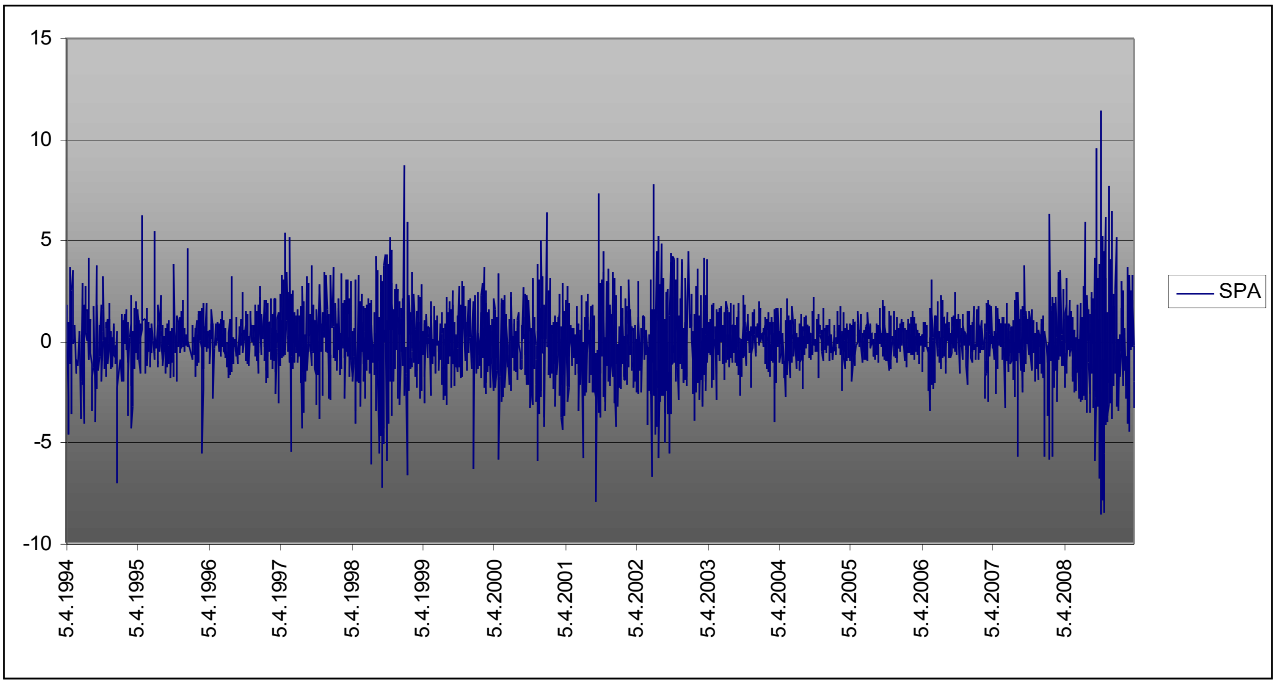
JAPAN - NIKKEI 225



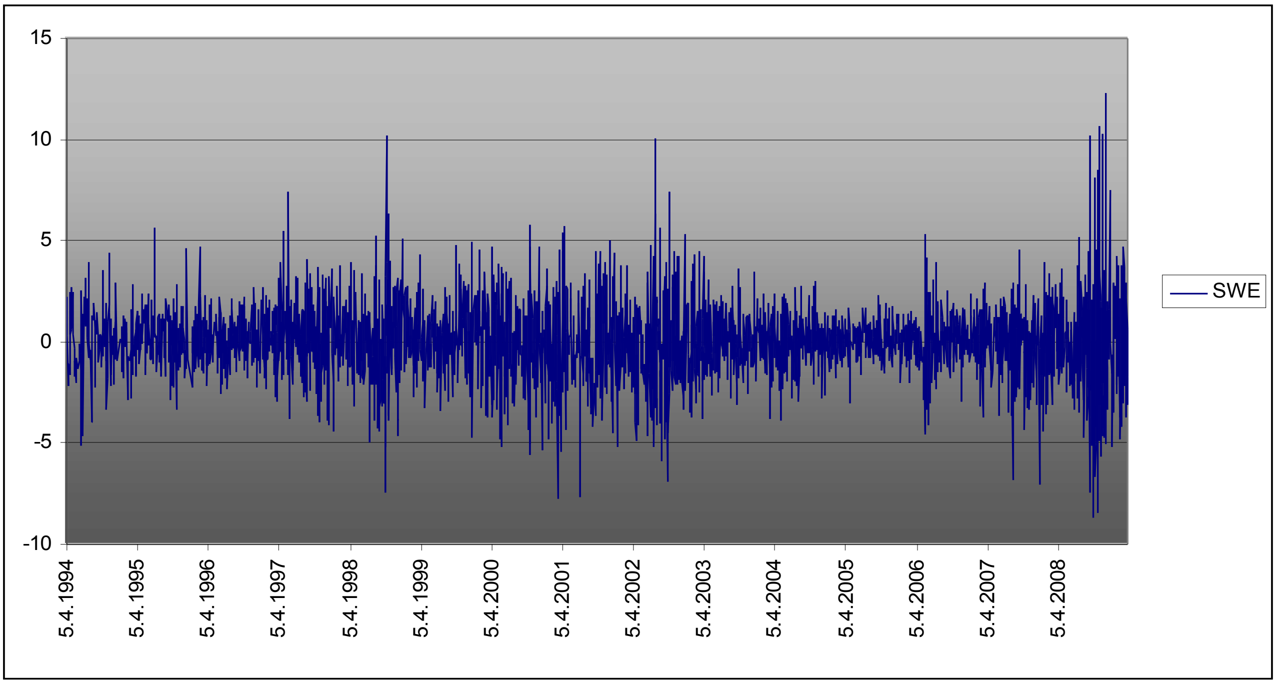
NETHERLANDS - AEX



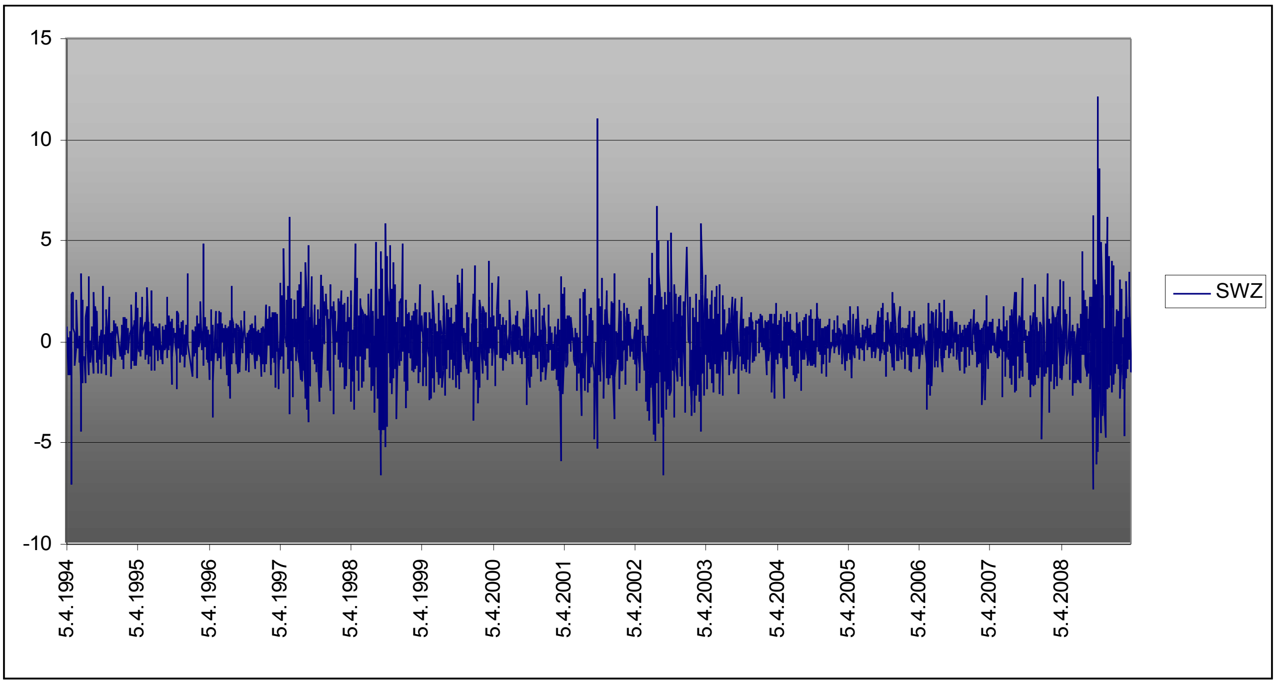
Spain - IGBM



Sweden - OMX SPI



Switzerland - SMI



United States of America - NYSE 100