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Northwestern University
Robert R. McCormick School of
Engineering and Applied Science
Department of Industrial Engineering
and Management Sciences
Technological Institute
2145 Sheridan Road
Suite C210
Evanston, Illinois 60208-3119

Office 847. 491. 3576
Fax 847. 491. 8005

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Opponent Report on Habilitation Thesis
Advances in Stochastic Programming Approaches to Optimization under Uncertainty
submitted by RNDr. Martin Branda, Ph.D.

I write in response to the request of Professor Jan Trlifaj, Vice Dean, dated 12 July 2018, that I serve as an opponent for the habilitation thesis of RNDr. Martin Branda, Ph.D.

The habilitation thesis, *Advances in Stochastic Programming Approaches to Optimization under Uncertainty*, is based on a collection of ten papers authored, and co-authored, by Martin Branda. The thesis organizes the discussion into work on: (i) chance-constrained optimization models; (ii) sample approximation of mixed-integer stochastic programs; and, (iii) extensions of the data envelopment analysis technique to capture diversification, with applications in portfolio optimization.

Chance-constrained optimization:

Mathematical programming models with chance constraints have been studied for nearly as long as stochastic programming. This began with work of Charnes and Cooper in the late 1950s. Then, chance-constrained optimization was studied deeply by Prékopa for over four decades, beginning in the 1970s. In his most influential work in stochastic programming, Prékopa focused on chance-constrained optimization under *continuous* probability distributions. This includes his seminal work on log-concave probability distributions, and their implications for convexity in chance-constrained models. The dissertation of James Luedtke in 2007 took chance-constrained programming in a new direction, with significant advances based on problems with probability distributions with finite support. This new direction applied mathematical tools from discrete optimization, formulating chance-constrained models as mixed-integer *linear* programs. It is clearly desirable to know when a chance-constrained optimization model is convex, e.g., for certain classes of models under a log-concave probability distribution. That said, from a computational perspective, such convex optimization models can be intractable because we cannot even evaluate the multidimensional integral involved in the constraint, let alone optimize over such functions. Thus, even though chance-constrained problems with discrete distributions are typically NP-hard when formulated as integer programs, in some cases these models indeed are computationally tractable. I briefly review this history because I believe it provides some context for the contributions of Martin Branda.

Ermoliev et al. (2000) used a positive-part penalty function for a single chance constraint. In Branda and Dupačová (2012) and Branda (2012), this type of penalty function was generalized to handle multiple chance constraints and a family of penalty functions that include smooth and non-differentiable functions. That said, these results were obtained under continuous probability distributions, i.e., distributions that do not lend themselves to numerically solving the optimization model. Importantly, Branda's 2013 paper in *Mathematical Methods of Operations Research* extends these earlier results to handle the case of discrete probability distributions. In this approach the chance constraint is effectively moved to the objective function via the expectation of a penalty function that is smoother than the indicator function. Then, as the weight on the penalty function grows large, an equivalence with the original chance-constrained model is obtained. The equivalence is based on a certain notion of "calmness" due to Burke, and importantly, an equivalence can be obtained with a finite weight on the penalty function. Moreover, useful and asymptotically tight, upper and lower bounds are provided on the optimal value via the parametric regularized models.

In his co-authored 2016 paper in *Journal of Optimization Theory and Applications* and his co-authored 2018 paper in *Computational Optimization and Applications*, the focus is on mixed integer *nonlinear* optimization models. The former paper directly concerns chance-constrained optimization models. The latter concerns models with a cardinality constraint which limits the number of decision variables that take nonzero value. (Cardinality constraints have application, for example, in asset allocation models in finance, when we limit the number of assets in which we can invest; in notions of sparsity in estimating statistical models; and, in chance-constrained optimization.) Both papers investigate continuous relaxations of the discrete nonlinear optimization model, coupled with appropriate notions of regularization. Interestingly in the 2016 paper, the global minima of the relaxation matches that of the original problem, although the relaxation introduces spurious local minima and stationary points. In both papers the regularization is parametrized, and the papers establish that, asymptotically, stationary points (i.e., necessary conditions for optimality) of the regularized model converge to those of the original model under an appropriate constraint-qualification hypothesis.

Sample approximation in stochastic mixed-integer optimization:

One of the primary ways that the discrete probability distributions just discussed arise in chance-constrained programming is through approximating a continuous distribution using a Monte Carlo sample. This is especially true in the moderate- to high-dimensional cases for the multivariate random vectors upon which practical stochastic programs are based. Thus, the first two parts of Branda's habilitation thesis, on (i) chanced-constrained models under discrete distributions and (ii) sample approximations in stochastic mixed-integer programming, are well integrated under the same thread of research.

When using a Monte Carlo approximation (sample approximation) in a chance-constrained model, the danger is that the solution is "over trained" to the finite sample, and is infeasible to the true problem over the original continuous probability distribution. This is circumvented by enforcing the chance constraint with a higher level of reliability in the Monte Carlo approximation. In this setting, it is possible to show the probability that the sample approximation solution is feasible to the original problem approaches one at an exponential rate that depends, in part, on the difference between the reliability levels. Such results had been previously established when the feasible region of the problem is finite (i.e., the pure integer programming case) and when the feasible region involves continuous decision variables and is bounded. In his 2012 paper in *Operations Research Letters*, Branda unifies these two results in the *mixed* integer programming case. His results are careful to distinguish how the continuous and discrete decision variables contribute to the rate of convergence or, equivalently, to the sample size needed to ensure a desired probability that the sample approximation solution is feasible to the original problem. His results yield computationally valuable upper and lower bounds on the model's optimal value, and assess the quality of the solution with out-of-sample reliability testing.

In Branda's 2014 paper in *Optimization Letters*, he again deals with sample approximation of problems with expected-value constraints. The assumptions on his expected-value constraints cover a range of problems of interest including models with CVaR constraints, integrated chance constraints, certain types of stochastic dominance, and certain penalty-based approximations to chance constraints, discussed above. In this setting, the 2014 paper generalizes earlier results of his 2012 paper and of Wang and Ahmed (2008). The generalization includes the ability to, again, handle the *mixed* case of both continuous and some discrete decision variables, as well as handling certain types of non-iid sampling. In addition, Lipschitz continuity is relaxed using a notion of calmness due to Hölder. While Hoeffding's inequality is the basis for the results in Branda's 2012 paper, the 2014 paper is rooted in large deviations theory. The result again lends itself to computing a sample size needed to ensure feasibility to the original problem with high probability.

DEA with diversification consistency:

Direct application of data envelopment analysis (DEA) fails when applied to asset allocation models because it fails to capture dependencies in the returns of the collection of assets. With this as motivation, Lamb and Tee (2012) introduced notions of diversification in DEA models. The approach of Lamb and Tee was limited to considering the positive part of risk measures, while the approach of Branda, first in his 2013 paper in *European Journal of Operational Research*, is more general. Using coherent risk measures and their return counterparts, in Branda's 2015 paper in *Omega*, he formulates a family of three fractional programs—and reformulates them as convex programs—which can identify Pareto efficient solutions that are, in turn, weakly, semi-strongly, and strongly Pareto efficient. Moreover, using the expected return and the coherent risk measure of CVaR, Branda (2015) shows, under a discrete probability distribution, that the resulting DEA fractional program can identify a solution that satisfies desirable second-order stochastic dominance efficiency properties. The work of Branda, joint with Miloš Kopa, published in *Operations Research Letters* in 2016 extends their earlier work on DEA-style models yielding second-order stochastic dominance tests of efficiency to variants that handle handle higher-order characterizations of stochastic dominance.

Summary:

I see Branda's research contributions as today's most important results in the area of chance-constrained optimization in combining discrete probability distributions with nonlinear functions in the constraints and the objective function. His work in this area is insightful, deep, original, and, importantly, accounts for computational tractability when it comes to numerically solving the approximating models that he proposes. This work is highly valued in the stochastic programming community. Branda's research in asset allocation models using DEA, coupled with notions of diversification: (i) shows his creativity in mathematical modeling, (ii) again displays his strength in developing mathematical rigorous characterizations of solutions to optimization models, and (iii) again confirms his ability to perform compelling computational experiments. His work on DEA and finance has attracted considerable attention from a community complementary to that in stochastic programming. He is publishing an an excellent rate, and the quality of the journals where he is publishing is also excellent. Most importantly, the quality of Martin Branda's scholarly work is outstanding. I strongly recommend that he be promoted to associate professor.

Sincerely,

[redacted]

David Morton
Chair, Industrial Engineering & Management Sciences
David A. and Karen Richards Sachs Professor