Report on the master thesis

**Daniel Jahn: Generalized random tessellations, their properties, simulation and applications**

Special models of random Delaunay tessellations and random Laguerre-Delaunay tessellations in $\mathbb{R}^3$ are investigated. These are induced by Gibbs point processes with interactions depending on the tetrahedra forming the tessellation. The idea stems from Dereudre, Drouilhet and Giorgi (Existence of Gibbsian point processes with geometry-dependent interactions, PTRF 153 (2012), 643–670), where only Delaunay tesselations are considered and the two-dimensional case is treated in detail. Daniel builds on this paper rather heavily, presents the existence theorems for Gibbs point processes in rather high generality and, as main theoretical contribution, verifies the assumptions on four particular models.

In the second part, simulation of models considered in the theoretical part is done. Finally, some numerical results are presented.

Daniel surely deserves high appreciation for the wide range of knowledge he had to acquire, including the mathematically demanding staff of Gibbs point processes, as well as techniques of simulation and the Metropolis-Hastings algorithm. From the mathematical point of view, there are no difficult new results. On the other hand, the theory is presented in a rather extensive way documenting a deep understanding. Especially the geometrical part introducing Laguerre tessellations is a beautiful introductory to this staff. The part introducing Gibbs point processes is more difficult to follow, the notation is complicated and, maybe, some simplifying assumptions could have been used at the beginning (e.g., to consider only "unary" potentials). In the simulation part, I wonder why no picture of a simulated tessellation has been provided.

A list of some concrete questions follows.

(1) p. 6, Def. 5: $\mathcal{D}_4(\mathbf{x})$ yields a tessellation only if $\mathbf{x}$ satisfies the "reinforced general position" assumption (if more than four points lie on the same sphere with no other points in its interior, $\mathcal{D}_4(\mathbf{x})$ contains different intersecting tetrahedra). A similar effect could appear in the case of Laguerre-Delaunay.
(2) p. 17, comment after Def. 19: This is not true. It can surely happen that $\varphi(\eta, \mathbf{x}) \neq 0$, but $\eta \notin \mathcal{E}_\Lambda(\mathbf{x})$ (if $\Lambda$ is away from the horizon of $(\eta, \mathbf{x})$).
(3) p. 18, Remark 7: By definition, $\mathcal{E}_\Lambda(\mathbf{x})$ depends on the potential $\varphi$ which is not specified in the two models considered here. Does it mean that the Remark 7 is true for any (unary) potential on the given hypergraph structures?
(4) p. 30, Proposition 13: This statement is not clear to me. What is $\tilde{\mathcal{N}}_{\Lambda^c}$?

I suggest Daniel to answer these questions during the defense.

Regardless of this, the thesis in my opinion fulfills the criteria for a master thesis.

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