# CHARLES UNIVERSITY IN PRAGUE

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# Portfolio diversification with cryptoassets

*Bachelor thesis*

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## Abstract

This thesis investigates diversification benefits of Bitcoin and Ethereum. Technological innovation that made them possible, is interesting but for investors hard to grasp. The more important question is whether they should buy the digital currency or avoid it. We analyze Bitcoin and Ethereum from point of view of an investor within compatible with mean-variance (and non-mean-variance respectively) framework. Both cryptoassets are alternately added to base portfolio consisting of global indices representing American, European and Asian markets. Statistically rigorous tests suggest that Bitcoin yields added value to investors with utility function consistent with mean-variance setting. Same holds for for investors with preferences described by exponential and power utility function. Ethereum shows similar results with exception of exponential utility. Performance benefits of both assets are preserved in the out-of-sample setting as size of test window reaches 28 weeks and increases. In the case of shorter test window, base assets show similar or slightly superior performance. Optimal allocation in out-of-sample framework is found by direct utility maximization with gradient based method.

## Keywords

**Bitcoin, Ethereum, digital currency, investment portfolio, diversification**

# Abstrakt

Táto téza skúma diverzifikačné benefity Bitcoinu a Etherea. Technologická inovácia stojaca za ich vznikom, je síce zaujímava no pre investorov ťažko pochopiteľná. Dôležitejšia otázka z ich pohľadu je, či by mali do kryptoaktív investovať alebo sa im vyhýbať. Analyzujeme Bitcoin a Ethereum z pohľadu investora kompatibilného s mean-variance (respektíve non-mean-variance) teóriou. Obe kryptoaktíva sú striedavo pridané do základného portfólia zloženého globálnych indexov reprezentujúcich americké, európske a ázijské trhy. Štatisticky rigorózne testy ukazujú, že Bitcoin disponuje pridanou hodnotou pre investorov s úžitkovou funkciou konzistentou s mean-variance teóriou. Rovnaký výsledok preukázali aj testy u investorov s exponenciálnou a power úžitkovou funkciou. Ethereum ukazuje rovnaké výsledky s výnimkou prípadu investora s exponenciálnou úžitkovou funkciou. Výkon oboch kryptoaktív je zachovaný taktiež mimo vzorku, ako náhle je veľkosť testovacieho okna väčšia ako 28 týždňov. V prípade kratšieho testovacieho okna, základné portfólio ukazuje porovnateľný alebo jemne lepší výkon. Optimálne váhy pre jednotlivé aktíva sú nájdené pomocou priamej maximalizácie úžitkovej funkcie cez metódu gradientov.



# Klíčová slova

**Bitcoin, Ethereum, digitálne meny, investičné portfolio, diverzifikácia**

## Declaration of Authorship

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# Bachelor Thesis Proposal

## Motivation

Even though according to [Ali Robleh et al., (2014)] the key innovation of Bitcoin is "distributed ledger" or "blockchain" which is already supported by many academic papers, its recent surge in market value can't be ignored. As of May 2017 Bitcoin's market cap reached as high as $47 billion. Since its creation (2008), Bitcoin has evolved from a mathematical proof of a concept to a rapidly expanding economic network. Its decentralized character, open source operating basis and limited supply securing impossibility of inflation policy made Bitcoin a very unique asset.

Unfortunately there are also negatives in decentralized ecosystems, for instance it makes it harder to reach a consensus in certain very important themes. Paradoxically the recent increased demand caused community disagreement on how the system should expand further to handle more transactions at the time for less costs . Lack of authority, apart from others, created room for growth of other cryptocurrencies such as Ethereum. Personage of co-founder Vitalik Buterin appears to be a very important asset in connecting decentralized cryptocurrency world with public. Formation of Enterprise Ethereum Alliance supported by J.P. Morgan Chase, Microsoft, and Intel or latest tokenization of Singaporean Dollar through Ethereum's Blockchain increased "Ether's" market cap as high as $34 billion which Bitcoin first reached just in May of 2017.

Even considering all this information, there is still a shortage of economic perspective on the topic. That's why we decided to perform one of the first such analyses with both Bitcoin and Ethereum from portfolio optimalization standpoint in mean-variance setting.

## Methodology

In previous years, many studies (e.g. [Briere, et al. (2015)]) evaluated the diversification impact of commodities in the mean–variance framework of [Markowitz (1952)] to test the impact of the introduction of additional N risky assets (test assets) on the efficient frontier of an investment opportunity set of K benchmark assets. However, [Daskalaki et al., (2011)] have shown that violation of two important assumptions often occurs. Namely that asset returns are not distributed normally (e.g. [Peiro, (1999)]) and that investor's preferences are not that well described by a quadratic utility function. With that being said we decided to follow [DeRoon et al., (1996)] rigorous robust in-sample spanning test.

This thesis improves upon [Chowdhury, (2016)] and [Briere, et al. (2015)] in two aspects. Firstly we extend the sample period by almost 4 years long period which means inclusion of the biggest Bitcoin bubble burst in first half of 2014. In contrast to previous research, we am able to analyze periods with considerable bearish sentiments and eliminate signs of early-stage behavior.

We consider investor holding a diversified portfolio comprising both traditional assets (worldwide stocks, bonds, currencies) represented by several liquid financial indices. Daily closing prices of Bitcoin and Ethereum are also used. Finally, we investigate relative performances of portfolio with and without cryptoassets using Wald test statistics and respective p-values. Descriptive statistic containing Sharpe ratio, coefficients of kurtosis and skewness are presented with additional comparison to previous papers.

## Hypotheses

- Does introduction of Bitcoin and Ethereum in the investor's asset universe yields diversification benefits within an in-sample mean-variance framework?
- Are the results robust for investors with utility function inconsistent with the mean-variance setting?
- Are there any benefits in investing in Bitcoin and Ethereum within an out-of-sample (non)mean-variance framework?

## Outline

1. Introduction
2. Literature Review
3. Features of Bitcoin and Ethereum
4. Portfolio Theory and Asset Diversification
5. Methodology
6. Data & Results
7. Conclusion

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# 1 Introduction

In just about eight years, a bare $1 purchase of Bitcoin at $0.05 in July 2010 has grown to an aggregate value of $147,667 or $7,383.39 per Bitcoin. The birth of virtual currency dates back to 2009, with the publication of white paper by [Nakamoto, (2008)], which was considered a pseudonym by [Castronova, (2001)]. Even though Bitcoin has existed for about a decade, it had a very limited liquidity and usage during its first years. Bitcoin is viewed simultaneously as a currency, a commodity, and an asset class according to [Trautman et al., (2017)]. During the year 2017, the value of Bitcoin rose from about $970 to $15,000, which is an increase of approximately 1,735 percent ([Vigna, (2018a)]). By mid-December 2017, Bitcoin was equivalent to the world's sixth largest currency in terms of capitalization according to [Trautman, (2018)].

Such rapid evolution attracted serious attention from investors, and consequently became closely observed by regulators. The latter, however, remain in dispute over the significance of Bitcoin. [Kar et al., (2018)] indicated there are nations such as the USA, Japan, Canada, and Australia, which have recognized and regulated the use of Bitcoin. On the other hand, countries like Saudi Arabia, Bolivia, or Bangladesh have declared usage of Bitcoin as illegal. On top of that, there are many countries like India, South Africa, South Korea, and Vietnam, where national agencies still have not adopted a clear standpoint with regards to Bitcoin.

Presumably, the story of Bitcoin was well covered by the mainstream media along the way. The Dutch newspapers suggested that Bitcoin was a hedge for stocks during Brexit as well as during the US elections of 2016 ([FD, (2016)]). [Shaffer, (2017)] from CNBC hypothesized about Bitcoin rivalling gold as a safe haven while [Ford, (2013)] from Bloomberg speculated about bitcoin being the last safe haven. Last year's studies demonstrated that Bitcoin can show signs of both a hedge and a safe haven. Nevertheless, [Bouri et al., (2017a)] indicated that Bitcoin is a poor hedge and so is only suitable for diversifying purposes. Only in cases of weekly extreme down movements Bitcoin can be a strong safe haven. Finally, [Dyhrberg, (2015a)] found that Bitcoin is a hedge against the FTSE and a short-term hedge against the US Dollar. Moreover, [Dyhrberg, (2015b)] found that Bitcoin can be useful in risk management, when a negative shock is expected. According to [Bouoiyour et al., (2017)], Bitcoin is a safe haven property that varies in time and has primarily been a weak safe haven in both short- and long-term.

Therefore, there is an evidence that Bitcoin has some hedging capabilities, even though its inclusion in traditional stock-bond portfolios has yielded mixed results so far. ([Briere, et al. (2015)], [Chowdhury, (2016)], [Kar et al., (2018)], [Wilmars et al., (2018)], [Eisl et al., (2015)]).

We explore virtual currencies Bitcoin and Ethereum (B&E) in particular as an alternative asset class being incorporated into global investment portfolios. Both may be an attractive investment from a diversification standpoint because of their low correlation with equities. However, B&E's characteristics of high volatility and potential illiquidity complicate their comparison make with more traditional asset classes such as equities and bonds.

This thesis proceeds in five parts. Firstly, we describe financial and technical properties of Bitcoin, namely its use cases, price determinants, market specifications, and unique risks posed to potential investors. Secondly, rise of Ethereum is put into the context of cryptocurrency space. Distinctive attributes as smart contracts are described with real world potential use cases. Thirdly, an overview of portfolio theory and asset diversification follows. In the fourth part, research is conducted and presented over 32 quarters (Q3 2010 to Q2 2018), examining the relationship between B&E and other asset classes, including global equities, bonds and currencies.

## 2 Literature Review

Bitcoin has charmed researchers from different fields since its inception. The use of 'blockchain' technology for payment systems has caught attention of academicians in the field of computer science whereas the decentralized and private nature of transactions on the Bitcoin network has become a matter of interest for researchers in the area of law and compliance. Studies concerning the economic and financial implications of Bitcoin picked up pace from 2013 when the currency touched $1000 mark. [Weber, (2015)] and [Bohme et al., (2015)] present an elaborative view of the governance and technological issues of Bitcoin and discuss the feasibility of Bitcoin as a potential medium of exchange. [Luther, (2014)] claim that the highly volatile nature of Bitcoin's value makes it unsuitable to become a popular means of exchange. [Lo et al., (2014)] also find Bitcoin to be a poor replacement for fiat currencies due to the economies of scale possessed by the latter. [Glaser et al., (2014)] conclude that digital currencies are more popular amongst users as an investment alternative than a payment mechanism.

Being one of the first studies to evaluate the diversification benefits of Bitcoin, [Briere, et al. (2015)] calculated that Bitcoin reached annualized return of more than 370% with 175% volatility (July 2010 - July 2013). They found that its returns had a weak but significant correlation with gold and inflation-linked bond. It was concluded that a small allocation up to 3% of Bitcoin to a well-diversified portfolio—could improve risk-return trade-off.

[Eisl et al., (2015)] test the relevance of addition of Bitcoin to an already well-diversified portfolio comprising of 12 different indices. With a perspective of a US investor, they calculate risk-return ratio similar to [Campbell et al., (2001)] to find the optimal combination of assets on a rolling basis for each month from July 2011 to April 2015. They conclude that average monthly returns and CVaRs are higher for a portfolio with Bitcoin and report superior risk-adjusted performance of portfolios containing Bitcoin.

[Chowdhury, (2016)] assesses Bitcoin's role in financial markets. As one of the initial attempts to evaluate inclusion of Bitcoin in a portfolio, it follows a more general approach than the traditional mean-variance in-sample setting. Results suggest Bitcoin offers diversification benefits to investors regardless of the performance measure used.

[Moore et al., (2016)] assess the potential benefits and costs of holding Bitcoins as a part of portfolio of international reserves by considering the case of Barbados. The study finds that if Bitcoin were included in the portfolio, it would have generated significant returns, but the volatility of the reserves would also increase.

[Bouri et al., (2017b)] find that Bitcoin may serve as a safe haven against severe declines in Asian stocks, and such properties of Bitcoin may vary across different horizons.

[Carpenter, (2016)], using a modified mean-variance framework, shows that Bitcoin appears to be an attractive investment that can substantially increase the return/risk ratios of an efficient portfolio–even when we impose considerable return penalties.

[Kajtazi et al., (2018)] similarly to [Eisl et al., (2015)], using mean-CVaR approach, try to asses diversification benefits of Bitcoin in portfolios consisting of U.S., European and Chinese assets. By adding Bitcoin, investors observe increased returns and better performance of the portfolios measured using Risk-Return, Sortino and Omega ratios.

[Symitsi et al., (2018)], using multivariate GARCH model, document significant diversification benefits of Bitcoin for equal-weighted and optimal minimum-variance portfolios in out-of-sample setting. An important finding is that the economic gains are not reduced after the consideration of transaction costs with daily and weekly re-balancing.

[Klein et al., (2018)] conclude from their ex-post minimum-variance portfolio exercise that in contrast to Gold, Bitcoin does not provide a hedge for equity investments which are under distress.

[Chan et al., (2018)] used pairwise GARCH models and constant conditional correlation models to demonstrate strong hedging capabilities of Bitcoin with respect to Euro STOXX, Nikkei, Shanghai A-Share and S&P 500 under monthly data frequency. Therefore, holding Bitcoin longer may benefit investors by providing risk management abilities to their equity portfolios.

[Guesmi et al. (2018)] explore the conditional cross effects and volatility spillover between Bitcoin and financial indicators using different multivariate GARCH specifications. They demonstrate that a short position in the Bitcoin market allows hedging the risk investment for all different financial assets and out-performs portfolio made up of gold, oil and equities only.

[Katsiampa, (2018)], using a bivariate Diagonal BEKK model, shows that Ethereum can be an effective hedge against Bitcoin, while the analysis of optimal portfolio weights indicates that Bitcoin should outweigh Ethereum.

# 3 Features of Bitcoin and Ethereum

## 3.1 Bitcoin

*"Virtual currencies, perhaps most notably Bitcoin, have captured the imagination of some, struck fear among others, and confused the heck out of the rest of us." – Thomas Carper, US-Senator*

Cryptocurrencies are digital assets intended to serve as alternative means of payment. They are created and managed via decentralized open source code, instead of authority such as a central bank. Most popular one is Bitcoin, which was introduced by [Nakamoto, (2008)]. The whitepaper states that Bitcoin is a "peer-to-peer version of electronic cash [which] would allow online payments to be sent directly from one party to another without going through a financial institution". Nakamoto likely a pseudonym, was inspired by 2008 Financial Crisis which suggests following quote:"The Times 03/Jan/2009 Chancellor on brink of second bailout for banks". Considerable amount of transactions started to appear on the network around 2010.

Recently, Bitcoin's market cap and number of transactions has been unheard-of. As a result, plenty of other cryptocurrencies (altcoins) usually based on Bitcoin's fundamentals started to enter the market. Currently more than 130 altcoins exceed $100 million market cap. This extreme growth has been very likely result of speculation and brought a lot of chaos between investors, financial institutions, policy makers and public.

### 3.1.1 Technical overview

Whereas traditional payment systems and fiat currencies rely heavily on hulking financial institutions to control supply and mediate transactions, Bitcoin distributes this burden on set of peers or so called miners who are compensated for their incurred costs by receiving payouts in the cryptocurrency. This process broadcasts trust and so allows users to rely rather on many different parties instead of placing it just on one single entity.

As summarized in [Bonneau et al., (2015)], Bitcoin consists of three components: transactions transferring ownership of coins, the consensus protocol, and the communications network.

*Transactions*: Bitcoin transactions are protocol messages that transfer currency from one wallet to another. Each wallet is represented by a public/private key pair, and the hash of the public key serves as an address that can be associated with transaction inputs and outputs. By creating a new transaction, a wallet owner can claim Bitcoins output from a prior transaction if they prove ownership of the corresponding private key, as outlined in [Kaiser et al., (2018)].

*Consensus protocol*: Shared public leger, implemented as series of blocks, serves as main tool for consensus maintenance. Each block includes set of transactions, a timestamp, an arbitrary number (nonce), a hash of the previous block, and other protocol metadata. Every block is cryptographically linked to the previous block, creating chain-like structure. Any alteration to an earlier block requires recalculation of all following hashes and although extremely unlikely would cause 'break' of the chain, as stated in white paper by [Nakamoto, (2008)]. Miners role is to collect transactions, validate them, organize them to blocks and publish them to the blockchain. They receive block reward which is special amount included in each block. This is only mean by which Bitcoins can be created. Computational puzzle called Proof-Of-Work (POW) is used in order to determine which block is the new one. As a result global consensus can be reached by all nodes in the network. A valid block is uniquely defined as one whose double SHA256 hash is below a target threshold value. Miners have to basically randomly guess different nonces until a valid block is found. In case multiple valid blocks would be found, something called fork occurs and miners have to decide through a vote which branch will stay. Typically miners choose the longer branch to continue mining. Miners usually collaborate in groups called mining pools in order to decrease reward variance. Pool members submit partial POWs, which are blocks that hash to a value close to the target but are not actually valid. Based on he submissions pool managers are able to proportionally reward miners based on computational power they preformed without the need to be actual founder of the valid hash.

*Communication network*: Bitcoin nodes use a peer-to-peer broadcast network to announce and propagate block and transactions. Set of rules is placed on all the nodes present in communication network in order to enhance performance and maintain consensus. The communication network's performance and degree of centralization have an effect on the consensus protocol. With respect to performance, higher latency between particular nodes can result in short-term forks which in turn cause instability of the network. In order to maintain fairness in the network, centralized control of nodes should be avoided. Radical cases can cause 51-49 attack where miner control majority of nodes and so can mine their own block and validate them. Furthermore, if anyone is able to censor transactions in the network, they can prevent transactions from spreading and cause unnecessary energy spends. Hence to ensure fairnes and stability, network should maintain low latency, sufficient level of decentralization and uncensorability.

### 3.1.2 Uses of Bitcoin

Independence of financial infrastructure and government helps Bitcoin to gain great popularity in countries suffering from higher degrees of inflation. In Venezuela, many citizens converted their unstable Bolivars to Bitcoin to store their value and purchase goods online. Subsequently mining has become more and more popular as a source of income, [Torpey, (2018)] argues. Comparable effect of unstable economic conditions have been observed in countries like Zimbabwe, Greece and Ukraine according to evidence provided by [Armario, (2018)]. Moreover, investment bank Goldman Sachs has begun to facilite trading by offering its own Bitcoin derivative products to clients shows [Popper, (2018)].

On the other hand, [Christin, (2013)] claims that Bitoin is also extensively used on black markets for illegal purposes. Main reasons being that it offers certain degree of anonymity, discussed by [Goldfeder et al., (2018)] and lack of clear rules on what goods and services could be bought ([Bohme et al., (2015)]; [Bryans, (2014)]). Although not completely anonymous, Bitcoin still leaves very little of physical evidence for authorities and so is often used for money laundering ([Bryans, (2014)]; [Moser et al., (2013)]). Most famous example would be online marketplace Silk Road as is described in US vs. Ulbricht, (2014), Government Exhibit 12.940. Other activities and industries, such as gambling and casinos, began to embrace Bitcoin. Last but not least, Bitcoin was used to avoid international capital controls. As a result People's Bank of China banned Bitcoin trading in 2014, [Van Alstyne, (2014)] claims.

Bitcoin steadily earned more and more popularity among retailers too. [Cuthbertson, (2015)] shows that Bitcoin came to be accepted as a method of payment by more than 100,000 merchants in 2015. The main reason being smaller transaction costs. Though broader use is restricted by scaling issues Bitcoin currently has. Since every transactions needs to be recorded in blockchain, millions of transactions could take longer to process argues [Houy, (2014)]. There are block size limits and difficulty requirements, which also take into account increasing computing power, that cause this unwanted property. Several ideas were proposed in order to fix these scaling issues recently. Including on-chain solutions as for example SegWit or Bitcoin Cash, which in the end was not a success or off-chain solution called Lightning Network which will be discussed later.

Ultimately, most people see Bitcoin as investment asset. This speculative use is heavily supported by literature ([Baek et al., (2015)]; [Cheah et al., (2015)]; [Glaser et al., (2014)]).

### 3.1.3 Bitcoin's value

Across the literature, multiple authors explained why Bitcoins have value. Simple reason being that there is just demand for it ([Ciaian et al., (2015)]; [Kroll et al., (2013)]). This demand is caused mainly by cheaper transaction costs related to no intermediaries, solid anonymity and fungibility ([Árnason, (2015)]; [Dwyer, (2015)], [Segendorf, (2014)]). It appears that Bitcoin is particularly interesting for risk loving investor due to its high degree of volatility.

Second reason enabling the Bitcoin to have value is peoples' trust in the system. Due to its open-source nature everybody is able to see and verify its source code and propose changes as is shown in [Buchholz et al., (2012)]. Unlike traditional banking there is no risk of double-spend and thus Bitcoin cannot be counterfeited as [Dwyer, (2015)] shows. Users' trust is enhanced also by limited total amount in the circulation. Number of Bitcoins is capped by 21,000,000 and will be reached approximately in 2140. [Nakamoto, (2008)] assumes that miners will be afterwards incentivised just by validating the blocks in order to preserve functioning ecosystem. Users basically place their trust in the scarcity and mathematics on which Bitcoin network runs, [Polasik et al., (2015)] claims.

Third reason that provides Bitcoin with value is that consensus exists between all users, miners and whole ecosystem. Decentralized nature secures that real consensus has to be in place, for system functioning properly. Every user agrees on sending/receiving payments in Bitcoin. Every user agrees on rules which determine validity of each block and as consequence transaction. This ensures that all users accept chain history as outlined in [Kroll et al., (2013)].

### 3.1.4 Determinants of Bitcoin's price

As we showed there is clearly unique combination of value determinants for Bitcoin, which makes it really appealing for researchers to study. Recently many academics tried to answer question of most influencing factor for Bitcoin's price formation. [Buchholz et al., (2012)] uncover that supply and demand are crucial factors. Other than that transaction volume shows to be serious driver as well.

However [Kristoufek, (2013)] disagrees with former statements of [Buchholz et al., (2012)] and proceeds to show that price cannot be really explained by supply/demand to a sufficient degree. Since Bitcoin has (semi-)fixed supply there is no underlying economy and so no macroeconomic expectations can be made. Notwithstanding what has just been said, author claims that price is solely driven by speculative investors that expect future price growth. Google and Wikipedia queries are suggested as possible predictors for these price movements.

Following [Buchholz et al., (2012)] and [Kristoufek, (2013)], [Ciaian et al., (2016)] examine different interaction terms in price formation using Vector Autoregressive model (VAR). First, they arrive to similar result as [Kristoufek, (2013)] with resepct to speculative investors forming the price. Second, they discover very good fit to standard fundamentals of fiat currency price formation. Mainly demand-side drivers meaning velocity of "coins" circulation and economy size.

[Kristoufek, (2015)] using continuous wavelet analysis, investigates potential price determinants of Bitcoin. First, study confirms findings of [Ciaian et al., (2015)] that standard fundamentals as trade usage, price level and money usage, play important role in the process. Second, heavy impact of speculative investors is outlined aligned with previous work of [Kristoufek, (2013)] and later study of [Ciaian et al., (2015)].

[Polasik et al., (2015)] expanding on correlation between Wikipedia queries and price shown by [Kristoufek, (2013)], similar relationship is shown with number of publications written in English media. result of [Garcia et al., (2014)] attribute movements in Bitcoin's price to growing are also aligned with hypothesis of public attention as a driver and conclude that major price downfalls are preceded by rise in information search for Bitcoin. Work of Kristoufek is later confirmed by [Ciaian et al., (2016)], [Ciaian et al., (2015)] and [Bouoiyour et al., (2017)] on both ends, since they find speculative forces and classic fundamentals to have strong impact on Bitcoin's price.

### 3.1.5 Bitcoin's Market

Although, there are hundreds of millions of dollars' worth of transactions crossing the network daily, Bitcoin is still functioning on its own without any central entity. This fact has convinced several researchers that there is no fundamental reason for its value, including [Hanley, (2013)] and [Yermack, (2013)]. Recent comments made by CEO of JPMorgan Chase, Jamie Dimon have been aligned with work of [Cheah et al., (2015)] claiming that there is no fundamental value of Bitcoin and entire value is a fraud. Challenging this claims, [Hayes (2016)] argues that Bitcoin has quantifiable intrinsic value and presents pricing model based on marginal costs of production (ie mining) which as is known requires high electric power spends for computation. Hence the value of Bitcoin is expressed by costs of production.

## 3.2 Ethereum

Currently, Bitcoin faces growing competition from Ethereum mainly caused by extensive community disagreement on how should network expand further. Ethereum is an open-source and public blockchain that anyone can employ as a decentralized ledger. It was developed with the main purpose of creating a more generalized blockchain platform, enabling to straightforwardly build applications that benefit from the decentralization and security properties of blockchains, and to evade the necessity to generate a novel blockchain for each new application. The Ethereum blockchain has its own cryptocurrency called Ether, which is similar to Bitcoin, but what attracts the attention of several companies is the underlying Ethereum network. Even though the Bitcoin blockchain has tended to be utilized for payment transactions, the adoption of Ethereum blockchain technology by the corporate world implies it could be much larger than its early stage rival. Ethereum technology is expected to highly enhance smart contract applications that can make automatic intricate physical and financial supply chain procedures.

The value of Ether has increased by about 4,500 percent since the start of the 2017. The industry publication CoinDesk claimed in June 2017 and based on a survey of 1,100 virtual currency users, that 94 percent were optimistic about the situation of Ethereum and its related cryptocurrency (Ether), while only 49 percent were positive about the state of Bitcoin.

### 3.2.1 Smart Contracts

The concept of smart contracts was introduced in the 1990s by Wei Dai. In a post on anonymous credit, Wei described an anonymous loan scheme with redeemable bonds and lump-sum taxes to be collected at maturity. Unlike traditional contracts that rely on the reputation of the counterparties, smart contracts can be made between untrusted, anonymous people. Also, the execution of contractual terms is automatic and does not rely on any intermediary. [Szabo, (1997)] later discussed the potential form of smart contracts and proposed to use cryptographic mechanisms to enhance the security. Smart contracts have become a reality with the boom of blockchain technology. Nxt, a new generation of public blockchain, provides users with several application templates, including Decentralized Asset Exchange, Marketplace, and Voting. In addition, there are also platforms that enable more complex functionalities and exibilities, e.g., Ethereum according to [Wood, (2014)], which has adopted turing-complete languages for smart contracts. Nowadays, smart contracts are also being built on consortium blockchains, e.g., the Hyperledger Fabrics.

*Why Smart Contracts?*

Firstly, blockchains and other distributed ledgers can maintain an immutable record of data and efectively mitigate single points of failure.

Secondly, since smart contracts inherit the encryption pseudonymity of blockchains, even with the code and data accessible to everybody, smart contracts are able to prevent unwanted monitoring and tracking.

Thirdly, thanks to the flexibility of programming languages, smart contracts also have a good interoperability among multiple instances and are able to tackle modifications better presents [Idelberger et al., (2016)].

Therefore, smart contracts have the potential to reduce risks, improve efficiency and save costs, when it comes to transaction settlement among different parties. Unlike traditional paper contracts that rely on middlemen and third-party intermediaries for execution, smart contracts help to automate contractual procedures, minimize interactions between parties, and reduce the cost of administration and service. Smart contracts on public blockchains appear to be attractive for start-ups and people in academia, while smart contracts on permissioned blockchains have the potential to speed up transaction settlement, reduce risk and costs in bussiness consortiums.

On public blockchains, fees are often charged to deploy and update the states stored in smart contracts as the computation is performed by all network nodes and these fees can be high. For example, based on a study by [Rimba et al., (2017)], the cost of maintaining smart contracts on the public Ethereum blockchain is nearly 360 times higher than using bussiness models deployed on Amazon SWF.

### 3.2.2 Use cases

*Health Care & Medical Records.* Major blockchain application is related to health care, access an version control of medical records. Blockchain technology and smart contracts are seen by many health care practitioners as a secure way of sharing and accessing patients' Electronic Health Records. Smart contracts can feature multi-signature approvals between patients and providers to only allow authorized users to access or edit the record. They also enable interoperability via collaborative version control to maintain the consistency of the record. According to [Hu et al., (2018)] smart contracts can also be used to grant researchers access to certain personal health data and enable micropayments to be automatically transferred to the patients for approval.

*Identity Management.* uPort 15 is an identity management framework that leverages Ethereum smart contracts to recover accounts and protect user privacy in the case of a device loss. The main component - uPort identiffieer is a unique 20-byte hexadecimal string representing the address of a Proxy contract that lies in-between a controller contract and an application contract. uPort enables users to replace their private key (saved off-chain) while maintaining an on-chain persistent identifier. If a valid user brings a new device, he can seek for approval from a list of existing recovery delegates, and replace the old user address with a new one. Identity management frameworks using blockchain still need to go through a number of enhancements before adoption. In the case of uPort, the publicity of the recovery delegates of a user poses the security risk of compromising user identities.

*Scaling Blockchains.* Scaling blockchains continue to be one of biggest bottlenecks which prevents them from mainstream use. Therefore, Bitcoin developers are developing the Lightning Network (LN) presented by [Poon et al., (2015)]. LN is smart contracts based off-chain payment protocol which is smart contracts based. The objective is to improve the scalability by reducing on-chain storage and verification requirements.The corresponding implementation in Ethereum is the Raiden Network. Such technologies could help with faster processing of transactions and thus allowing broader mainstream use. *Banking.* Banking remains a main focus area for smart contracts use cases. According to a report from [Capgemini Consulting, (2016)], consumers could save $480-$960 per loan and banks would be able to cut US$3 billion-$11 billion of annual costs in the US and Europe, given smart contracts would be used. Another possible scenario would be usage in streamline clearing and settlement processes. Know your customer (KYC) and anti money laundering (AML) policies can be also very easily implemented within smart contracts logic. Nonetheless, interoperability with legacy systems and scalability issues remain to be main handicap in actual production use of smart contracts. What remains essential is nearly unbreakable level of security against attacks aimed at stealing assets and damage of contract code, [Atzei et al., (2017)] argues.

*Provenance & Supply Chain.* Accroding to [Sadouskaya et al., (2017)] blockchain may help solve supply chain and logistics related problems as optimization,transparency, visibility and security of various operations during transport. Serialization and provenance of the product could be considerably improved by reducing risk of spoilage and wastage. Example of such solution is IBM's blockchain-based food supply-chain.

*Voting.* Another application that can benefit from smart contracts is voting. [McCorry et al., (2017)] suggested a boardroom voting scheme that is different from existing e-voting proposals. Mccorry's system works under the assumption of a small group of voters with known identities responsible for nodes and provides maximum voter privacy and verifiability. Mccorry et al. have also tested the system's feasibility on a Ethereum private network and estimated the cost of $0.73 per voter for running it. The statistics have shown that public blockchains are more feasible for small polls whereas permissioned ones will be required to run national elections. Some efforts were already made in Denmark where political party has implemented a smart contract to ensure the fairness and transparency for internal election purposes.

*Insurance.* In the insurance industry, smart contracts can perform error checking, routing, approve workflows, and calculate payouts based on the type of claim and the underlying policy. For example, there can be automatic verification of fight delays in case of travel insurance settlements. Administrative costs can be rapidly decreased by removing intermediaries in such process, [Capgemini Consulting, (2016)] claims.

However, legal regulations is major challenge to be addressed before shifting to smart contracts for insurance policies. Another drawback is the inflexibility of smart contracts. Traditional contracts can be amended or terminated upon agreement between both parties, but smart contracts have no such property.

# 4 Portfolio Theory and Asset Diversification

*"Diversification cannot eliminate all variance" ([Markowitz (1952)] p.79)*

The risk-reward ratio is an important ratio for each investor. Important is that there are two main risk factors which investors are concerned with. That is market risk and firm-specific risk.

*Firm-specific risk* also known as non-systematic risk, unique risk or diversifiable risk. Unique risk is determined by micro-economic factors. Each micro-economic factor only influences the specific firm. By means of diversification can the firm-specific risk be eliminated. However, this is not entirely true and will be explained after the market risk.

*Market risk* also known as systematic risk or non-diversifiable risk. Characteristics for market risk is that it is inherent to the investment in the market. By no means can this form of risk be lowered by any amount. Market risk is exposed to macro-economic factors such as conjuncture cycles and interest rates. Each factor influences the market as a whole ([Bodie et al., (2014)]). No matter the number of assets the market risk will always be the same. In other words, diversification cannot eliminate all risk.

A side-note on the risk part is that there is a third form of risk, namely *industrial risk*. This form of risk can be placed between market risk and firm-specific risk. Industrial risk only influences a certain industry. For example, when there is a really dry year with lots of sun, this might be bad for the agriculture sector but at the same time this is great for the energy sector which specifies on solar panels. Other firms will likely not be affected by the unpredictable weather. This implies that just investing in more assets does not mean that all firm-specific risk can be eliminated. Which [Markowitz (1952)] p.89 already mentioned: the 'right kind' of diversification for the 'right reason.'

The principle of the right kind of diversification for the right reason is based on the following: Assume there is a portfolio with two risky assets, Asset $X$ and Asset $Y$. With expected return $r_1$ and $r_2$ and variance of $\sigma_1$ and $\sigma_2$. The expected return and variance is as follows:

$$\mathbb{E}(r_p) = w_1\mathbb{E}(r_1) + w_2\mathbb{E}(r_2) \tag{1}$$

$$var(r_p) = w_1^2 var(r_1) + w_2^2 var(r_2) + 2w_1 w_2 cov(r_1, r_2) \tag{2}$$

Where $w$ denotes the weight imposed on the asset. The variance can be noted as $\sigma^2$. The covariance is the correlation between the asset 1 and 2, denoted as $\rho$, and their variance.

$$\rho = \frac{cov(r_1, r_2)}{\sqrt{var(r_1) * var(r_2)}} \tag{3}$$

Rewriting formula (3) and substituting to (2) gives us:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 \tag{4}$$

When there is a perfect positive correlation $\rho_{12}$ is equal to 1. There is a linear trade-off between the two risk assets in return and variance because the variance is as follows:

$$\sigma_p^2 = [w_1\sigma_1 + (1 - w_1)\sigma_2]^2 \tag{5}$$

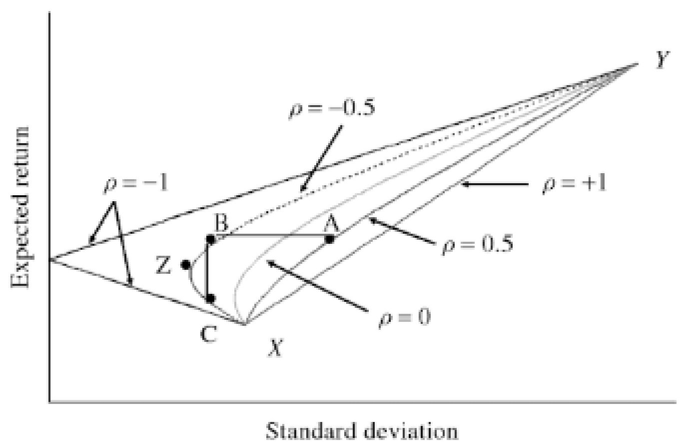


Fig. 1: Efficient frontier and correlations ([Cuthbertson et al., (2004)])

Figure 1 shows how the portfolio of the two-risky asset varies when the correlation between the two assets varies. The lower the correlation between the two assets the higher the return risk ratio. An important aspect is the mean-variance dominance criteria, which implies that point B is preferred to point C because the standard deviation is the same only the return of B is higher. Furthermore, B is preferred to A since both have the same return but the standard deviation of B is lower than A. In short, the upperpart (the concave line from Z upwards) is the line with the efficient portfolios. This part is also known as the efficient frontier.

The two-asset portfolio framework can be extended to a N assets portfolio. Where the weights are $w_i; (i = 1, 2, ..., N)$ Assume that $n$ expected returns $\mu_i$, variance $\sigma_i^2$ and $\frac{n(n-1)}{2}$ covariances $\sigma_{ij}$ (or as formula 3 showed the correlation coefficients $\rho_{ij}$) are known. The formula for the return and the variance of the portfolio are as follows:

$$\mu_p = \sum_{i=1}^{n} w_i\mu_i \tag{6}$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} w_iw_j\rho_{ij}\sigma_i\sigma_j \tag{7}$$

For an investor only concerned about risk/return when determining all $w_i$, the investor's budget constraint is $\sum_{i=1}^{n} w_i = 1$ In this model are short sales permitted ($w_i < 0$). The following figure can be drawn:

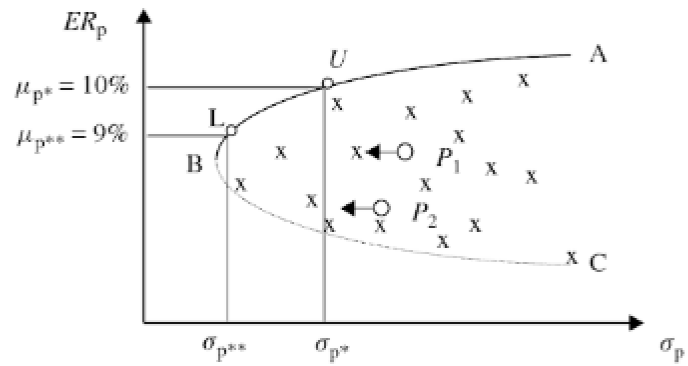


Fig. 2: Efficient frontier N assets ([Cuthbertson et al., (2004)])

Figure 2 shows the range of portfolios and their efficiency. Every portfolio on the line B-A, efficient frontier, should be preferred to portfolios on line B-C because of the previously mentioned mean-variance dominance criteria.

Increasing the number of stocks in a portfolio can decrease the risk of a portfolio. An important remark is that this does not necessarily lead to an optimal portfolio. For an optimal portfolio, efficient diversification is very important. Efficient diversification can be achieved through investments in low correlated assets or even negative correlated assets ([Bodie et al., (2014)]). The lower the correlation, $\rho_{ij}$ in formula (7), the more efficient the diversification effect is. The most important contribution of [Malkiel, (1999)] is that the number of shares is not that important but that the correlation between asset is very important in achieving a well-diversified portfolio. Coming back to the side note about industrial risk, because all companies in the same industries are partly affected by the same factors their correlation is relatively high. Increasing the number of shares of the same portfolio does not have to change the portfolio's risk.

To measure the risk-return ratio the Sharpe ratio will be used, introduced by [Sharpe, (2007)]. The Sharpe ratio is the excess return i.e. the expected return ($\mu_p$) of the portfolio from which is subtracted the risk-free rate ($r_F$), divided by the standard deviation ($\sigma_p$). In other words, the Sharpe ratio is the excess returns, i.e. desirable for an investor, divided by volatility i.e. undesirable for an investor under the assumption of Markowitz that the investor is risk averse. The higher this ratio the better the risk-return of the portfolio.

$$Sharpe\ ratio = \frac{\mu_p - r_F}{\sigma_p}$$

However, the Sharpe ratio is suitable to assess the performance of a allocation strategy only in the case of normally distributed returns. Thus, we use opportunity cost measure proposed by [Simaan, (1993)] to assess the economic significance of the difference in performance of the two (base and augmented by cryptoasset) optimal portfolios. Let $r_{wc}$ ,$r_{nc}$ denote the optimal portfolio realized returns obtained by an investor with the expanded investment opportunity set that includes cryptoassets and the investment opportunity set restricted to the base asset classes, respectively. The opportunity cost $\theta$ is defined as the return that needs to be added to the portfolio return $r_{nc}$ so that the investor becomes indifferent (in utility terms) between the two mentioned strategies.

$$E[\ U(1 + r_{nc} + \theta)\ ] = E[\ U(1 + r_{wc})\ ]$$

Therefore, a negative opportunity cost implies that the investor is worse off in case of an investment opportunity set that allows cryptoasset investing. Opportunity cost takes into account all the characteristics of the utility function and hence it is suitable to evaluate strategies even when the return distributions are not normal.

# 5 Methodology

In recent years, several studies have used the [Markowitz (1952)] mean-variance (MV) static asset allocation framework to study whether addition of new asset to pre-defined asset bundle improves investment opportunities ([DeRoon et al., (2001)]). There are several scenarios which may occur.

Firstly, if the MV frontier of benchmark portfolio and the frontier of the benchmark portfolio plus newly added assets have exactly one common point, this is known as *intersection.* In this case, there is just one MV utility function for which there are no benefits from adding new assets.

Secondly, if the MV frontier of benchmark portfolio coincides with the frontier of benchmark portfolio with new assets included, there is *spanning.* This means that no MV investor can benefit from adding the new assets to the benchmark portfolio, which was shown by [DeSantis, (1995)].

However, [Daskalaki et al., (2011)] have shown several shortcomings of this approach. First, Markowitz framework is based on two assumptions which might not reflect precisely the gains form investing in a new assets. Violations may occur when distribution of asset returns is non-normal and investor's utility function is non-parametric (not quadratic). There is serious evidence, especially in short horizons, that asset return are not distributed normally ([Peiro, (1999)]; [Gorton et al., (2006)]; [Kat et al., (2007)]). Investors usually show signs of risk aversion, which causes preference of positively skewed distributions with low level of kurtosis. In such cases, [Jondeau et al., (2006)] showed serious utility loss and therefore we should take into account higher moments of the distribution. Additionally, quadratic utility function shows negative marginal utility after a certain finite wealth level and increasing absolute risk aversion with respect to wealth which is again inconsistent with rational behavior.

Second, based on [DeRoon et al., (2001)], many researchers assessed diversification benefits of adding new assets to the portfolio by visual inspection of the relative position of efficient frontiers. According to [Chowdhury, (2016)], comparison of the relative position of efficient frontiers should be examined using more rigorous statistical tests.

Third, most recent studies have been focused on in-sample investigation of newly added assets. Given that realized portfolio returns are uncertain over investment time horizon, [Daskalaki et al., (2011)] argue that portfolio selection should be examined in out-of-sample setting.

With that being said, we follow [Daskalaki et al., (2011)] robust approach investigating whether Bitcoin & Ethereum (B&E) should be included in portfolio which consists of equities, bonds, currencies in a standard *static asset allocation* context and make the following changes.

First, we perform analysis in-sample context so results can be compared to [Briere, et al. (2015)] and [Chowdhury, (2016)]. Instead of visual analysis of efficient frontiers conducted in [Briere, et al. (2015)], we follow [Chowdhury, (2016)] more statistically rigorous approach. Typically standard regression-based spanning techniques are used in case of utility function consistent with MV framework, as well as more robust ones in non-MV scenario ([Huberman et al., (1987)]; [DeRoon et al., (2001)]); for MV spanning; [DeRoon et al., (1996)], for non-MV spanning tests).

Second, following [deMiguel et al., (2009)] and [Kostakis eet al., (2010)], we conduct out-of-sample analysis. As a result, several performance measures are calculated for static one-period optimal portfolios.

In order to counter utility loss shown by [Jondeau et al., (2006)], we consider higher moments of distribution of returns. For this task, non-parametric maximization of direct utility is conducted following [Cremers et al., (2005)], [Adler et al., (2007)] and [Sharpe, (2007)].

## 5.1 Spanning tests & Out-of-sample analysis

Spanning was first introduced by ([Huberman et al., (1987)]) and initially restricted to a mean-variance framework. In short, mean-variance spanning studies the effect of additional risky assets (test assets) on the mean-variance frontier of a set of base assets ([DeRoon et al., (2001)]). Spanning occurs when the frontier derived from the augmented investment set (base plus test assets) coincides with the frontier of the base assets. This implies that the mean-variance investors cannot improve their risk/return trade-off by adding the test assets, regardless of their risk aversion level. We analyze economic benefits from investing in Bitcoin and Ethereum by means of spanning tests in both mean-variance and non-mean-variance framework. For this we follow work of [DeRoon et al., (1996)].

### 5.1.1 Mean-variance in-sample spanning tests

Proposition 1 from [DeRoon et al., (1996)] shows that returns of test asset are spanned by returns of base asset if and only if returns can be written as the return of a portfolio of the base assets, and a zero-mean error term. Additionally efficiency proof of such method by [Hansen et al., (1991)] allows us to formulate mean-variance spanning test as linear regression in following manner:

$$R_{t+1}^{test} = \alpha + \beta R_{t+1} + \epsilon_{t+1} \tag{8}$$

where $R_{t+1}^{test}$ are gross returns of test assets (in our case one cryptoasset at the time); $R_{t+1}$ gross returns of $K$ base assets (stocks, bonds, currencies and risk-free asset) and $\epsilon_{t+1}$ zero mean error term. Consequently null hypothesis for spanning is ([Huberman et al., (1987)]; [Bekaert et al., (1996)])

$$H_0 : \alpha = 0 \; and \; \beta_{l_k} = 1 \tag{9}$$

Due to inclusion of risk free asset to our K-base asset universe we need to reformulate spanning tests in terms of excess returns. To fix this, we subtract risk-free rate $R_t^f$ from return terms on both side of equation. From financial theory it would be legitimate use $R_t^f$ as another regressor. On the other hand, given its persistence reformulation is preferred in econometric framework and so we arrive to:

$$R_{t+1}^{test} - R_t^f = \alpha_j + \beta(R_{t+1} - R_t^f) + \epsilon_{t+1} \tag{10}$$

where $E(\epsilon_{t+1}) = E(\epsilon_{t+1} R_{t+1}) = 0$. [Daskalaki et al., (2011)] prove equivalence of intercept terms in equations (8) and (10) i.e.:

$$\alpha_j = \alpha - R_t^f (1 - \beta_{l_k}) \tag{11}$$

Hence, in excess return formulation, null hypothesis of spanning is equal to testing only significance of intercept term:

$$H_0 : \alpha_j = 0 \tag{12}$$

### 5.1.2 Non mean-variance in-sample spanning tests

In case where investor's preferences cannot be described by quadratic utility we need to use more general approach. For that we define non mean-variance utility function $U(.)$ and consider it to be equal either an exponential or power (isoelastic) utility function, with different levels of risk aversion.

The negative exponential utility function is defined as:

$$U(W) = -\frac{exp(-\eta W)}{\eta},\ \eta > 0 \tag{13}$$

where $\eta$ is the coefficient of risk aversion (ARA). The exponential utility function occurs when one models an investor with constant absolute risk aversion.

The power (isoelastic) utility function is defined as:

$$U(W) = \frac{W^{1-\gamma} - 1}{1 - \gamma},\ \gamma \neq 1 \tag{14}$$

where $\gamma$ is the coefficient relative risk aversion (RRA).The isoelastic function assumes that risk-averse decision-makers maximize the expected value of a concave von Neumann-Morgenstern utility function.

As a result we apply a wide range of risk aversion coefficients for each non mean-variance $U_i(.)$ with $i = 2, 4, 6, 8, 10$ corresponding to $i^{th}$ risk aversion value. As outlined in [DeRoon et al., (1996)],[DeRoon et al., (2001)] test for spanning have to be carried out by examining whether the relative restrictions hold for any value of risk aversion. Hence we estimate equation again in excess terms according to Appendix B of [Daskalaki et al., (2011)] in following manner:

$$R_{t+1}^{test} - R_t^f i_K = \alpha_j + \beta(R_{t+1} - R_t^f) + \sum_{i=1}^{n} \gamma_i U_i'(w_i'^* R_{t+1}) + \epsilon_{t+1} \tag{15}$$

Thus, the restrictions that need to hold for the joint existence of mean-variance and non-mean-variance spanning, are

$$H_0 : \alpha_j = 0\ and\ \gamma_i = 0\ \forall i \tag{16}$$

Finally, we test restrictions (12) and (16) by Wald test [DeRoon et al., (2001)] and correct standard errors of the estimators by [Newey et al., (1994)] HAC in (12) and MacKinnon and White's HC3 alternative heteroskedasticity robust standard errors in (16). [Long et al., (2000)] recommended usage of HC3 because it can keep the test size at the nominal level regardless of the presence or absence of heteroskedasticity (and there is only a slight loss of power associated with HC3 when the errors are indeed homoskedastic). Importantly, in order to estimate regressions we need to estimate the unobserved regressors (i.e. marginal utilities).

#### 5.1.3 Out-of-sample benefits of cryptoassets

In order to study potential diversification benefits of cryptoassets we also perform out-of-sample analysis. For this task we employ 'rolling-sample' approach which should mimic more realistic scenario that investors usually face. Let dataset consist of $T$ daily observations for each asset and $K$ be the size of rolling window in weeks, that we use to calculate portfolio weights. Weights estimated by direct utility optimization are then used for calculation of realized returns in period $[t, t+1]$. In order to ensure good generalization we use different window sizes $K = 14, 28, 42, 56$. Eventually comparison of Sharpe ratio and Opportunity cost is done for both base and augmented portfolios for all $K's$.

### 5.2 Portfolio formation

We use two approaches to portfolio formation: mean-variance optimization and direct utility optimization. Classic investors typically prefer mean-varaince approach mainly because it requires only knowledge of expected returns, standard deviations with correlations of portfolio assets.

#### 5.2.1 Mean-variance optimization

For this approach, our objective is to minimize variance $w^T \Sigma w$ with arbitrarily chosen minimal returns $r_{min}$. Due to problem convexity we're able to formulate quadratic program, to find its minimum. Additionally we condition the problem on $\sum_{i=1}^{n} w_i = 1$, $\forall i : 0.25 \geq w_i \geq 0$ and $r_{avg}^T w \geq r_{min}$. Limiting $w_i$ to have a maximum of 0.25 was chosen based on [Conover et al., (2009)]. Mentioned conditions are applied to direct utility optimization too.

#### 5.2.2 Direct utility optimization

Mean-variance approach is adequate only if either investors are well described by quadratic utility or returns are distributed normally. First assumption suggests that investors are equally averse to deviations away from mean. Quadratic utility function also exhibits increasing absolute risk aversion and achieves a satiation point, beyond which return begins to have negative value. Second assumption was proved to be usually wrong for Bitcoin ([Molnar et al., (2015)]; [Baur et al., (2017)]) and eventually non-normal distribution of Bitcoin was implied by [Eisl et al., (2015)]. That being said, we perform utility maximization using second order approximation.

Let the mean value of future wealth, $\overline{W}_{t+1}$ be defined as

$$\overline{W}_{t+1} = E_t(W_{t+1}) = 1 + \sum_{i=1}^{N} w_i \mu_{i,t+1} = 1 + \mu_{p,t+1} \tag{17}$$

where $\mu_{i,t+1}$ stands for mean rate of return on the individual asset $i$ and $1 + \mu_{p,t+1}$ the mean return of allocated portfolio. [Garlappi et al., (2009)] showed that Taylor expansion around $\overline{W}_{t+1}$ can be, under certain conditions, expressed in terms of $k$ partial derivatives of the utility function and all the central moments of the distribution, i.e.

$$E[U(W_{t+1})] = E\left[\sum_{k=0}^{\infty} U^k(\overline{W}_{t+1})\frac{[W_{t+1} - \overline{W}_{t+1}]^k}{k!}\right] = \sum_{k=0}^{\infty} \frac{U^k(\overline{W}_{t+1})}{k!} E\left[(W_{t+1} - \overline{W}_{t+1})^k\right] \tag{18}$$

According to proposition of [Markowitz (1952)] we stick to $k = 2$. Consequently, under assumption of power utility described in equation (14), we are able to formulate objective function as

$$E[U(W_{t+1})] \approx \frac{\overline{W}_{t+1}^{1-\gamma} - 1}{1-\gamma} - \frac{\gamma \overline{W}_{t+1}^{-1-\gamma} \sigma_{p,t+1}^2}{2} \tag{19}$$

and under assumption of negative exponential utility from equation (13) as

$$E[U(W_{t+1})] \approx -\frac{exp(-\eta \overline{W}_{t+1})}{\eta} \left(1 + \frac{\eta^2 \sigma_{p,t+1}^2}{2}\right) \tag{20}$$

From implementation point of view we then minimize equations (19) and (20) multiplied by factor $(-1)$ and estimate means and covariance matrix of the asset returns by their sample counterparts. In contrast with [Daskalaki et al., (2011)] we replace computationally expensive grid search by Sequential Least-Squares Quadratic Programming (SLSQP) algorithm for nonlinearly constrained, gradient-based optimization which was initially proposed by [Kraft, (1988)]. The algorithm treats problem as a sequence of constrained least-squares problems with first-order approximations of the constraints.

# 6 Dataset & Results

The dataset consists of daily closing prices of number of stock, bond, currency indices, Bitcoin and Ethereum. B&E absolute prices originate in https://www.cryptocompare.com but are retrieved from https://finance.yahoo.com.

The stock indices are chosen to represent the most important stock markets. First, we employ the *S&P 500* return index tracked by GSPC to capture American market based on the capitalizations of 500 largest companies listed on the NYSE or NASDAQ. Secondly, to track prosperity of businesses regulated by UK company law, we incorporate *FTSE 100 Index.* This index covers 100 companies listed on the London Stock Exchange with the highest market capitalization. At the same time *GDAX*, a blue chip stock market index consisting of the 30 major German companies traded on the Frankfurt Stock Exchange is included. Risking high levels of correlation between last two mentioned we also bet on over time more and more diverging UK and EU economies. In order to gain insight into the Asian economy we add *Nikkei 225* and *SSE Composite Index.* Former is the most widely quoted average of Japanese equities, similar to the Dow Jones Industrial Average. Latter, also called Shanghai A-shares which are shares of mainland China-based companies, and were historically only available for mainland citizens because China restricts foreign investment. Since 2003, it is not longer the case. Finally, the MSCI world index is added to ensure global exposure.

The bonds are selected based on the three most important markets too: the American market, the European market and the Asian market. First one being options on the 10-year rate (*TNX*) which are based on the yield-to-maturity of the most recently auctioned 10-year Treasury note issued by United States government. The notes are usually auctioned every three months. Second being iShares EUR Govt Bond 15-30yr ETF (*IEGL.L*), an exchange-traded fund established in Ireland. The Fund aims to track the performance of the Bloomberg Barclays Euro Government Bond 15-30 Year Term Index. Last used is ABF Pan Asia Bond Index Fund (*2821.HK*), a unit trust incorporated in Singapore. The Fund invests in debt obligations issued or guaranteed by the governments of China, Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore and Thailand.

The fiat currencies are covered in similar fashion as bonds with proper exposure to 3 different continents. Since American Dollar is nearly every time used as point of reference it is not so easy express its value meaningfully. For this task we decide to use Invesco DB US Dollar Bullish (*UUP*). The investment seeks to establish long positions in ICE U.S. Dollar Index futures contracts with a objective to track the changes, positive and negative, in the level of the Deutsche Bank Long USD Currency Portfolio Index. As far as Japanese Yen (our currency of choice to represent Asian continent) and Euro go, we include Invesco CurrencyShares Japanese Yen ($FXY$) and its companion Euro Currency ($FXE$) respectively. Former represents a cost-effective way relative to traditional means of investing in Japanese Yen. Later, in similar fashion, reflects the price of Euro in USD terms.

As far as commodities go we decide to limit ourselves to just single instrument and that being Energy Select Sector SPDR ETF ($XLE$). The instrument seeks to provide investment results that, before expenses, correspond generally to the price and yield performance of publicly traded equity securities of companies in the Energy Select Sector Index mainly focussing on oil, gas and consumable fuels.

Finally, augmented portfolio will alternately include Bitcoin or Ethereum. Both cryptoassets use USD are reference currency. As is the case for all previous assets, historical prices are obtained from https://finance.yahoo.com. Main reason being requirement of easier reproducibility of our work.

The sample period runs from $1^{st}$ of July 2010 to $1^{st}$ of August 2018 (excluded). Only exception being Ethereum due to its creation dated to $30^{th}$ of July 2015. Whenever Ethereum is used to augment portfolio, all other assets' sample sizes are adjusted to mimic later start date. To perform spanning tests in excess returns terms, risk-free asset has to be subtracted. Following [Daskalaki et al., (2011)] we employ Libor 1-month rate as a proxy and by simple transformation obtain daily rates. Importantly, semi-definitivity of covariance matrix is checked after its estimation. Due to missing data (supposedly at random), returned covariance matrix is an unbiased estimate of the variance and covariance between the assets. This could in worst case lead to estimate correlations having absolute values which are greater than one, and/or a non-invertible covariance matrix.

In Table 1 we report descriptive statistics of all assets over the sample period. We can see right away exceptionally higher average daily returns on Bitcoin and Ethereum, 0.68% and 0.78% respectively. In the mildest case, this rate is as high as 10 times average returns of second best performing asset return-wise MSCI. Due to the inclusion of nearly exponentially bullish period at times, Bitcoin's number over performs previous research of [Briere, et al. (2015)] and [Chowdhury, (2016)] (after conversion to daily returns). Sharp increase in price came with high degree of fluctuations in both cryptoassets. Standard daily deviation reaches 9.1% for Bitcoin and 7.87% for Ethereum, which is again 5-6 times than other assets at minimum. As [Frehen et al., (2013)] outline, assets linked to financial innovations are very likely to exhibit bubble-like behaviour. That said, investors should stay careful when predicting future expected returns of Bitcoin and Ethereum. Extremely high levels of kurtosis (650.38) can reflect presence of severe risks. This, also called leptokurtic, behaviour is more commonly denoted as "fat tails" which indicates higher probability of extreme outcomes ([Verhoven et al., (2004)]). [Briere, et al. (2015)]

and [Kajtazi et al., (2018)] also found leptokurtic behaviour of the Bitcoin. Kurtosis of Ethereum (7.4) landed in middle range of all used assets. Surprisingly second highest value of kurtosis is recorded on IEGL. Although it is not the case here, these levels of kurtosis are sometimes reached by bonds of emerging governments. Notable levels of skewness are again reached by Bitcoin (650.38) and usually require advanced investing strategies to get to (Ethereum has skewness of 0.4). As a result of these extreme price movements, the Sharpe ratio is considerably greater for cryptoassets than all the others. The reported evidence is unsurprisingly in line with previous research, claiming that Bitcoin (and newly Ethereum) out perform other asset classes in excess returns terms. However it needs to be noted, that none of the used assets was able to pass Jarque-Bera test for normality of the returns even on 1% level. This fact slightly downgrades Sharpe ratio as a valuable comparison metric.

Table 1: Summary statistics

| Asset | Average Return | Standard Deviation | Sharpe Ratio | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Shanghai A-shares | 0.013% | 1.119% | -0.003 | -1.02 | 10.48 |
| ABF Pan Asia Bond Index | 0.004% | 0.355% | -0.033 | -0.11 | 3.8 |
| BTC-USD | 0.684% | 9.108% | 0.073 | 18.41 | 650.38 |
| ETH-USD | 0.776% | 7.873% | 0.095 | 0.4 | 7.4 |
| Invesco CurrencyShares EUR | -0.002% | 0.478% | -0.037 | -0.02 | 3.82 |
| Invesco CurrencyShares JPY | -0.008% | 0.496% | -0.048 | 0.09 | 6.97 |
| iShares Euro Gov. Bond 15-30yr | 0.01% | 0.633% | -0.01 | 0.99 | 155.42 |
| MSCI All Country World Index | 0.073% | 1.348% | 0.042 | -2.44 | 58.62 |
| Invesco DB US Dollar Bullish | 0.001% | 0.396% | -0.039 | 0.06 | 3.76 |
| Energy Select Sector SPDR ETF | 0.028% | 1.121% | 0.01 | -0.24 | 5.31 |
| FTSE 100 Index | 0.018% | 0.78% | 0.003 | -0.13 | 4.7 |
| DAX Performance-Index | 0.032% | 1.02% | 0.016 | -0.25 | 5.12 |
| S&P 500 Index | 0.037% | 0.753% | 0.028 | -0.47 | 8.24 |
| Nikkei 225 | 0.037% | 1.099% | 0.019 | -0.49 | 9.46 |
| CBOE 10 Year Trsry Note Index | 0.017% | 1.837% | 0.001 | 0.31 | 3.6 |

*Note: Entries report the descriptive statistics for the alternative asset classes used in this work. The dataset spans the period from from* $1^{st}$ *of July 2010 to* $1^{st}$ *of August 2018, with exception of Ethereum which ws created on* $30^{th}$ *of July 2015. Table reports daily mean returns, standard deviations and Sharpe Ratios as well as skewness and kurtosis. Results of Jarque-Bera test were excluded due to clear rejection on almost zero levels.*

Similarly, pairwise correlations are shown in Table 2 and Figure 3. Consistently with [Briere, et al. (2015)] and [Chowdhury, (2016)] we see very low levels of correlation between cryptoassets and the others. Highest among the all being Ethereum's relationship with FTSE (-0.07). This is barely surprising as cryptoassets are often presented as an inflation hedge ([Harper, (2013)]), which is obviously alluring to investors ([Bodie et al., (2014)]). Notably we see high level of correlation between UUP and FXE which may be caused by a nature Ivesco Dollar Bullish Index, whose reference bundle is heavily weighted by Euro currency.

Table 2: Correlation matrix

| | -.SS | -.HK | BTC | ETH | FXE | FXY | IEGL | MSCI | UUP | XLE | FTSE | GDAX | GSPC | N225 | TNX |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -.SS | 1.00 | 0.07 | 0.01 | -0.03 | -0.01 | -0.08 | 0.01 | 0.09 | 0.01 | 0.11 | 0.18 | 0.14 | 0.12 | 0.24 | 0.04 |
| -.HK | 0.07 | 1.00 | -0.00 | -0.03 | -0.00 | -0.03 | 0.01 | 0.03 | -0.00 | 0.04 | 0.10 | 0.07 | 0.04 | 0.08 | -0.01 |
| BTC | 0.01 | -0.00 | 1.00 | 0.34 | 0.02 | 0.00 | 0.00 | 0.00 | -0.02 | 0.01 | -0.00 | 0.00 | 0.02 | -0.01 | 0.00 |
| ETH | -0.03 | -0.03 | 0.34 | 1.00 | 0.05 | 0.03 | -0.01 | -0.01 | -0.04 | 0.04 | -0.07 | -0.05 | -0.02 | -0.00 | -0.05 |
| FXE | -0.01 | -0.00 | 0.02 | 0.05 | 1.00 | 0.31 | -0.07 | 0.16 | -0.96 | 0.24 | 0.09 | 0.06 | 0.23 | -0.03 | 0.03 |
| FXY | -0.08 | -0.03 | 0.00 | 0.03 | 0.31 | 1.00 | 0.11 | -0.16 | -0.45 | -0.16 | -0.23 | -0.25 | -0.26 | -0.28 | -0.45 |
| IEGL | 0.01 | 0.01 | 0.00 | -0.01 | -0.07 | 0.11 | 1.00 | -0.07 | 0.05 | -0.10 | -0.05 | -0.04 | -0.09 | -0.03 | -0.28 |
| MSCI | 0.09 | 0.03 | 0.00 | -0.01 | 0.16 | -0.16 | -0.07 | 1.00 | -0.15 | 0.47 | 0.40 | 0.40 | 0.64 | 0.08 | 0.29 |
| UUP | 0.01 | -0.00 | -0.02 | -0.04 | -0.96 | -0.45 | 0.05 | -0.15 | 1.00 | -0.26 | -0.08 | -0.07 | -0.22 | 0.05 | 0.02 |
| XLE | 0.11 | 0.04 | 0.01 | 0.04 | 0.24 | -0.16 | -0.10 | 0.47 | -0.26 | 1.00 | 0.52 | 0.48 | 0.79 | 0.11 | 0.39 |
| FTSE | 0.18 | 0.10 | -0.00 | -0.07 | 0.09 | -0.23 | -0.05 | 0.40 | -0.08 | 0.52 | 1.00 | 0.82 | 0.60 | 0.26 | 0.37 |
| GDAX | 0.14 | 0.07 | 0.00 | -0.05 | 0.06 | -0.25 | -0.04 | 0.40 | -0.07 | 0.48 | 0.82 | 1.00 | 0.61 | 0.25 | 0.40 |
| GSPC | 0.12 | 0.04 | 0.02 | -0.02 | 0.23 | -0.26 | -0.09 | 0.64 | -0.22 | 0.79 | 0.60 | 0.61 | 1.00 | 0.14 | 0.44 |
| N225 | 0.24 | 0.08 | -0.01 | -0.00 | -0.03 | -0.28 | -0.03 | 0.08 | 0.05 | 0.11 | 0.26 | 0.25 | 0.14 | 1.00 | 0.10 |
| TNX | 0.04 | -0.01 | 0.00 | -0.05 | 0.03 | -0.45 | -0.28 | 0.29 | 0.02 | 0.39 | 0.37 | 0.40 | 0.44 | 0.10 | 1.00 |

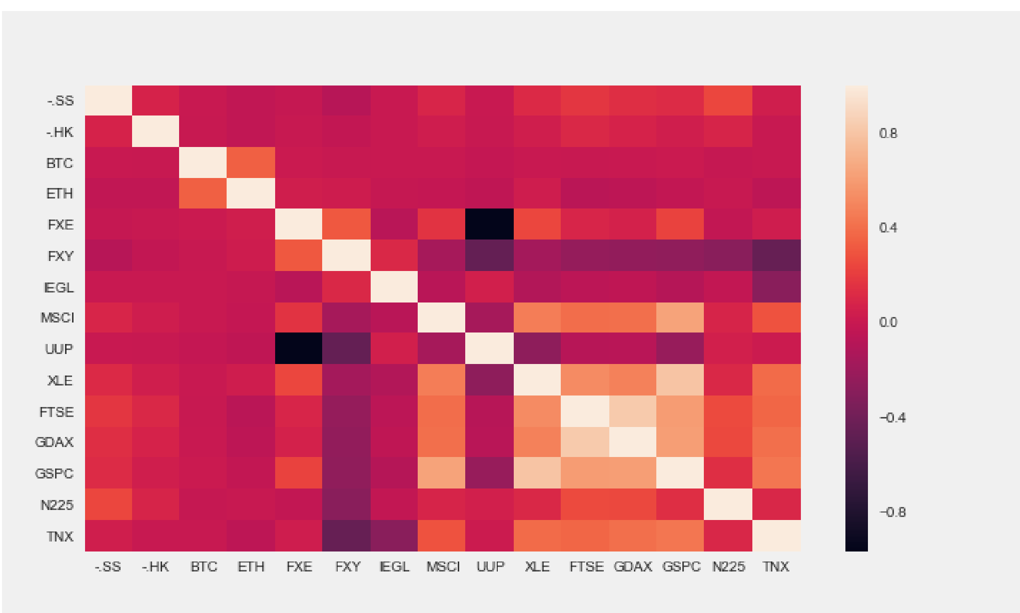


Fig. 3: Correlation plot

In Table 3 we report results of testing of the spanning hypothesis, when Bitcoin and Ethereum are alternately added and included in base asset universe. Moreover, the Wald test statistic and respective p-values for testing null hypothesis of spanning are shown. Newey-West's heteroscedasticity and autocorrelation robust covariance matrix is used with lags determined by the rule of thumb $L = \frac{3}{4}T^{\frac{1}{3}} - 1$ ; ($T$ being length of time series) as applied in [Newey et al., (1994)]. We test individually for MV spanning, joint MV, and non-MV spanning with exponential and power utility respectively. We are able to reject null hypothesis at 5% level for all cases except for the joint MV and the non-MV with exponential utility. In the MV setting and in the case of Ethereum with investor subject to power utility we are able to reject null hypothesis of in-sample spanning at an even 1% level. Therefore, there is a significant evidence that Bitcoin and Ethereum can improve investing opportunity of investor holding a portfolio of stocks, bonds, and currencies in an in-sample mean-variance framework. In the case of Bitcoin, these findings are in line with [Eisl et al., (2015)] and [Briere, et al. (2015)], who assess the diversification benefits from the standpoint of an American investor. However, our results contradict with those of [Chowdhury, (2016)], who did not find diversification capability of Bitcoin for an MV investor within in-sample.

Table 3: Spanning tests

| Test Asset | Mean - Variance (MV) | MV & Exponential | MV & Power |
|---|---|---|---|
| Bitcoin | 13.10 | 15.69 | 10.21 |
| | $(2.95 * 10^{-4})^{*}$ | $(1.54 * 10^{-2})^{**}$ | $(1.68 * 10^{-2})^{**}$ |
| Ethereum | 9.81 | 11.35 | 11.57 |
| | $(1.73 * 10^{-3})^{*}$ | $(7.78 * 10^{-2})$ | $(9.00 * 10^{-3})^{*}$ |

*Entries report the Wald test statistics and respective p-values (* Significant at 1% level; ** Significant at 5% level) for the null hypothesis that a set of benchmark assets consisting of stocks, bonds and the risk-free asset spans a given test asset from the commodities market. The first column reports results for the null hypothesis that there is mean-variance spanning. The next column reports results for the null hypothesis that there is both mean-variance and exponential utility spanning with risk aversion coefficient ranging from 2 to 10. The third column reports results for the null hypothesis that there is both mean-variance and power utility spanning with risk aversion coefficient ranging from 2 to 10.*

Furthermore, our results are robust for joint MV and non-MV setting. The only exception arise in the case of Ethereum with investor being subject to the power utility function. The finding related to Bitcoin is aligned with [Chowdhury, (2016)], which contradicts with the case of MV investor reported above.

In Table 4 we show results of testing whether benefits are preserved also out of sample. After weights' optimization, performed while excluding the last $K$ weeks, we evaluate performance of the portfolio on the last $K$ weeks of our dataset and report Sharpe Ratio and Opportunity costs. Since this particular period exhibits bearish sentiments in most of the markets, our optimized portfolios show negative Sharpe ratio in absolute terms. Nevertheless, all augmented portfolios demonstrate the capability of improving the performance measured by Sharpe ratio in such occasion. Generally, the 5% allocation (found by quadratic program) of Bitcoin appears to yield more than the one of Ethereum (allocation oscillates around 4%). When looking at the Opportunity cost measure, we can conclude that an investor who includes Ethereum in his portfolio constructed within MV framework is worse off in all time windows except for $K = 14$. In the case of Bitcoin, we report improvement over base portfolio reaching as much as 0.02% in average daily returns.

Table 4: Mean-variance optimization: Quadratic utility function

| | Metric | BTC | ETH | Base |
|---|---|---|---|---|
| K=14 | Sharpe ratio | -0.096 | -0.126 | -0.154 |
| | Opportunity cost | 0.02 | 0.001 | |
| K=28 | Sharpe ratio | -0.137 | -0.163 | -0.19 |
| | Opportunity cost | 0.013 | -0.001 | |
| K=42 | Sharpe ratio | -0.057 | -0.072 | -0.083 |
| | Opportunity cost | 0.008 | -0.002 | |
| K=56 | Sharpe ratio | -0.018 | -0.058 | -0.063 |
| | Opportunity cost | 0.015 | -0.003 | |

*Entries report the performance measures (daily Sharpe Ratio, average daily Opportunity Cost) for the case where the quadratic utility is maximized by ways of risk minimization in quadratic program. Results are reported for different sizes of the rolling window (K=14,28,42,56 weeks) and different degrees of relative risk aversion (RRA=2,4,6,8,10)..*

Table 5 shows out of sample performance of portfolios formed by direct exponential utility maximization. Firstly, it stands out that short-term ($K = 14$) Bitcoin starts to overperform base asset space only as Absolute Risk Aversion coefficient reaches value 6 and continues to move as ARA grows. For this $K$, Ethereum fails to perform better for any ARA. For $K = 28$ same positive trend continues for Bitcoin and Ethereum, with an exception that both cryptoassets are already performing better for all values of ARA. The difference tends to increase in $K > 28$, where both assets finally reach positive Sharpe Ratio and Opportunity cost. It is important to note that Bitcoin augmented portfolios consistently over-perform those augmented by Ethereum in Sharpe ratio terms (for greater $K's$ Ethereum is superior in terms of Opportunity costs). With respect to the optimal allocation weights that are chosen they oscillate around $[0.25, 0.20, 0.14, 0.08]$ ($[0.25, 0.25, 0.22, 0.17, 0.13]$) for $2, .., 10$ values of ARA in case of Bitcoin and Ethereum respectively. Intuitively, this suggests that investors who are more risk-averse prefer lower weights of cryptoassets than those who are more risk-loving. Notably, bearish period is visible on negative Sharpe ratios for base portfolios and therefore positive performance of augmented portfolios (in some cases) suggests great benefits for investors asymmetrical to risk.

Table 5: Direct utility maximization: Exponential utility function

| | | ARA=2 | | | ARA=4 | | | ARA=6 | | | ARA=8 | | | ARA=10 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Metric | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base |
| K=14 | Sharpe ratio | -0.030 | -0.041 | -0.010 | -0.029 | -0.041 | -0.014 | -0.032 | -0.041 | -0.022 | -0.034 | -0.043 | -0.027 | -0.040 | -0.048 | -0.033 |
| | Opp. cost | -0.028 | -0.079 | | -0.021 | -0.077 | | -0.012 | -0.063 | | -0.008 | -0.047 | | -0.007 | -0.039 | |
| K=28 | Sharpe ratio | -0.048 | -0.050 | -0.062 | -0.053 | -0.050 | -0.066 | -0.061 | -0.050 | -0.070 | -0.066 | -0.057 | -0.071 | -0.071 | -0.061 | -0.071 |
| | Opp. cost | -0.010 | -0.064 | | -0.007 | -0.062 | | -0.007 | -0.063 | | -0.005 | -0.053 | | -0.005 | -0.043 | |
| K=42 | Sharpe ratio | 0.067 | 0.029 | -0.017 | 0.058 | 0.029 | -0.023 | 0.039 | 0.028 | -0.032 | 0.019 | 0.022 | -0.037 | 0.005 | 0.014 | -0.042 |
| | Opp. cost | 0.080 | 0.075 | | 0.066 | 0.077 | | 0.045 | 0.071 | | 0.031 | 0.053 | | 0.025 | 0.039 | |
| K=56 | Sharpe ratio | 0.113 | 0.036 | 0.001 | 0.099 | 0.036 | -0.003 | 0.077 | 0.035 | -0.009 | 0.057 | 0.032 | -0.011 | 0.038 | 0.024 | -0.023 |
| | Opp. cost | 0.106 | 0.078 | | 0.085 | 0.081 | | 0.058 | 0.078 | | 0.040 | 0.057 | | 0.033 | 0.042 | |

*Entries report the performance measures (daily Sharpe Ratio, average daily Opportunity Cost) for the case where the Taylor expansion of exponential utility is maximized by gradient based SLSQP. Results are reported for different sizes of the rolling window (K=14,28,42,56 weeks) and different degrees of relative risk aversion (ARA=2,4,6,8,10)..*

Table 6 shows out of sample performance of portfolios formed by power utility maximization. Almost all that holds for exponential utility holds also for power utility with a few interesting facts. Superior performance of base assets remains true for $K = 14$ and the development of Bitcoin superiority is same as the above, despite Bitcoin outperforming Ethereum on both grounds, the Sharpe ratio and the Opportunity cost. Interestingly enough, the performance is identical across all $K's$ for Relative Risk Aversion coefficient equal to 6 and 8. Further inspection reveals that optimal weights are alike for those two cases for any value of $K$. With respect to optimal allocation weights, all weights chosen for both cryptoassets are always maxed-out to 0.25. The only exception here is the case of Ethereum with RRA=10, where the weight is 0.07.

Table 6: Direct utility maximization: Power utility function

| | | RRA=2 | | | RRA=4 | | | RRA=6 | | | RRA=8 | | | RRA=10 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Metric | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base | BTC | ETH | Base |
| K=14 | Sharpe ratio | -0.030 | -0.041 | -0.025 | -0.030 | -0.042 | -0.010 | -0.066 | -0.061 | -0.010 | -0.066 | -0.061 | -0.010 | -0.066 | -0.108 | -0.010 |
| | Opp. cost | -0.021 | -0.072 | | -0.028 | -0.081 | | -0.065 | -0.117 | | -0.065 | -0.117 | | -0.065 | -0.066 | |
| K=28 | Sharpe ratio | -0.048 | -0.050 | -0.071 | -0.048 | -0.050 | -0.062 | -0.069 | -0.060 | -0.062 | -0.068 | -0.060 | -0.062 | -0.134 | -0.113 | -0.062 |
| | Opp. cost | -0.014 | -0.067 | | -0.010 | -0.064 | | -0.028 | -0.082 | | -0.028 | -0.082 | | -0.019 | -0.034 | |
| K=42 | Sharpe ratio | 0.067 | 0.029 | -0.036 | 0.067 | 0.028 | -0.017 | 0.041 | 0.017 | -0.017 | 0.041 | 0.017 | -0.017 | 0.041 | -0.025 | -0.017 |
| | Opp. cost | 0.087 | 0.082 | | 0.080 | 0.072 | | 0.051 | 0.047 | | 0.051 | 0.048 | | 0.051 | -0.005 | |
| K=56 | Sharpe ratio | 0.113 | 0.036 | -0.011 | 0.113 | 0.035 | 0.001 | 0.089 | 0.022 | 0.001 | 0.089 | 0.022 | 0.001 | 0.089 | -0.009 | 0.001 |
| | Opp. cost | 0.113 | 0.085 | | 0.106 | 0.075 | | 0.075 | 0.048 | | 0.075 | 0.048 | | 0.075 | -0.007 | |

*Entries report the performance measures (daily Sharpe Ratio, average daily Opportunity Cost) for the case where the Taylor expansion of isoelastic utility is maximized by gradient based SLSQP. Results are reported for different sizes of the rolling window (K=14,28,42,56 weeks) and different degrees of relative risk aversion (RRA=2,4,6,8,10)..*

# 7 Conclusion

In this thesis, the analysis focuses on the extent and of the significance of diversification gains, which accrue to an already diversified portfolio due to the inclusion of Bitcoin and Ethereum from a perspective of investor holding global portfolio. We use global indices representing stock, bond, energy, and currency market for adequate representation of existing investment choices. We assess performance of Bitcoin and Ethereum while using a more general approach than the mean-variance (MV) setting which is used primarily in the previous literature. First novelty brought to the research is the application of rigorous in-sample tests of spanning to investor described not only by quadratic but also by exponential and power utility function. Our regression models use Newey-West's robust covariance matrix with non-arbitrary number of lags in difference to [Chowdhury, (2016)]. Additionally, performance of portfolios augmented by cryptoassets is analyzed out-of sample. For this, direct utility maximization using gradient-based optimization is employed, as opposite to a grid search which was applied in previous studies.

Results are consistent for classic mean-variance and also non-mean-variance frameworks, suggesting that both cryptoassets can significantly improve investing opportunities of investors. This is well aligned, as far as Bitcoin's case is concerned, with previous research of [Briere, et al. (2015)], [Eisl et al., (2015)] and [Kajtazi et al., (2018)]. However, results contradict with in-sample findings of [Chowdhury, (2016)] MV case, even though they are in line for non-MV framework. The only exception to our significant results is Ethereum in the case of joint MV and non-MV exponential utility based spanning. Here we failed to reject the null hypothesis. As a consequence, investor subject to these preferences is not advised to invest in this asset if he seeks an optimal diversification.

Out of sample results suggest improved performance of portfolios augmented by cryptoassets mainly on mid-to-longterm scale (meaning more than 28 weeks), which is in line with [Symitsi et al., (2018)]. This contradicts with findings of [Chowdhury, (2016)], where base portfolios were shown to be categorically superior. Our analysis demonstrates that allocation of around 4-8% should generalize best out-of sample. This is a very important note, since a recent high rate of returns could convince investors to max-out on these assets. These results are well aligned with [Eisl et al., (2015)], who report average investment of 1.65% to 5%. Analysis of portfolio weights indicates that in general Bitcoin outweighs Ethereum which is in agreement with [Katsiampa, (2018)].

We suggest that further research should focus on general performance of both cryptoassets as they mature and hopefully become less volatile. Secondly, bigger emphasis should be placed on examining also transaction costs as additional factor. From our point of view, the base asset space could be selected with more caution in order to capture the choices of standard investors more realistically. One of our priorities was replicability and thus asset choice suffered from the unavailability of a sufficient number of indices. Given that our aim was to assess cryptoassets from out-of-sample setting in order to provide possible investment advice, we conclude that both assets offer performance benefits, which are mainly present in mid- to long-term time periods.

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# 8 Appendix

## 8.1 A

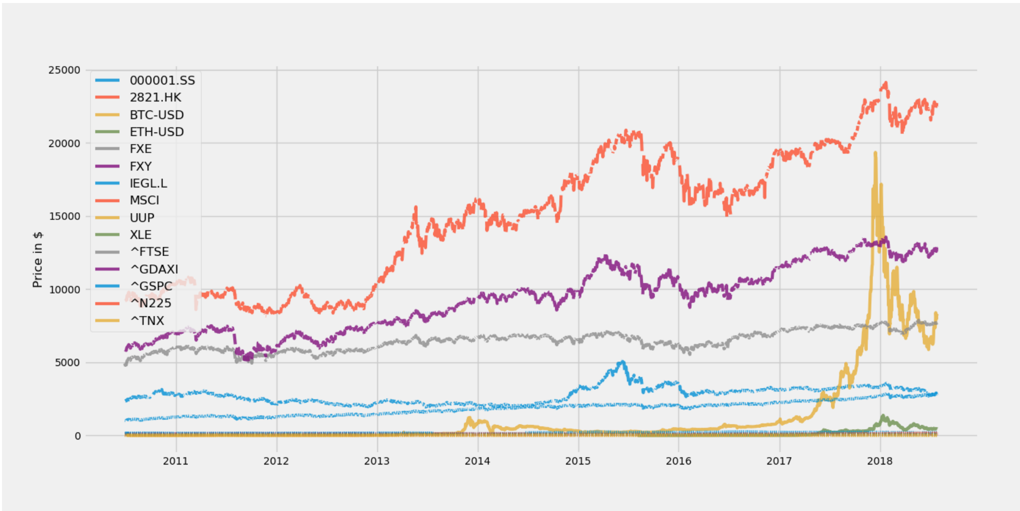


Fig. 4: Asset prices

### 8.2 B

```python
######################## data_dwnld.py ###################################
import quandl
import pandas as pd
pd.core.common.is_list_like = pd.api.types.is_list_like
import numpy as np
import requests
import datetime
import pandas_datareader as pdr
import fix_yahoo_finance as yf

# has to be done due to my firewall setu otherwise regular session can be used
session = requests.Session()
session.verify = False
web = pdr.data

# init neccessary keys
start = datetime.datetime(2010, 7, 1)
end = datetime.datetime(2018, 7, 31)
quandl_key = 'QLcxbL3_wshHUcC8973a'
tiingo_key = '1bb7da5fa10a67a81c2123ab7000d8835ee44877'


# download and write to dir
stocks = ["^GSPC","^FTSE","^N225","000001.SS","^GDAXI","MSCI",
          "^TNX","CL=F","BTC-USD","ETH-USD","GC=F","UUP","FXE",
          "FXY","IEGL.L","2821.HK","XLE"]
for stock in stocks:
    data = yf.download(stock, start=start, end=end)
    data.to_csv(f"{stock}.csv")
```

```python
########################### utils.py ###########################
# laod libs
import numpy as np
from cvxopt import matrix, solvers, blas
import pandas as pd
import os
import glob

# Load data
def load_returns():
    extension = 'csv'
    result = [i for i in glob.glob('*.{}'.format(extension))]
    result.remove('GC=F.csv')
    result.remove('LIBOR.csv')

    df = pd.DataFrame()
    for file in result:
        data = pd.read_csv(file)
        data = data.set_index(pd.to_datetime(data.Date).dt.date)
        data = data['Adj Close']
        df = pd.concat([df,data],axis=1)
    df.columns = [file.replace(".csv","") for file in result]
    returns = df.pct_change()
    return returns


# function to create random weights with heuristic normalization for sum(x)==1
def rand_weights(n):
    ''' Produces n random weights that sum to 1 '''
    k = np.random.rand(n)
    return k / sum(k)

# function returning mean and sd of random portfolio
def random_portfolio(returns):
    '''
    Returns the mean and standard deviation of returns for a random portfolio
    '''
```

```python
    p = np.asmatrix(np.mean(returns, axis=0))
    w = np.asmatrix(rand_weights(returns.shape[1]))
    C = np.asmatrix(returns.cov())

    mu = w * p.T
    sigma = np.sqrt(w * C * w.T)
    return mu, sigma



# function solves the QP, where x is the allocation of the portfolio:
# minimize   x'Px + q'x
# subject to Gx <= h
#            Ax == b
#
# Input:  n       - # of assets
#         avg_ret - nx1 matrix of average returns
#         covs    - nxn matrix of return covariance
#         r_min   - the minimum expected return that you'd
#                   like to achieve
#         max_w   - max weight which can be assigned to one asset
# Output: sol - cvxopt solution object


def optimize_portfolio(n, avg_ret, covs, r_min, max_w):
    P = matrix(covs)
    q = matrix(np.zeros((n, 1)), tc='d')

    # inequality constraints Gx <= h
    # captures the constraints (avg_ret'x >= r_min),(x >= 0) and (x <= 0.25)
    G = matrix(np.concatenate((
        -np.transpose(matrix(avg_ret)),
        -np.identity(n),
        np.identity(n)), 0))
    h = matrix(np.concatenate((
        -np.ones((1,1))*r_min,
        np.zeros((n,1)),
```

```python
        np.ones((n,1))*max_w), 0))
    # equality constraint Ax = b; captures the constraint sum(x) == 1
    A = matrix(1.0, (1,n))
    b = matrix(1.0)
    opts = {'maxiters' : 22, 'feastol': 1e-14, 'abstol': 1e-14,
            'reltol': 1e-14, 'show_progress':False}
    sol = solvers.qp(P, q, G, h, A, b, options = opts)
    return sol

# MV framework optimization
# Input: returns_vec     - DataFrame of returns
#        drop            - test asset to drop
#        r_min           - arbitrary min returns that you optimize for
#        max_w           - max weight to be assigned to one asset
#
# Output: numpy array of optimal weights


def MV(returns_vec, drop = None , r_min = 0.0006, max_w = 0.25):
    if drop is None:
        covs = np.asmatrix(returns_vec.cov())
        avg_ret = np.mean(returns_vec, axis=0).T
    else:
        returns_vec = returns_vec.drop(drop + '-USD', axis=1)
        covs = np.asmatrix(returns_vec.cov())
        avg_ret = np.mean(returns_vec).T

    # solve
    sol_aug = optimize_portfolio(returns_vec.shape[1], avg_ret, covs, r_min, max_w)
    return np.array(sol_aug['x'])

# Mazimization of Utility expanded by Taylor series with up to k=2 order moments

def exp_utility(x, xRA = 2, exp = True, power = False, base = False, asset = 'BTC',
                return_vec = load_returns()):

    # Computes Expected Utility with second order moments included according to
```

```
# Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all?
# New evidence", pages 28-30

# Instead of grid search Sequential Least SQuares Programming (SLSQP) Algorithm used
# Originally implemented by Kraft, D. A software package for sequential quadratic
# programming. 1988. Tech. Rep. DFVLR-FB 88-28, DLR German Aerospace Center -
# Institute for Flight Mechanics, Koln, Germany.



# Power utility is not scaled properly so we use scale factor 10**(RRA*4);
# found experimentally
# More on the issue can be found:
#https://stackoverflow.com/questions/11155721/
#                         positive-directional-derivative-for-linesearch/11177146

scaler = 10**(xRA*4)

if not base:
    ret = return_vec.drop(asset + '-USD',axis=1)
    avg_ret = np.mean(ret, axis=0).T
    w = np.asmatrix(x)*np.asmatrix(avg_ret).T
    cov = np.asmatrix(ret.cov())
    if exp:
        eu = ((np.exp(-xRA*w)/xRA)*(1+(xRA**2)*(np.asmatrix(x)*cov*np.asmatrix(x).T)/2))
    elif power:
        eu = -((w**(1-xRA)-1)/(1-xRA) -
               (xRA*(w**(-xRA-1))*(np.asmatrix(x)*cov*np.asmatrix(x).T))/2)/scaler
else:
    ret = return_vec.drop(['BTC-USD','ETH-USD'],axis=1)
    avg_ret = np.mean(ret, axis=0).T
    w = np.asmatrix(x)*np.asmatrix(avg_ret).T
    cov = np.asmatrix(ret.cov())
    if exp:
        eu = ((np.exp(-xRA*w)/xRA)*(1+(xRA**2)*(np.asmatrix(x)*cov*np.asmatrix(x).T)/2))
    elif power:
        eu = -((w**(1-xRA)-1)/(1-xRA) -
```

```python
                   (xRA*(w**(-xRA-1))*(np.asmatrix(x)*cov*np.asmatrix(x).T))/2)/scaler
    return eu

def apply_sum_constraint(inputs):
    total = 1.0 - np.sum(inputs)
    return total

cons = ({'type': 'eq', "fun": apply_sum_constraint})
```

```python
################################# opt.py #################################
# libs and utils

import numpy as np
import pandas as pd
from cvxopt import matrix, solvers, blas
import os
import glob

import statsmodels.api as sm
from statsmodels.formula.api import ols

import matplotlib.pyplot as plt
import seaborn as sns
import quandl
plt.style.use('fivethirtyeight')
np.random.seed(777)
%matplotlib inline
%config InlineBackend.figure_format = 'retina'

from scipy import optimize
from scipy.stats import jarque_bera

from utils import *

# load libor from csv
libor = pd.read_csv('LIBOR.csv')
libor = libor.set_index(pd.to_datetime(libor.Date).dt.date)
libor = libor['1M']/100
# trnasform to daily rate
libor = (1 + libor)**(1/30) - 1

# initialize
n = load_returns().shape[1]
return_vec = load_returns()
returns = load_returns()
```

```python
# simulate and plot random portfolios
n_portfolios = 1000
means, stds = np.column_stack([random_portfolio(return_vec) for _ in range(n_portfolios)])

plt.plot(stds, means, 'o', markersize=5)
plt.xlabel('std')
plt.ylabel('mean')
plt.title('Mean and standard deviation of returns of randomly generated portfolios')

##################################################################################
# Test for spanning for MV spanning
# BTC
# For each asset, separate X matrix is created due to shorter sample for ETH

# Create non-NaN subset and Libor
sbset = returns.drop(['ETH-USD'],axis=1).dropna().drop(['BTC-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 14

X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['ETH-USD'],axis=1).dropna()[['BTC-USD']].sub(libor,axis=0))

est = sm.OLS(Y, X)
est2 = est.fit(cov_type='HAC',cov_kwds={'maxlags':int(0.75*X.shape[0]**(1/3) - 1)})
#print(est2.summary())

# Hypothesis
# hypothesis = '(const = 0, x1+x2+x3+x4+x5+x6+x7+x8+x9+x10+x11+x12+x13 = 1)'
hypothesis = '(const = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of MV spanning with Bitcoin.")
print(wald)
```

```
####################################################################################
# Test for spanning for MV spanning
# ETH
# For each asset, separate X matrix is created due to shorter sample for ETH

# Create non-NaN subset and Libor
sbset = returns.drop(['BTC-USD'],axis=1).dropna().drop(['ETH-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 14

X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['BTC-USD'],axis=1).dropna()[['ETH-USD']].sub(libor,axis=0))

est = sm.OLS(Y, X)
est2 = est.fit(cov_type='HAC',cov_kwds={'maxlags':int(0.75*X.shape[0]**(1/3) - 1)})
#print(est2.summary())

# Hypothesis
hypothesis = '(const = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of MV spanning with Ethereum.")
print(wald)

####################################################################################
# Power Utility
# Base portfolio optimization

k = n-2
bnds = [(0,.25) for i in range(k)]
init_g = 1/k
```

```
for RRA in range(2,12,2):
    args = (RRA, False, True, True, _, returns)
        #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    print("Portfolio given RRA =",RRA)
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Base portfolio weight search by power utility optimization.")
    print(np.round(res.x,4))

####################################################################################
# Power Utility
# Augmented portfolio optimization
# BTC

k = n - 1
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for RRA in range(2,12,2):
    args = (RRA, False, True, False, 'ETH', returns)
        #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    print("Portfolio given RRA =",RRA)
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Augmented portfolio by Bitcoin weight search by power utility optimization.")
    print(np.round(res.x,4))

####################################################################################
# Power Utility
# Augmented portfolio optimization
# ETH

k = n - 1
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for RRA in range(2,12,2):
```

```python
    args = (RRA, False, True, False, 'BTC', returns)
        #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    print("Portfolio given RRA =",RRA)
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Augmented portfolio by Ethereum weight search by power utility optimization.")
    print(np.round(res.x,4))

####################################################################################
# Exponential utility maximization
# Base portfolio optimization

k = n-2
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for ARA in range(2,12,2):
    args = (ARA, True, False, True, _, returns)
        #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    print("Portfolio given ARA =",ARA)
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Base portfolio weight search by exponential utility optimization.")
    print(np.round(res.x,4))

####################################################################################
# Exponential utility
# Augmented portfolio optimization
# BTC

k = n - 1
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for ARA in range(2,12,2):
    args = (ARA, True, False, False, 'ETH', returns)
    print("Optimal portfolio given ARA=",ARA)
```

```python
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Augmented portfolio by Bitcoin weight search by exponential utility optimization.")
    print(np.round(res.x,4))

###################################################################################
# Exponential utility
# Augmented portfolio optimization
# ETH

k = n - 1
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for ARA in range(2,12,2):
    args = (ARA, True, False, False, 'BTC', returns)
    print("Optimal portfolio given ARA=",ARA)
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)
    print("Augmented portfolio by Ethereum weight search by exponential utility optimization.")
    print(np.round(res.x,4))

###################################################################################
###################################################################################
######################## Non Mean Variance spanning ###############################
######################## Exponential with Bitcoin #################################
###################################################################################

# Test for spanning for non-MV spanning with Exponential Utility
# BTC
# For each asset, separate X matrix is created due to shorter sample for ETH

# Create non-NaN subset and Libor
sbset = returns.drop(['ETH-USD'],axis=1).dropna().drop(['BTC-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
```

```
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 16


X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['ETH-USD'],axis=1).dropna()[['BTC-USD']].sub(libor,axis=0))

# Get optimal portfolio given maximizing Exponential Utility with 2nd order moment
k = n-2
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for ARA in range(2,12,2):
    args = (ARA, True, False, True,  _, returns)
            #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)

    # Compute Expected marginal utility
    X2 = np.asmatrix(np.exp(-ARA*(np.sum(sbset*res.x,axis=1)))).T

    # Append to previous ARA & R(t+1)
    X = np.concatenate((X2,X),axis=1)

# OLS estimation using MacKinnon and White's (1985) alternative heteroskedasticity
# robust standard errors.
# Defined as (X.T X)^(-1)X.T diag(e_i^(2)/(1-h_ii)^(2)) X(X.T X)^(-1)
# where h_ii = x_i(X.T X)^(-1)x_i.T
#
# HC3 weights each squared OLS residual by a factor of 1/(1-hii)**2 rather than 1/(1-hii).
# Using simulations, Long and Ervin (2000) evaluated the empirical power functions of the
# t tests of the regression coefficients, using both the ordinary OLS estimator and the
# four HC methods; they recommended that HC3 always be used because it can keep the test
# size at the nominal level regardless of the presence or absence of heteroskedasticity
# (and there is only a slight loss of power associated with HC3 when the errors are indeed
```

```python
# homoskedastic).
est = sm.OLS(Y, X)
est2 = est.fit(cov_type='HC3')

# Hypothesis
hypothesis = '(const = 0, x1 = 0,  x2 = 0, x3 = 0, x4 = 0, x5 = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of joint MV and non-MV (Exponential) spanning with Bitcoin.")
print(wald)


####################################################################################
####################################################################################
######################## Non Mean Variance spanning ################################
######################## Exponential with Ethereum ################################
####################################################################################

# Test for spanning for non-MV spanning with Exponential Utility
# ETH
# For each asset, separate X matrix is created due to shorter sample for ETH

# Create non-NaN subset and Libor
sbset = returns.drop(['BTC-USD'],axis=1).dropna().drop(['ETH-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 16


X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['BTC-USD'],axis=1).dropna()[['ETH-USD']].sub(libor,axis=0))

# Get optimal portfolio given maximizing Exponential Utility with 2nd order moment
k = n-2
```

```
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for ARA in range(2,12,2):
    args = (ARA, True, False, True,  _, returns)
            #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)

    # Compute Expetcted Marginal utility
    X2 = np.asmatrix(np.exp(-ARA*(np.sum(sbset*res.x,axis=1)))).T

    # Append to previous ARA & R(t+1)
    X = np.concatenate((X2,X),axis=1)

# OLS estimation using MacKinnon and White's (1985) alternative heteroskedasticity
# robust standard errors.
# Defined as (X.T X)^(-1)X.T diag(e_i^(2)/(1-h_ii)^(2)) X(X.T X)^(-1)
# where h_ii = x_i(X.T X)^(-1)x_i.T
#
# HC3 weights each squared OLS residual by a factor of 1/(1-hii)**2 rather than 1/(1-hii).
# Using simulations, Long and Ervin (2000) evaluated the empirical power functions of the
# t tests of the regression coefficients, using both the ordinary OLS estimator and the
# four HC methods; they recommended that HC3 always be used because it can keep the test
# size at the nominal level regardless of the presence or absence of heteroskedasticity
# (and there is only a slight loss of power associated with HC3 when the errors are indeed
# homoskedastic).est = sm.OLS(Y, X)
est2 = est.fit(cov_type='HC3')

# Hypothesis
hypothesis = '(const = 0, x1 = 0,  x2 = 0, x3 = 0, x4 = 0, x5 = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of joint MV and non-MV (Exponential) spanning with Ethereum.")
print(wald)

##############################################################################
##############################################################################
```

```python
######################## Non Mean Variance spanning ################################
############################ Power with Bitcoin ####################################
####################################################################################

# Test for spanning for non-MV spanning with Power Utility
# BTC
# For each asset, separate X matrix is created due to shorter sample for ETH

# Create non-NaN subset and Libor
sbset = returns.drop(['ETH-USD'],axis=1).dropna().drop(['BTC-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 16


X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['ETH-USD'],axis=1).dropna()[['BTC-USD']].sub(libor,axis=0))

# Get optimal portfolio given maximizing Exponential Utility with 2nd order moment
k = n-2
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for RRA in range(2,12,2):
    args = (RRA, False, True, True, _, returns)
            #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)

    # Compute Expetcted Marginal utility
    # Log10 of is taken in order to counter numerical instability & bad scaling of the problem
    # Final Hypothesis shouldn't be invalidated since we only create semi-elastic relationship
    X2 = np.asmatrix(np.log10((np.sum(sbset*res.x,axis=1))**(-RRA))).T
```

```
    # Append to previous RRA & R(t+1)
    X = np.concatenate((X2,X),axis=1)

X = pd.DataFrame(X)
X = X[~X.isin([np.nan, np.inf, -np.inf]).any(1)]

# OLS estimation using MacKinnon and White's (1985) alternative heteroskedasticity
# robust standard errors.
# Defined as (X.T X)^(-1)X.T diag(e_i^(2)/(1-h_ii)^(2)) X(X.T X)^(-1)
# where h_ii = x_i(X.T X)^(-1)x_i.T
#
# HC3 weights each squared OLS residual by a factor of 1/(1-hii)**2 rather than 1/(1-hii).
# Using simulations, Long and Ervin (2000) evaluated the empirical power functions of the
# t tests of the regression coefficients, using both the ordinary OLS estimator and the
# four HC methods; they recommended that HC3 always be used because it can keep the test
# size at the nominal level regardless of the presence or absence of heteroskedasticity
# (and there is only a slight loss of power associated with HC3 when the errors are indeed
# homoskedastic).est = sm.OLS(Y[X.index],np.asmatrix(X))
est2 = est.fit(cov_type='HC3')

# Hypothesis
hypothesis = '(const = 0, x1 = 0,  x2 = 0, x3 = 0, x4 = 0, x5 = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of joint MV and non-MV (Power) spanning with Bitcoin.")
print(wald)


###############################################################################
###############################################################################
####################### Non Mean Variance spanning ############################
########################### Power with Ethereum ###############################
###############################################################################
# Test for spanning for non-MV spanning with Power Utility
# ETH
# For each asset, separate X matrix is created due to shorter sample for ETH
```

```python
# Create non-NaN subset and Libor
sbset = returns.drop(['BTC-USD'],axis=1).dropna().drop(['ETH-USD'],axis=1)
libor = libor[sbset.index].fillna(method='bfill')

# K-benchmark asset universe includes also the risk-free asset, so test for MV spanning is
# reformulated in excess returns terms according to Daskalaki and Skiadopoulos;
# "Should investors include commodities in their portfolios after all? New evidence", pages 16


X = np.asmatrix(sbset.sub(libor,axis=0))
X = sm.add_constant(X)

Y = np.asmatrix(returns.drop(['BTC-USD'],axis=1).dropna()[['ETH-USD']].sub(libor,axis=0))

# Get optimal portfolio given maximizing Exponential Utility with 2nd order moment
k = n-2
bnds = [(0,.25) for i in range(k)]
init_g = 1/k

for RRA in range(2,12,2):
    args = (RRA, False, True, True, _, returns)
            #(ARA/RRA, exp, power, base); scipy.optimie.minimze accepts only *args not **kwargs
    res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                            args=args, method='SLSQP', bounds=bnds, tol=1e-14, constraints=cons)

    # Compute Expetcted Marginal utility
    # Log10 of is taken in order to counter numerical instability & bad scaling of the problem
    # Final Hypothesis shouldn't be invalidated since we only create semi-elastic relationship
    X2 = np.asmatrix(np.log10((np.sum(sbset*res.x,axis=1))**(-RRA))).T

    # Append to previous RRA & R(t+1)
    X = np.concatenate((X2,X),axis=1)

X = pd.DataFrame(X)
X = X[~X.isin([np.nan, np.inf, -np.inf]).any(1)]

# OLS estimation using MacKinnon and White's (1985) alternative heteroskedasticity
```

```
# robust standard errors.
# Defined as (X.T X)^(-1)X.T diag(e_i^(2)/(1-h_ii)^(2)) X(X.T X)^(-1)
# where h_ii = x_i(X.T X)^(-1)x_i.T
#
# HC3 weights each squared OLS residual by a factor of 1/(1-hii)**2 rather than 1/(1-hii).
# Using simulations, Long and Ervin (2000) evaluated the empirical power functions of the
# t tests of the regression coefficients, using both the ordinary OLS estimator and the
# four HC methods; they recommended that HC3 always be used because it can keep the test
# size at the nominal level regardless of the presence or absence of heteroskedasticity
# (and there is only a slight loss of power associated with HC3 when the errors are indeed
# homoskedastic).
# ref http://www.afhayes.com/public/BRM2007.pdf
est = sm.OLS(Y[X.index],np.asmatrix(X))
est2 = est.fit(cov_type='HC3')

# Hypothesis
hypothesis = '(const = 0, x1 = 0,  x2 = 0, x3 = 0, x4 = 0, x5 = 0)'
wald = est2.wald_test(hypothesis)
print("Hypothesis of joint MV and non-MV (Power) spanning with Ethereum.")
print(wald)

####################################################################################
####################################################################################
###################### Out of sample performance evaluation ########################
########################## Quadratic with BTC/ETH/Base #############################
####################################################################################

# Quadratic Out of Sample
# BTC

MV_BTC_returns = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    x = list(np.array(MV(drop='ETH',returns_vec = return_vec)).flat)

    ret = x*returns.iloc[(returns.shape[0] - t*7 + 1):,:].drop(['ETH-USD'],axis=1)
```

```
    excess_return = (np.mean(np.sum(ret,axis=1)) -
                     np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
    var = (np.asmatrix(x)*
          np.asmatrix(returns.iloc[:(returns.shape[0] -
            t*7),:].drop(['ETH-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
            t*7)],axis=0).cov())*np.asmatrix(x).T)
    MV_BTC_returns.append(np.mean(np.sum(ret,axis=1)))
    #print("K: ",t)
    #print("Sharpe ratio: ",np.round(excess_return/np.sqrt(var),3))
    #print("Return: ",np.mean(np.sum(ret,axis=1)))

####################################################################################
# Quadratic Out of Sample
# ETH

MV_ETH_returns = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    x = list(np.array(MV(drop='BTC',returns_vec = return_vec)).flat)

    ret = x*returns.iloc[(returns.shape[0] - t*7 + 1):,:].drop(['BTC-USD'],axis=1)
    excess_return = (np.mean(np.sum(ret,axis=1)) -
                     np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
    var = (np.asmatrix(x)*
          np.asmatrix(returns.iloc[:(returns.shape[0] -
            t*7),:].drop(['BTC-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
            t*7)],axis=0).cov())*np.asmatrix(x).T)
    MV_ETH_returns.append(np.mean(np.sum(ret,axis=1)))
    #print("K: ",t)
    #print("Sharpe ratio: ",np.round(excess_return/np.sqrt(var),3))
    #print("Return: ",np.mean(np.sum(ret,axis=1)))

####################################################################################
# Quadratic Out of Sample
# Base
```

```
MV_base_returns = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 2
    x = list(np.array(MV(returns_vec = return_vec)).flat)

    ret = x*returns.iloc[(returns.shape[0] - t*7 + 1):,:]
    excess_return = (np.mean(np.sum(ret,axis=1)) -
                     np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
    var = (np.asmatrix(x)*
          np.asmatrix(returns.iloc[:(returns.shape[0] -
                                        t*7),:].sub(libor.iloc[:(libor.shape[0] -
                                        t*7)],axis=0).cov())*np.asmatrix(x).T)
    MV_base_returns.append(np.mean(np.sum(ret,axis=1)))
    #print("K: ",t)
    #print("Sharpe ratio: ",np.round(excess_return/np.sqrt(var),3))
    #print("Return: ",np.mean(np.sum(ret,axis=1)))


#####################################################################################
#####################################################################################
###################### Out of sample performance evaluation ########################
######################### Exponential with BTC/ETH/Base ###########################
#####################################################################################

# Exponential Out of Sample
# BTC

BTC_EXP = []
BTC_EXP_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for ARA in range(2,12,2):
```

```python
        args = (ARA, True, False, False, 'ETH', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given ARA=",ARA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                args=args, method='SLSQP',
                                bounds=bnds, tol=1e-14, constraints=cons)


        ret = res.x*returns.iloc[(returns.shape[0] - t*7 + 1):,:].drop(['ETH-USD'],axis=1)
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                         np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
              np.asmatrix(returns.iloc[:(returns.shape[0] -
                t*7),:].drop(['ETH-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                                t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        BTC_EXP.append(excess_return/np.sqrt(var))
        BTC_EXP_r.append(np.mean(np.sum(ret,axis=1)))
        #print("K: ",t)
        #print("Sharpe ratio: ",excess_return/np.sqrt(var))
        #print("Return: ",np.mean(np.sum(ret,axis=1)))

####################################################################################
# Exponential Out of Sample
# ETH

ETH_EXP = []
ETH_EXP_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for ARA in range(2,12,2):
        args = (ARA, True, False, False, 'BTC', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given ARA=",ARA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                args=args, method='SLSQP',
```

```
                                bounds=bnds, tol=1e-14, constraints=cons)


        ret = res.x*returns.iloc[(returns.shape[0] - t*7 + 1):,:].drop(['BTC-USD'],axis=1)
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                         np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
              np.asmatrix(returns.iloc[:(returns.shape[0] -
                t*7 ),:].drop(['BTC-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                                t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        ETH_EXP.append(excess_return/np.sqrt(var))
        ETH_EXP_r.append(np.mean(np.sum(ret,axis=1)))
        #print("K: ",t)
        #print("Sharpe ratio: ",excess_return/np.sqrt(var))
        #print("Return: ",np.mean(np.sum(ret,axis=1)))

########################################################################################
# Exponential Out of Sample
# Base

base_EXP = []
base_EXP_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 2
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for ARA in range(2,12,2):
        args = (ARA, True, False, True, 'ETH', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given ARA=",ARA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                args=args, method='SLSQP',
                                bounds=bnds, tol=1e-14, constraints=cons)

        ret = res.x*returns.iloc[(returns.shape[0] -
                                  t*7 + 1):,:].drop(['ETH-USD','BTC-USD'],axis=1)
```

```
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                         np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
              np.asmatrix(returns.iloc[:(returns.shape[0] -
            t*7 ),:].drop(['ETH-USD','BTC-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                                    t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        base_EXP.append(excess_return/np.sqrt(var))
        base_EXP_r.append(np.mean(np.sum(ret,axis=1)))
        #print("K: ",t)
        #print("Sharpe ratio: ",excess_return/np.sqrt(var))
        #print("Return: ",np.mean(np.sum(ret,axis=1)))


####################################################################################
####################################################################################
###################### Out of sample performance evaluation ########################
############################ Power with BTC/ETH/Base ###############################
####################################################################################
# Power Utility Out of Sample
# BTC

BTC_POW = []
BTC_POW_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for RRA in range(2,12,2):
        args = (RRA, False, True, False, 'ETH', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given RRA=",RRA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                  args=args, method='SLSQP',
                                  bounds=bnds, tol=1e-14, constraints=cons)

        ret = res.x*returns.iloc[(returns.shape[0] -
```

```
                                 t*7 + 1):,:].drop(['ETH-USD'],axis=1)
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                        np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
              np.asmatrix(returns.iloc[:(returns.shape[0] -
                t*7),:].drop(['ETH-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                       t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        BTC_POW.append(excess_return/np.sqrt(var))
        BTC_POW_r.append(np.mean(np.sum(ret,axis=1)))
        #print("K: ",t)
        #print("Sharpe ratio: ",excess_return/np.sqrt(var))
        #print("Return: ",np.mean(np.sum(ret,axis=1)))

##################################################################################
# Power Out of Sample
# ETH

ETH_POW = []
ETH_POW_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 1
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for RRA in range(2,12,2):
        args = (RRA, False, True, False, 'BTC', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given RRA=",RRA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                args=args, method='SLSQP',
                                bounds=bnds, tol=1e-14, constraints=cons)

        ret = res.x*returns.iloc[(returns.shape[0] -
                                 t*7 + 1):,:].drop(['BTC-USD'],axis=1)
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                        np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
```

```
            np.asmatrix(returns.iloc[:(returns.shape[0] -
              t*7),:].drop(['BTC-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                        t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        ETH_POW.append(excess_return/np.sqrt(var))
        ETH_POW_r.append(np.mean(np.sum(ret,axis=1)))
        #print("K: ",t)
        #print("Sharpe ratio: ",excess_return/np.sqrt(var))
        #print("Return: ",np.mean(np.sum(ret,axis=1)))

##################################################################################
# Power Out of sample
# Base

base_POW = []
base_POW_r = []
for t in range(14, 70, 14):
    return_vec = returns.iloc[:(returns.shape[0] - t*7),:]
    k = n - 2
    bnds = [(0,.25) for i in range(k)]
    init_g = 1/k

    for RRA in range(2,12,2):
        args = (RRA, False, true, True, 'ETH', return_vec)
        #print("Last ", t, "weeks is left out of training.","Optimal portfolio given ARA=",ARA)
        res = optimize.minimize(exp_utility, np.array([init_g for i in range(k)]),
                                args=args, method='SLSQP',
                                bounds=bnds, tol=1e-14, constraints=cons)

        ret = res.x*returns.iloc[(returns.shape[0] -
                                  t*7 + 1):,:].drop(['ETH-USD','BTC-USD'],axis=1)
        excess_return = (np.mean(np.sum(ret,axis=1)) -
                         np.mean(libor.iloc[(libor.shape[0] - t*7 + 1):]))
        var = (np.asmatrix(res.x)*
            np.asmatrix(returns.iloc[:(returns.shape[0] -
          t*7 ),:].drop(['ETH-USD','BTC-USD'],axis=1).sub(libor.iloc[:(libor.shape[0] -
                                                t*7)],axis=0).cov())*np.asmatrix(res.x).T)
        base_POW.append(excess_return/np.sqrt(var))
```

```python
base_POW_r.append(np.mean(np.sum(ret,axis=1)))
#print("K: ",t)
#print("Sharpe ratio: ",excess_return/np.sqrt(var))
#print("Return: ",np.mean(np.sum(ret,axis=1)))
```