Abstract

The continuum function is a function which maps every infinite cardinal κ to 2^κ. We say that a regular uncountable cardinal κ has the tree property if every κ-tree has a cofinal branch, or equivalently if there are no κ-Aronszajn trees. We say that a regular uncountable cardinal κ has the weak tree property if there are no special κ-Aronszajn trees. It is known that the tree property, and the weak tree property, have the following non-trivial effect on the continuum function: $(*)$ If the (weak) tree property holds at κ^{++}, then $2^\kappa \geq \kappa^{++}$. In this thesis we show several results which suggest that $(*)$ is the only restriction which the tree property and the weak tree property put on the continuum function in addition to the usual restrictions provable in ZFC (monotonicity and the fact that the cofinality of 2^κ must be greater than κ; let us denote these conditions by $(**)$). First we show that the tree property at $\aleph_{2n}$ for every $1 \leq n < \omega$, and the weak tree property at $\aleph_n$ for $2 \leq n < \omega$, does not restrict the continuum function below $\aleph_\omega$ more than is required by $(*)$, i.e. every behaviour of the continuum function below $\aleph_\omega$ which satisfies the conditions $(*)$ and $(**)$ is realisable in some generic extension. We use infinitely many weakly compact cardinals (for the tree property) and infinitely many Mahlo cardinals (for the weak tree property) as the optimal large cardinal assumption. In the second result we show that the tree property at the double successor of a singular strong limit cardinal κ with countable cofinality does not limit the size of 2^κ except for conditions $(*)$ and $(**)$. We use the assumption of the existence of a supercompact cardinal with a weakly compact cardinal above it for the result. In the final result we show that the tree property at $\aleph_{\omega+2}$ with $\aleph_\omega$ strong limit is consistent with $2^{\aleph_\omega}$ being equal to $\aleph_{\omega+2+n}$ for any prescribed $0 \leq n < \omega$. We use the existence of a strong cardinal of a suitable degree and a weakly compact cardinal above it for this result.