

Report on Dr. Radek Honzik Habilitation thesis.

January 23, 2018

Dear Professor Ian Trlifaj,

I think that it is an excellent thesis and it certainly should be approved.

Dr. Radek Honzik deals with variety of interesting problems and obtains excellent results. The methods used are deep and an outstanding technical sophistication is demonstrated.

Let me address some specific results which are most appealing for me.

Radek Honzik in a series of papers obtained an extremely interesting result which is parallel to the classical result by W. Easton. Thus, Easton showed that any function F over regular cardinals can be realized as the power function provided

- (1) $\alpha \leq \beta$ implies $F(\alpha) \leq F(\beta)$
- (2) $\text{cof}(F(\alpha)) > \alpha$.

But what if we like to preserve more structure. Namely, can we preserve a measurability of cardinals? His work gives an almost complete answer.

Thus in the paper [32] (joint with S. Friedman) a surprising combination of Sacks and Cohen forcings is used in order to blow up powers of $H(F(\kappa))$ -strong cardinals and to preserve their measurability. In the same paper, but with an additional restriction on F called *a local definability*, it was shown that a full strongness can be preserved.

Paper [34](joint with S. Friedman) deals with optimal assumptions (of the form $\text{cof}(\kappa) = \kappa^{++}$) need in order to realize above type situations. The combination of Sacks and Cohen forcings is replaced by a more involved forcing notion.

The above results and their extensions were successfully applied to instances of the Singular Cardinal Problem, which is one of the central problems in Set Theory. The simplest case of it is called the Singular Cardinal Hypothesis (SCH):

whenever κ is a singular strong limit cardinal, then $2^\kappa = \kappa^+$. The classical method, due to Silver and Prikry, to obtain a model of \neg SCH is first to construct a model with a measurable κ so that $2^\kappa > \kappa^+$, and then to change its cofinality to ω using the Prikry forcing. There is an extensive body of work on this problem.

In the paper [52], Honzik uses his previous methods of constructing models with measurable cardinals violating the Generalized Continuum Hypothesis in combination with an other technique called the extender based Prikry forcing, in order to violate SCH simultaneously at many singular cardinals. This extends previous results by various authors including myself.

Paper [33] (joint with S. Friedman) deals with definability of a well order on $H(\kappa^+)$ (the sets which are hereditarily of cardinality $\leq \kappa$), where κ is either a measurable or a singular strong limit and $2^\kappa = \kappa^{++}$.

Connonical inner models $L[E]$ are used in the construction together with the Sacks forcing in a very nice way in order to code things and to provide a definability.


Finally the construction is pushed down to the first singular cardinal \aleph_ω and culminates with a model in which

1. \aleph_ω is a strong limit,
2. $2^{\aleph_\omega} = \aleph_{\omega+2}$,
3. there is a definable well order on $H(\aleph_{\omega+1})$.

It looks to me a striking and uninspected result.

In conclusion, I think that Dr. Radek Honzik demonstrated in his thesis a deep understanding of various aspects of set theory, suggested original ways to deal with variety of problems and produced excellent results.

Very truly yours,



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