Swansea University
Prifysgol Abertawe

Swansea, 2 December 2018

External Examiner's Report

Candidate: Amirhossein Akbar Tabatabai

This doctoral thesis investigates two loosely connected topics in the area of Proof Theory within Mathematical Logic.

The first topic contributes to the study of what arithmetical theories can express in a restricted language about their provability predicates. Guided by Gödel's results [12] which give a provability interpretation of intuitionistic propositional logic, the thesis introduces a new concept of *provability models*, and uses this as a tool to provide soundness and completeness for a number of modal logics in terms of provability models with additional properties tied to the modal logic in question, as well as a negative result that such soundness and completeness is impossible for strong modal logics (extensions of so called KD45). Furthermore, the thesis extends Solovay's results by providing soundness and completeness for a number of logics in terms of BHK interpretations in provability models with additional properties tied to the logic in question, plus a negative result that such soundness and completeness is impossible for classical logic.

The second topic contributes to the study of characterising arithmetical theories in terms of computation. Building on a long history of a variety of approaches, that reaches to the present day, the thesis is concerned with characterising the computational content of certain arithmetical theories (denoted *bounded theories of arithmetic* in the thesis) using a new concept denoted *computational flow*. The thesis develops non-deterministic and deterministic computational flows, and applies these to various examples to demonstrate the usefulness of computational flows as a tool.

Contribution

The main contributions of this thesis, in both parts, are to provide new concepts that aim for a unifying approach which distills and builds on exisiting approaches and results.

Athro Arnold Beckmann * Pennaeth yr Adran Gyfrifiadureg * Y Coleg Gwyddoniaeth • Prof Arnold Beckmann * Head of Department of Computer Science * College of Science
Parc Singleton, Abertawe SA2 8PP, Cymru, DU • Singleton Park, Swansea SA2 8PP, Wales, UK
Tel/Ffôn: +44 (0)1792 602370 Fax/Ffacs: +44 (0)1792 295708 Email/Ebost: a.beckmann@swansea.ac.uk

These new concepts will help to deepen the understanding of the areas. It also leads to new results: Besides reproving existing, the thesis provides new provability interpretations of modal logics (K4, KD4, and S4) plus new BHK interpretations in the first part. In the second part, computational flows are used to reprove existing characterisation, and provide new ones for strong theories $\mathrm{PA} + \mathrm{TI}(\alpha)$.

Recommendation

The thesis makes a substantial contribution to the field. I strongly recommend the thesis for acceptance.

Further Comments

Given the nature of the thesis, it would have been appropriate to carefully put the approach taken here into context of existing approaches and results. I assume this will be done in any follow-up publications.

There are some gaps in the exposition of the second part, which I trust will be closed in any follow-up publication. For example, the proof system G3 eliminates certain structural rules at the cost of modifying other rules. The author should carefully check whether the system as it stands is complete, in particular whether explicit contraction for propositional rules is needed or not. This has implications for the proof of Thm 2.3.10, 'only if' direction, which does not consider contraction.

Arnold Beckmann

Prof Arnold Beckmann