Universiteit Utrecht

In the Light of Intuitionism: Two Investigations in Proof Theory

Doctoral Thesis

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Review

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Short Summary

This thesis consists of two attempts to specify a correct interpretation for certain mathematical theories, intuitionistic and modal logics on the one hand and weak arithmetical theories on the other hand. In the first case, the aim is to provide an interpretation of intuitionistic logic in terms of provability, and in the second case the aim is to provide a computational interpretation of proofs in mentioned weak theories.

The problem concerning intuitionistic logic that is addressed is an old and intriguing one. How to give a formal interpretation of intuitionistic logic, where the intuition that intuitionistic truth is provability is adhered to. There have been previous attempts to solve this problem, but all have somehow failed. The most famous one, the Logic of Proofs, is discussed in the thesis and correctly dismantled as not an actual answer to the problem.

The approach developed in this thesis is ingenious and beautiful. The main idea is to clearly distinguish theories and meta-theories, where facts about the former can be proven in the latter. It is shown (Theorem 1.10.9) that intuitionistic logic indeed has an interpretation in these terms. Moreover, similar results are obtained for various other logics.

The issue of the computational content of weak arithmetical theories is approached in a way that uses methods from Skolemization and Herbrand's Theorem. The aim is to model the flow of computational content in derivations. To this end, reduction steps are defined, which are then combined to (computational) flows between formulas. Once the techniques are in place, various results are presented that relate provability in concrete arithmetical theories with this form of computational content.

Evaluation

I can fully certify the high quality and originality of the doctoral thesis of Amirhossein Tabatabai. Let me give some reasons.

The two problems that are addressed are fundamental and the solutions that are provided are useful and insightful. These are new scientific results in the truest sense of the word, which makes the thesis of very high quality.

The problems that the thesis addresses are well-motivated, some are problems that many have thought about before without success. The results are important for the field of logic, in particular for proof theory, (bounded) arithmetic, and philosophy.

In logic, one often encounters solutions to problems that although formally correct are not the intended or meaningful solution. An example of this phenomenon from the literature is given in Section 1.9 of the thesis. In this thesis, on the other hand, all provided solutions are meaningful solutions, which makes the thesis a mathematical as well as a philosophical contribution, which again illustrates its high quality.

A minor point of critique is the absence of a discussion of the rich literature on Skolemization and Herbrand's Theorem, in particular the expansions in the work of Baaz, Hetzl, and Weller. That body of work should have been mentioned and it might have been used to simplify the exposition in Section 2.3.1. But this is a small matter.

The doctoral thesis shows that the author excels in scientific work, is able to contribute in a creative way to science, tackles important problems, has broad interests, and all this without losing sight of the philosophical implications of his work. In short, someone who deserves to receive the title of *doctor*.

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