Abstract

A synthetic approach to the construction of projective geometry, its methods and selected results are given in the proposed thesis. The main historical drawbacks of the original proof of Chasles's theorem for non-developable ruled surfaces and von Staudt's formalization of projective geometry are commented. The corresponding theoretical background is elaborated on visual demonstrations with the accent to interrelations of classical synthetic, axiomatic and analytic points of view. Synthetic methods of projective geometry and their mixture with analytic methods are described on examples including numerous alternative proofs and generalizations of some theorems. A method of four-dimensional visualization is introduced in details. Elementary constructions of images of points, lines, planes and 3-spaces are followed by models of polychora, their sections and shadows. Chasles's theorem is proven for non-developable ruled quadrics on synthetic visualizations, then generalized and proven within the pure projective framework for algebraic surfaces. The synthetic classification of regular quadrics is derived from descriptive geometry constructions of sections of four-dimensional cones and analytically verified in the projective extension of the real space. An integral part of the thesis is a supplementary online book with interactive constructions.