



# COLUMBIA UNIVERSITY

IN THE CITY OF NEW YORK

DEPARTMENT OF INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

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Dear Prof. Dr. Jan Kratochvil:

This letter serves as a review of the Ph.D. thesis of Pavel Veselý entitled *Online Algorithms for Packet Scheduling*.

This thesis considers the following basic scenario. You have packets arriving online, but you cannot schedule them all. You need to decide, on-line (without knowing about future arrivals) which packets to send.

The thesis considers two different problems in this area. In the first, packets have weights and deadlines. At each time step you schedule one packet whose deadline has not yet passed. The goal is to maximize the total weight of packets scheduled. The problem is studied via the competitive ratio, which measures how an online algorithm performs in comparison to an offline algorithm which has complete knowledge of the future. This problem has been studied actively for over 15 years, and there have been many papers on this problem and various restricted versions. The greedy algorithm has a competitive ratio of 2, there is a lower bound of  $\Phi \approx 1.618$ . The best known algorithm is 1.828, and that result was obtained about 10 years ago. There are also many special cases studied, mainly agreeable deadlines and cases where there is a bound on the time interval between the release date and deadline of a packet. In many of the special cases, there is a competitive ratio of  $\Phi$ .

Thus, the question of whether there is an online algorithm with a competitive ratio of  $\Phi$  has been an important open question for a while and there was not much recent progress on this problem.

This thesis resolves the complexity of the problem by giving a  $\Phi$ -competitive online algorithm. The algorithm is new and contains several innovative ideas. The basic idea of the algorithm is to always have a "plan", which is a tentative schedule of packets in the case that there are no new arrivals. When a new packet does arrive, the decision then becomes whether to add it into the plan. The decision is made by considering both the value of the arriving packet and which packet would have to be dropped from the plan if the packet is kept. This explicit combination is new. To actually make this idea work they actually need to modify deadlines in order to achieve a monotonicity problem.

The analysis of this algorithm is rather involved and required mathematical sophistication.

The second model considered in the thesis is called Packet Scheduling under Adversarial Jamming. Here, the packets have different sizes but no deadlines. The channel is now unreliable, and there is an adversary who at any time may interrupt the current transmission by a jamming error. The corrupted packet is lost completely but may be retransmitted immediately or at any

time later. The goal is to maximize the total size of successfully transmitted packets.

Here, the thesis gives online algorithms in a resource augmentation model, which means that he allows his algorithm to run at a faster speed than the offline solution against which it is compared. He proposes an algorithm which is 1-competitive at speed 4, which means that, given a machine that is 4 times faster, it sends as much total packets size as an optimal offline algorithm. They also show that to achieve this performance, a speed of at least  $\Phi + 1 \approx 2.618$  is necessary.

The thesis is well written and clearly organized. The results in it are of sufficient depth and he has shown a mastery of the related work.

I recommend that the Ph.D. degree be awarded.

Regards,



Clifford Stein  
Professor of IEOR  
Professor of Computer Science