

Structural Properties of Graphs and Efficient Algorithms: Problems Between Parameters

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Parameterized complexity became over last two decades one of the most important subfield of computational complexity. Structural graph parameters (widths) play important role both in graph theory and (parameterized) algorithm design. By studying some concrete problems we exhibit the connection between structural graph parameters and parameterized tractability. We do this by examining tractability and hardness results for the TARGET SET SELECTION, MINIMUM LENGTH BOUNDED CUT, and other problems.

In the MINIMUM LENGTH BOUNDED CUT problem we are given a graph, source, sink, and a positive integer L and the task is to remove edges from the graph such that the distance between the source and the sink exceeds L in the resulting graph. We show that an optimal solution to the MINIMUM LENGTH BOUNDED CUT problem can be computed in time $f(k)n$, where f is a computable function and k denotes the tree-depth of the input graph. On the other hand we prove that (under assumption that $\text{FPT} \neq \text{W}[1]$) no such algorithm can exist if the parameter k is the tree-width of the input graph. Currently only few such problems are known.

The TARGET SET SELECTION problem exhibits the same phenomenon for the vertex cover number and neighborhood diversity. In its specialized variant (MAJORITY TARGET SET SELECTION) for neighborhood diversity and twin-cover on one side and modular width on the other side. Currently we are not aware of other result of this type.