REFEREE REPORT FOR THE DOCTORAL THESIS
"Numerical Methods in Discrete Inverse Problems" by Marie Kubinova

The doctoral thesis presented by Marie for obtaining the degree of Doctor of Philosophy at the Charles University deals with several aspects of the numerical solution of linear discrete inverse problems through iterative regularization methods.

The thesis is built around four peer-reviewed publications, and further material is included to complement the topics explored in each paper. The thesis has a nice and coherent structure, and it reads fluently (although a few typos and very minor english usage inaccuracies occasionally appear).

The work contained in Marie's doctoral thesis is original and relevant. Indeed, the thesis contributes to answer important open questions linked with the regularization properties of a popular class of iterative methods (mainly based on the Golub-Kahan bidiagonalization algorithm or on the Lanczos symmetric process). Moreover, some derivations contained in the thesis are useful to further understand the inherent performance of some Krylov subspace methods as linear solvers (beyond the regularization task). The provided analysis is insightful and shows a deep understanding of the researched topics, including shortcomings and possible directions for future work. The surveyed bibliography, as provided at the end of each section, is very complete.

Some of the results obtained within Marie's thesis can be straightforwardly employed when considering several imaging applications, such as image deblurring and tomography (indeed, some realistic and successful numerical tests related to these applications are already reported in the thesis). However, since the point of view adopted throughout Marie's thesis is mostly theoretical, I can expect the new results to be applicable to a wider class of linear inverse problems (which are ubiquitous in many areas of Science and Engineering), such as those arising in geophysical surveying.

In my opinion only the following few points need clarification (some of them are probably typos).

- §3.1, in the middle of p. 333: when stating "It relies on the assumptions that the model (1.1) satisfies the discrete Picard condition [...]", do you mean that only the component b^{exact} of the right-had-side vector is taken into account?
- I think that the image deblurring experiments in §3.1 can be made a bit more realistic by choosing boundary conditions (instead of avoiding boundary conditions by artificially cropping the images).
- §3.2, at the end of p. 53: the discussion is slightly confusing. Perhaps there is a typo, and you mean "becomes steeper if we decrease the number of rows" (instead of columns).
- §4.2.1: even if this subsection surveys results that are already available in the literature, I think that a bit more discussion may have been provided. For instance, a sentence like "Combining (4.4) with other results, we can conclude that orthogonality can be lost only in the directions of converged Ritz vectors" (p.77) is not so transparent to me.

- §5.1: can you dwell a bit more on the reasons behind the choice of the entries of the diagonal matrix in (21)?
- §5.2: according to my understanding, there is no guarantee that the functional in (5.1) is convex for a generic loss function (see the arguments in §5.1). In the numerical experiments of §5.2.2, how do you address the fact that there could be several local minima?

Overall, I am confident in stating that this thesis proves Marie's ability for creative scientific work.

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September 10, 2018