NUMERICAL METHODS IN DISCRETE INVERSE PROBLEMS

by Marie Kubínová

This thesis manuscript is concerned with some aspects of the iterative solution of linear discrete inverse problems such as those arising from the discretization of integral equations or by removing the blur in images. These problems are characterized by the fact that the right-hand side is perturbed by some errors denoted as noise whose amplification spoils the solution when it is computed straightforwardly. The methods which are considered are mainly based on iterative bidiagonalization algorithms.

After a short introduction this work is divided in four chapters which are reproduction of already published papers in journals or proceedings followed by some comments and extensions.

Chapter 2 studies the propagation of the noise in several iterative methods: LSQR, LSMR and CRAIG. When handling discrete ill-posed problems with iterative methods it is necessary to stop the iterations before there is too much amplification of the noise. This is done by using the resemblance of the residual vector with the unknown noise vector.

First, the authors study the Golub-Kahan bidiagonalization algorithm with a perturbation of the right-hand side. Then they prove that LSQR and LSMR residual vectors are given by a linear combination of the bidiagonalization vectors with the coefficients related to the amount of propagated noise in the corresponding vector. For CRAIG, things are even simpler since the residual vector is only a multiple of a particular bidiagonalization vector. The approximation at iteration k is the exact solution of a problem where the right-hand side is a (known) perturbation of the exact right-hand side. Moreover the norm of the residual vector is obtained from the inverse of the amplification factor. This is an interesting result.

Contrary to some previous works on this topic this paper do not make any particular assumptions on the distribution of noise. It can be considered as an extension and improvement of the reference [20] where a white noise was assumed.

Exact arithmetic is assumed and simulated in the numerical experiments using double reorthogonalization.

This paper is an noteworthy contribution to the understanding of algorithms based on bidiagonalization processes for the solution of discrete ill-posed problems.

A discussion of how to simulate exact bidiagonalization using finite precision arithmetic is done in the comments of this chapter.

In chapter 3 it is shown how to estimate the level of noise which is used in some regularization methods, for instance, those using the Morozov principle. An inexpensive method for estimating the unknown white noise level was introduced in reference [11] of this paper. The iteration where the most high-frequency dominated vector is obtained is called the noise revealing iteration. This contribution studies experimentally the applicability of these estimates of the noise level to image deblurring problems with various types and amount of noise.

The authors compare implementations with and without reorthogonalization and demonstrate that reorthogonalization does not improve the quality of the estimate. They show that the performance of the estimator does not significantly depend on the particular type of blur. They also describe cases where the estimator of the noise level has not been reliable. All this is done with carefully designed numerical experiments. The contribution of this paper is to show that these techniques can be used to solve some practical problems.

The comment part of this chapter considers the extension of the noise level estimator to problems with rectangular matrices.

The bidiagonalization algorithms use short recurrences which are prone to the growth of rounding errors in finite precision computations. This leads to losses of orthogonality and delays in convergence. In chapter 4 it is studied how to relate some mathematical entities in the algorithms to their finite precision counterparts in later iterations. The authors consider the Lanczos implementation of the conjugate gradient algorithm (CGL) and the MINRES algorithm for symmetric positive definite matrices. They show that finite-precision CGL residual vectors and Lanczos vectors have to be aggregated over the intermediate iterations (related to the differences in the iteration numbers in the exact and finite precision computations) to form a counterpart to exact entities. They also study how possible stagnation of MINRES influence the correspondence and propose three approaches for the determination of the subsequence of relevant finite precision iterations. These findings are illustrated by interesting numerical experiments.

In the comment part the relationship between the exact and finite-precision Ritz vectors in the Lanczos process is considered.

For some noise characteristics the least squares formulation is not appropriate. In chapter 5 a functional is proposed for problems with mixed noise and also taking care of data corruptions known as outliers. The authors investigate convexity, regularization parameter selection schemes and incorporation of non-negative constraints. The resulting constrained optimization problem is solved with a projected Newton algorithm. They also propose a preconditioner to be used in the inner linear solves with the conjugate gradient algorithm and show that a variant of the GCV method can perform well in estimating regularization parameters in robust regression. The proposals are illustrated by numerical experiments for image deblurring problems.

The comment section discusses the Gauss-Newton algorithm for robust regression problems with some more numerical experiments.

The research presented in this thesis manuscript is original and quite interesting. Some of results increase our understanding of the behaviour of bidiagonalization-based algorithms for solving discrete ill-posed problems. It is particularly interesting to be able to estimate the noise level in practical problems. The papers are well written, the numerical experiments are carefully designed and the results nicely discussed.

Therefore Marie Kubínová certainly deserves to obtain her doctoral thesis since she demonstrated that she has the skills for developing interesting results in linear algebra and, more generally, in scientific computing.

Gérard Meurant, August 28, 2018