



Report on the Ph. D. Thesis
Rich Families of projections and retractions
by Jacopo Somaglia.

Valdivia and Corson compact spaces are two classes of topological spaces that arise naturally, and have proven useful for many purposes, when studying nonseparable Banach spaces. In recent years, the research on this topic has been pushed forward by a new point of view that focuses on skeletons of retractions. This has given rise, in particular, to new classes of closely related compact spaces by looking at what kind of skeletons they possess, like the class of noncommutative Valdivia compacta.

The thesis makes an interesting and substantial contribution to this line of research. The author makes a systematic study of continuous images of some of these related classes, and focuses on studying certain specific constructions, like the so called $[0, \eta]$ -sums, Aleksandrov duplicates and trees endowed with the coarse wedge topology. In this way, the author provides a collection of examples that are (or are not) in these classes, that allow to answer some natural questions. For example, it is a well known key result in this theory that the ordinal interval $[0, \omega_1]$ is the critical example of a Valdivia compact space that is not Corson (in the sense that any other example must contain a copy of $[0, \omega_1]$). It would be a natural conjecture, by analogy, that $[0, \omega_2]$ may be the critical example of non-commutative Valdivia compact space that is not Valdivia. The author's construction on Chapter 3 shows that this is not the case. A more extensive study of compact spaces of the same kind, made from trees, is carried out in Chapter 4, where it is determined for which trees the corresponding compact space belongs to several related classes relevant in this context.



The thesis presents original, interesting and at times technically complicated mathematical research. Part of its content is already published or accepted in two serious mathematical journals such as *Topology and its Applications* and *Fundamenta Mathematicae*. The work meets any international standards for obtaining a Ph. D. degree, and it certainly shows the ability of the author for creative scientific work. The thesis is definitely suitable to be submitted for the final exam.

A handwritten signature in black ink, reading "Antonio Avilés", written in a cursive style.

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