Title: Rich Families of Projections and Retractions

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Abstract: We deal with problems on non-separable Banach spaces and non-metrizable compact spaces. In particular these problems concern Banach spaces with a projectional skeleton and compact spaces with a retractional skeleton. A projectional (resp. retractional) skeleton is a family of continuous projections (resp. retractions) on a Banach (resp. compact) space, which satisfies certain compatibility properties. Banach spaces with projectional skeleton and compact spaces with retractional skeleton can be viewed as non-commutative version of Plichko Banach spaces and Valdivia compact spaces respectively.

The thesis is split into three chapters. Each chapter consists of a submitted/published paper concerning different problems in this area.

In the first chapter, *On the class of continuous images of non-commutative Valdivia compacta*, we investigate the stability of some topological properties in the class of weakly non-commutative Valdivia compacta (i.e. the class of spaces that are image of a non-commutative Valdivia compact space). We deal, among others, with arbitrary products, $[0, \eta)$-sums, Aleksandrov duplication.

In the second chapter, *New examples of non-commutative Valdivia compact spaces*, we characterize compact trees with a retractional skeleton. This characterization answers in the negative the following question:

Let X be a non-commutative Valdivia compact space that does not contain any copy of the ordinal space $[0, \omega_2]$. Is X necessarily Valdivia?

In the third chapter, *On compact trees with the coarse wedge topology*, we investigate in more detail the class of compact trees. We study the properties of Radon measures on compact trees, proving that each tree has the property (M). We characterize compact trees to be Valdivia and finally we prove that $C(T)$, the space of continuous functions on a compact tree T, is Plichko whenever T has height less than $\omega_1 \cdot \omega_0$.

Keywords: retractional skeleton, projectional skeleton, Valdivia compacta, Plichko spaces, tree.