IMPLICITLY CONSTITUTED FLUIDS AND THEIR FLOWS IN COMPLICATED GEOMETRIES

BY ADAM JANEČKA

My cooperation with Adam Janečka started during his undergraduate studies when I supervised his master thesis entitled *The motion of a fluid with pressure dependent material moduli under a surface load*. The topic of his research was numerical simulation of a flow of a piezoviscous fluid, and the problem was solved numerically using a spectral collocation method. (As a part of the solution of the problem Adam has also derived an analytical solution for the surface load of a finite-depth layer of Navier–Stokes fluid.) Adam wrote a nice master thesis, and the results obtained in the thesis were later published in Janečka and Průša (2014). Already at that time Adam has clearly shown his ability to work independently and also to aim at multiple targets – he was simultaneously studying at the University of Economics.

When looking for a topic of his PhD thesis, we decided to focus on flows of implicitly constituted fluids, which was, and still is, a topic intensively studied in our group. (For example papers Průša and Rajagopal (2012) and Bulíček et al. (2012) were published in the same year when Adam started his PhD studies.) The topic of Adam's thesis finally settled down to "Implicitly constituted fluids and their flows in complicated geometries" and Adam started to work. After spending two years at Charles University, Adam was awarded by *Fulbright Scholarship*, and he worked for a year in professor Rajagopal's group at Texas A&M University, College Station, United States. We were expecting a lot from his stay at Texas A&M University, but unfortunately the expectations did not fully materialise because of professor Rajagopal's health problems.

In the mean time the mathematical modelling group at Charles University was contacted by the company Glass Service, which is a company providing expert counselling the the field of glass manufacturing. This started our investigation of the float glass forming process (Pilkington process), and Adam played an important role in the early stages of the project. This amongst other showed Adam's ability to work on practically oriented scientific projects.

After his return from the United States, Adam started to summarise his research results and he started to work on his thesis. The thesis is mainly based on his already published papers Janečka and Průša (2015), Janečka and Pavelka (2018) and his recently submitted paper Janečka et al. (2018).

In the first chapter Adam introduces the so-called implicit constitutive relations and gives references to experimental data that can be explained using this class of constitutive relations. In particular he focuses on fluids that exhibit an S-shaped response in the shear stress/shear rate plot.

In the second chapter he interprets (some of) the implicit constitutive relations as consequences of a non-convex dissipation potential, and he investigates the stability of various branches of the constitutive relation. This part of thesis provides a testimony regarding Adams's fruitful cooperation with Michal Pavelka.

The non-convex potentials have been for a long time studied with respect to the energy storage mechanisms, while nothing has been done, to my best knowledge, regarding the non-convex potentials driving entropy producing processes. Although the mathematical tools remain largely similar as in the energetic case, the entropic case is nevertheless *very interesting and potentially very fruitful field* of research.

The next chapter is devoted to the stability study of some generic flow problems. (Stability of solutions to the complete set of governing equations.) Here Adam uses mainly linearised stability theory and he numerically solves the corresponding eigenvalue problems using a variant of spectral collocation method.

The last chapter is devoted to the development of a numerical scheme for simulation of flows of fluids with implicit constitutive relations. This part of thesis documents Adam's fruitful cooperation with Giordano Tierra. Manipulating the governing equations Adam shows that it is possible rewrite the system of governing equations for an implicitly constituted fluid as a system that resembles the standard Navier–Stokes system coupled with a *single scalar equation for the apparent viscosity*. This manipulation then allows him to develop and implement a relatively simple numerical scheme, and test it in various scenarios. This is an *important achievement*, since it allows one to *in silico* investigate extensional flows in complicated geometries, which is typically a problem that is beyond the reach of analytical methods.

The thesis is concluded by a discussion of other possible approaches to the modelling of the response of fluids with an S-shaped response in the shear stress/shear rate plot.

Besides the aforementioned papers, Adam has also published a paper focused on the mechanics of solids, see Janečka et al. (2016), and a paper on general thermodynamics, see Janečka and Pavelka (2018).

The topic of the thesis was from the very beginning very challenging. It was hard to *a priori* foresee viable research directions in such an interesting field that is still in its infancy. I am glad that Adam Janečka has been able to bring some order to the field. In my opinion his thesis clearly proves his ability to work independently and creatively, to work on long term projects, and to present his results in a concise and well-organised form. For me it was a pleasure to work with him, and I recommend Adam Janečka to be awarded by PhD degree.

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Vít Průša

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