Abstract

Finding a topologically accurate approximation of a real planar algebraic curve is a classic problem in Computer Aided Geometric Design. Algorithms describing the topology search primarily the singular points and are usually based on algebraic techniques applied directly to the curve equation. In this thesis we propose a more geometric approach, taking into account the subsequent high-precision approximation.

Our algorithm is primarily based on the identification and approximation of smooth monotonous curve segments, which can in certain cases cross the singularities of the curve. To find the characteristic points we use not only the primary algebraic equation of the curve but also, and more importantly, its implicit support function representation. Using the rational Puiseux series, we describe local properties of curve branches at the points of interest and exploit them to find their connectivity.

The support function representation is also used for an approximation of the segments. In this way, we obtain an approximate graph of the entire curve with several nice properties. It approximates the curve within a given Hausdorff distance. The actual error can be measured efficiently. The approximate curve and its offsets are piecewise rational. And the question of topological equivalence of the approximate and precise graphs of the curve is addressed and resolved using tangent triangles and axis projections. Special attention was devoted to the study of the behavior of the support function in the neighborhood of a curve inflection. Based on these results, we are able to change the subdivision scheme near the inflections to obtain the optimal approximation order.

The theoretical description of the entire procedure is accompanied by examples which demonstrate the efficiency of our method.