We study physical systems composed of at least two immiscible fluids occupying different regions of space, the so-called phases. Flows of such multi-phase fluids are frequently met in industrial applications which rises the need for their numerical simulations. In particular, the research conducted herein is motivated by the need to model the float glass forming process. The systems of interest are in the present contribution mathematically described in the framework of the so-called diffuse interface models. The thesis consists of two parts.

In the modelling part, we first derive standard diffuse interface models and their generalized variants based on the concept of multi-component continuous medium and its careful thermodynamic analysis. We provide a critical assessment of assumptions that lead to different models for a given system. Our newly formulated class of generalized models of Cahn–Hilliard–Navier–Stokes–Fourier (CHNSF) type is applicable in a non-isothermal setting. Each model belonging to that class describes a mixture of separable, heat conducting Newtonian fluids that are either compressible or incompressible. The models capture capillary and thermal effects in thin interfacial regions where the fluids actually mix.

In the computational part, we focus on the development of an efficient and robust numerical solver for a specific isothermal model describing incompressible fluids. The proposed numerical scheme, which is based on the finite element method, partly decouples the system of governing equations on the level of time discetization. We carefully discuss the advanced design of the preconditioner for the computationally most demanding part of the scheme, given by the system of incompressible Navier–Stokes equations with variable coefficients. The numerical scheme has been implemented using the FEniCS computing platform. The code capable of running parallel 2D and 3D multi-phase flow simulations is available in the newly developed FEniCS-based library MUFLON.