Opponent's Review on Dissertation Thesis

"Structure and Approximation of Real Planar Algebraic Curves" by Eva Blažková

Eva Blažková's dissertation thesis, entitled *Structure and Approximation of Real Planar Algebraic Curves*, is focused on finding a topologically accurate approximation of a real algebraic curve newly based on a more geometric approach. The main research contribution of the thesis is the proposition of a novel algorithm for rational approximating of algebraic curves with the minimal approximation error and as well as the evaluation of the suitability and the correctness of the obtained topology. The importance of the topic is evident from the frequent use of approximate techniques in Computer Aided Geometric Design.

The thesis is made up of an introduction, four main chapters, and a concluding section. After a brief description of the dissertation's content, Chapter 2 provides the background to the study of planar curves. Basic notations, definitions, and the state of the art with respect to the parametrized curves, algebraic curves, and the techniques for the approximate parametrization are presented. All the fundamental properties of studied curves are accompanied by illustrative examples. Sometimes I was surprised that some underlying theory was explained rather briefly (for instance Dual Curve in the section 2.3.4 or Resultant and Discriminants in the section 2.3.5). The first appearance of some notations isn't obvious at first sight (for instance on page 17, Example 4 – the coordinates of normal vector). In addition to the approximate parametrization I would also appreciate at least a short section here about the algorithm for the exact rational parametrization of the algebraic curves having genus 0. On the other hand, I have to say that all relevant information and specific methods are cited so the reader can use the references.

Chapter 3 is dedicated to the study of the local properties of algebraic curves at a given point, specifically the branches going through a singular point of a curve. The first part of the chapter presents the Duval's algorithm for computing rational Puiseux series with author's own contributions. This part consists of the vast amount of information, definitions and propositions related to the construction of the Puiseux tree. The second part of the chapter describes the geometric properties of a curve which can be derived from the so called singular Puiseux tree. I would put more emphasis here on distinguishing the author's contributions and the adopted results from work previously done by others. The last part of the chapter newly introduces the solution of an interesting inverse problem using the so called qualitative Puiseux tree, i.e. the construction of an algebraic curve with the prescribed singularities of given properties. The construction steps are described in detail and are illustrated by an example. The implementation of rational Puiseux series as a package in Mathematica software together with the detailed documentation of this package represent a nice addition to the thesis.

The main contribution of the thesis is in Chapters 4 and 5. In Chapter 4 a novel original algorithm for finding of the global topology of a real planar curve based on the decomposition of the curve into smooth monotonous segments is presented. This new algorithm uses an identification of characteristic points such as boundary points, extremal points, inflection points, and cusps. The first part of the chapter addresses the existing approaches to this problem and this in turn prepares the basis for the next sections and the description of a new method. Next, the decomposition of a boxed curve into smooth monotonous segments is determined. The third part of the chapter analyzes all types of points of a curve where the monotonicity or smoothness of the curve segment can be broken. The

determination of characteristic points is clearly demonstrated by two examples. The algorithm of the geometric nature connecting the found characteristic points is introduced and discussed in the fourth part of the chapter. It is also demonstrated by the specific examples. Finally, a new concept for verifying the correct topology and reducing the area with the topology problems to arbitrarily small is provided. The main results of this chapter were published in two internationally recognized journals.

Chapter 5 is focused on the approximation techniques of algebraic curves. Firstly, support function based approximation of the curve segments is introduced. The advantages of this method are discussed in detail. The second part of the chapter deals with the G^1 approximation of a curve using circular splines. Next, the G^2 approximation is studied. These three parts of the chapter follow two journal publications of the author. The last very interesting part of the chapter is devoted to the approximation of inflection points. The author provides an original analysis of the support function behavior in the neighborhood of an inflection point and suggests an interpolation scheme to receive the best possible approximation order 4 also for the inflection points. The results regarding the inflection points and their approximation are submitted in the proceedings paper to the CAGD journal.

The summary of the main contribution and results of the research is given last. Some notes regarding the possible directions for future work in the field of approximate techniques could be briefly proposed.

Author's Publications:

The results of the author's research have been published in the internationally recognized journals and in the proceedings of the international conferences. The dissertation thesis contains significant novel results which is evident from the two published papers in CAGD and in LNCS which are listed in the Scopus database and a paper in CAGD also in WOS.

Specific Comments and Questions:

- On page 12, in Example 1: Why isn't the Tschirnhausen cubic given by its specific parametrization regular everywhere?
- Even though it is not the scope of the thesis, it would be convenient to shortly mention the main techniques and approaches how to compute rational parametrization of a rationally parametrizable algebraic curve at least for the comparable reasons.
- In which sense besides the geometric nature is your connectivity algorithm (in section 4.4) better than the mentioned existing types of approaches? Could you discuss a little bit more the advantages of your approach? Have you conducted any comparative evaluation for specific types of curves?
- Is there any other approach to the approximation of a curve inflection which preserves the approximation degree besides applying a non-uniform subdivision scheme?
- Could any of your approach of the approximation lead to the approximation algorithms for certain surfaces used in Computer Aided Geometric Design?
- In the introduction of the thesis there is a remark: "the G^2 interpolation in Chapter 5 is studied using hypocycloids and epicycloids". Could you explain it as it is not clear to me since the hypocycloids and epicycloids aren't mentioned later in the text of the Chapter 5?
- Can you see any direction in which it would be promising to continue regarding the topic of approximation of planar algebraic curves?

General Comments:

The dissertation thesis is compact nevertheless it covers a lot of information of the area studied. The necessary preliminaries are mentioned in the introductory parts which allow the reader to understand and to orient in the topic quickly. Too, every chapter starts with the particular brief introduction. New ideas and methods are well presented, and represent a clear contribution to the approximation of real planar algebraic curve. Nevertheless, I dare to express my criticism regarding the thesis which pertains to the lack of any deeper discussion about the thesis's merit and its impact, I would expect the discussion of the contribution of suggested methods at the end of the both Chapters 4 and 5. More detailed description of the research goals would be appreciated too. While reading the thesis it wasn't always clearly evident which propositions were taken from the literature and which were original or in which sense they were changed and improved by the author. However, as the contribution of the thesis is of a high quality and sufficient for a doctoral dissertation, my criticisms should be taken only as the suggestions for future work.

Overall, the topic of the thesis is significant for further development in this research field. The author proved her capability regarding systematic research work and the high quality in the presenting scientific results.

The dissertation thesis is written in good English with only a few typos (the word "through" seems to be problematic), the grammar and spelling are very good; all source publications are cited correctly and are complete. The overall layout of the thesis is systematic, logical, and clear. The text is accompanied by very well chosen examples and illustrations.

Conclusion:

The thesis contains significant novel results and scientific material of high quality therefore I recommend the thesis for the defense. After the successful defense I strongly recommend that Eva Blažková is awarded the Ph.D. degree.

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