Report on the Thesis of

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Bonn, September 18th 2018

UNIVERSITÄT BONN
Rheinische Friedrich-Wilhelms-Universität

18.09.2018

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Argumentation in mathematics teachers' beliefs and practices in the context of the Czech Republic

This thesis is an investigation of the question how teachers' beliefs and practices influence the way of mathematical argumentation. The research was conducted in lower secondary mathematics classroom within the Czech curricular context. The author presents her results in two separated research reports (Study I and Study II) and three case studies. First she provides a characterization of the Czech curricular context, in particular, the national curricular document and aspects of justification of mathematical statements in selected series of mathematics textbooks, second a report on characteristics of teachers' beliefs and practices as related to argumentation at the example of six teachers, via interviews and observations of their lessons. For the case studies, she selected three teachers, which are supposed to have differing approaches to teaching and describe specific observed instances in their practices in relation to classroom norms regarding argumentation, justification of general mathematical truths, and aspects of arguments. In this part, the candidate tries to explicate how teachers' beliefs, textbooks and pupils as they are may influence the observed arguments. Here she uses a dichotomy between efficiency (fulfilling school curriculum demands) and sense-making. The study shows that these different goals lead to distinct curricula. The teachers see justification of general truths generally as important but do not relate clear cognitive aims for the pupils with it.

The thesis begins with an inviting introduction with an anecdotical personal account of a classroom situation about the division by zero in a high school in Chicago.

In total, the author divides her thesis into five chapters. The first chapter provides a theoretical framework. Argumentation, justification, warrant and proof in mathematics classrooms are described according to theories of Bell, de Villiers, Ernest, Hanna and others. Tolumin's model of argumentation is presented and applied, although maybe mathematical reasoning, arguing, proving, etc. is more subtle and different from general argumentation. Especially mathematical justification is expanded in the aspects modes of reasoning, cognitive engagement, planned and enacted arguments (Harel and Sowder). The chapter ends with a quick tour through Paul Ernest's philosophy of mathematics education and argumentation. The research questions then ask for the role of argumentation and justification with respect to 1) curricular context, 2) teacher orientations, 3) actual classroom practices.

The author distinguishes in this first chapter between proof in mathematics and mathematics education (p.17), in which she follows in particular Stylianides et al. (2013). To the reviewer, this appears to be a dangerous dichotomy. The example of division by zero from the introduction is taken on in this chapter in order to explain this difference. The question, why not to divide by zero is answered by two seemingly different arguments, the first is supposed to be more acceptable for first-year students and the second being apparently more acceptable in year 9 (cf. p. 18). Argument 1 sounds: "We do not divide by zero because it is impossible to divide anything into zero parts." This arguments is problematic since zero is – like all notions in mathematics – an abstract or noetic notion. And why should it not be possible to divide anything into zero parts? Certainly zero can be divided into zero parts. In some sense you can go zero times to school. Would it be a valid argument to say that there are no negative numbers since there is no room being empty after two people entered?

Argument 2 is that division by zero would cause a contradiction, when considering the sequence of fractions: 1/0.1 = 10; 1/0.01 = 100; 1/0.001 = 1000; etc. and on the other side: 1/−0.1 = −10; 1/−0.01 = −100; 1/0.001 = −1000, etc. This is supposed to be inconsistent. But why should these sequences behave continuously? Is here a reason for $1 \div 0$ not to exist? The “non-division by zero” is certainly not just a convention as stated on p. 22.

Maybe the only valid argument in this case is valid *because* it is valid in mathematics: Why not divide by zero? Let's try a definition. For a number x we put $x \div 0 = a$ for some number a . Then it follows $x = 0 \cdot a$. This cannot be – unless $x = 0$. And here it is indeed possible to define the division by zero! But the problem is that there are many choices. Calculus can be seen as the science of the division by zero where you have all these choices.

This example shows that it s certainly not sufficient to let a mathematical argumentation depend on the socio-mathematical norm aspect of mathematical justification by letting the acceptance of some argumentation within a community to be crucial. Right from the beginning of any mathematical activity, there is also a mathematical norm, that becomes more and more explicit and precise. Didactical reduction requires intellectual honesty, too.

The example of Yackel (p. 16) for instance is an example of a compression of a mathematical procedure (in the sense of David Tall) rather than an example for a change of social factors. And hopefully, the teacher does not argue just on the basis of her/his believes (p. 30).

The author uses this example also to attribute at least two different cognitive objectives to a teacher: a) the development of conceptual understanding of particular content and/or b) the development of the ability to argue and reason mathematically (p. 23). While the first objective can be considered as central for mathematical teaching, the second can be questioned, since there is no “ability to argue and reason mathematically” as such. Of course, we can enhance the student's experience with mathematical argumentation and reasoning. We can even teach som heuristics. But this does not mean that he can deliver this kind of general ability. Mutatis mutandis, this assertion holds the same for competencies in chapter 2 as they are analyzed in the curricular programs with a reference to Kilpatrick and Niss (p. 62f).

Despite these reservations to the separation of mathematics and mathematics education and to the related socio-mathematical norm aspects, the reviewer considers the authors attention and observations about argumentation in the classroom to be very important and will interpret the findings as was explicated above. Also very helpful for the later analysis of the teacher case studies are the interest groups of Paul Ernest, which indeed help to clarify the influence of certain attitudes on mathematics teaching.

Chapter 2 then is concerned with the first study on the curricular context of justification in Czech lower secondary school mathematics. First the author explains the specific Czech school situation and the different roles of the several curricula therein. She then analyzes the formulation of different (key) competences in these curricula as far as justification/ argument is concerned. The method here is just to select passages on the role and intended function of arguments (p. 39). The differentiation of a curriculum into three manifestations: the intended curriculum; the implemented curriculum; and the attained curriculum as it is usually ascribed to Goodlad, Klein and Tye (1979) could have been useful here to structure this analysis a little more.

The method for the analysis of textbooks is more subtle. It starts with a survey on comparable investigations in other countries and textbook-based comparative studies. Then the author describes an elaborated framework for the investigation of the school books. She especially analyzes how they deal with six topics: a) Non-division by zero, b) The zeroth power, c) Square root of zero and negative real numbers, d) *Minus times Minus*

is Plus, e) Division of fractions, f) Area of a circle. Moreover she applies a framework of Stacey and Vincent (2009): a) The presence of mathematical justification, b) Participation, c) Modes of representation and number of arguments, d) Modes of reasoning.

The results of the school book analysis are illustrated in Tables 2.1 – 2.4 and they give a differentiated impression on how Czech textbooks deal with argumentation and justification. The findings are varied.

Chapter 3 presents the results of the inquiries to Study 2: Argumentation in teachers' beliefs and practices. The study of mathematics education beliefs, and in particular on their beliefs about mathematical argumentation, was conducted with six lower secondary teachers.

The chapter starts with a detailed survey on research on argumentation in teachers' beliefs and practices, argumentation as observed in teachers' practices, beliefs about the role of argumentation and justification in a classroom, teachers' beliefs and practices, argumentation and its characteristics, teachers' beliefs about pupil dispositions, teachers' pedagogical content knowledge and resources .

The method is a multiple-case study according to Yin (1994). The researcher designed a sequence of interviews, woven around a sequence of lesson observations. In section 3.4 on Study 2A, we find a very detailed analysis of the respective six teacher personalities (p. 83-115), which is worthwhile for its own sake. The six candidates were moreover classified in terms of Ernst's interest groups (Figure 3.2, p. 84).

Chapter 4 then looks into some more detailed questions as they arose from Study 2. This Study 2B strives for a better understanding of argumentation in teachers' classrooms by three case studies. The most difficult part in such a study of concrete lessons is to have a systematic data analysis. For this the author distinguishes some terminology that was already introduced in Section 1.1.1: (mathematical) argument, claim, warrant, modes of representation, modes of reasoning and social norms in the classrooms.

With this elaborate framework, she analyzed the three textbooks used by the teachers, the transcripts of observations of lessons, that were moreover analyzed for episodes of argumentation. The identified episodes of argumentation were divided into individual arguments and identified warrants, modes of reasoning, and modes of representation in order to determine differences between arguments. Moreover, she tried to classify the observed arguments and looked at the teachers' own comments about particular arguments, warrants, reasoning or representations in class. During interviews she attempted to gain insight into the beliefs behind their decisions.

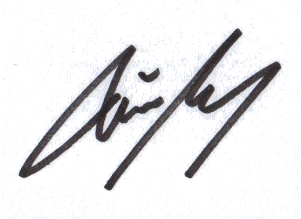
The results of Study 2B give a deep insight into the activities concerning argumentation in the class room as is shown in section 4.3 (p. 121-160). The analysis of all these data must have been an enormous work. The discussion of the results in section 4.4 (p. 160-170) is elaborate as well.

In the conclusion in chapter 5 the author extracts from the results the main findings on argumentation with respect to general mathematical statements and their justification, problem solving, sense making, efficiency, ways of thinking, understanding and justifying. In the appendices, we find some materials and tables for the orientation of the reader.

To summarize this review: The research questions are interesting and touch urgent problems of mathematics teaching since argumentation and proof are threatened in the mathematics teaching of many countries. The thesis contributes to the questions about the role of argumentation in mathematics teachers' beliefs and practices in the context of the Czech Republic in a substantial way. This kind of research questions cannot be tackled by simply doing quantitative empirical research. Therefor, the candidate chose an appropriate methodology by applying procedures of qualitative research and went through the tedious

process of analyzing many different kind of data. The result is a very precise view on the status of argumentation in Czech mathematics teaching.

The thesis shows moreover, how much mathematics education depends on national and local circumstances. Supporting mathematics teaching by the science of mathematics education can hardly mean to find general insights for all cultural and local situations, all forms of schools and times, that then have to be implemented in a top-down manner. To the contrary, these branches of mathematics education, that really support teachers and help to understand their working circumstances by developing a better understanding of what is going in classroom is tedious and needs great staying power. This thesis is a step forward in this long march and satisfies the standards that have to be imposed on scientific research in mathematics education on a high level. Particularly, it impresses by its extensive and precise (multiple)case studies.



(Prof. Dr. Rainer Kaenders)