Abstract

Graph communication protocols are a generalization of classical communication protocols to the case when the underlying graph is a directed acyclic graph. Motivated by potential applications in proof complexity, we study variants of graph communication protocols and relations between them.

The main result is a comparison of the strength of two types of protocols, protocols with equality and protocols with a conjunction of a constant number of inequalities. We prove that protocols of the first type are at least as strong as protocols of the second type in the following sense: For a Boolean function \( f \), if there is a protocol with a conjunction of a constant number of inequalities of polynomial size solving \( f \), then there is a protocol with equality of polynomial size solving \( f \). We also introduce two new types of graph communication protocols, protocols with disjointness and protocols with non-disjointness, and prove that the first type is at least as strong as the previously considered protocols and that the second type is too strong to be useful for applications.