Ramsey theory looks for regularities in large objects. Model theory studies algebraic structures as models of theories. The structural Ramsey theory combines these two fields and is concerned with Ramsey-type questions about certain model-theoretic structures. In 2005, Nešetřil initiated a systematic study of the so-called *Ramsey classes of finite structures.* This thesis is a contribution to the programme; we find Ramsey expansions of the primitive 3-constrained classes from Cherlin's catalogue of metrically homogeneous graphs. A key ingredient is an explicit combinatorial algorithm to fill-in the missing distances in edge-labelled graphs to obtain structures from Cherlin's classes. This algorithm also implies the *extension property for partial automorphisms (EPPA),* another combinatorial property of classes of finite structures.