Bachelor thesis is divided into four chapters. In the first chapter essential facts about the brachistochrone problem are presented, especially its history and solution. Second chapter concerns specific methods of solution that can be found in literature. Classic methods as well as less common solutions are described. In the third chapter theoretical calculations including numerical results in specific cases are presented and other problems encountered in solving the brachistochrone problem are discussed. The last chapter is devoted to experiments. Firstly, energy losses during rolling with slipping were investigated. Secondly, a time comparison for balls rolling along three real curves was performed. For each experiment, a video or photo is available on the enclosed DVD. The thesis is suitable for students of teaching physics but it may also serve as motivation for students in compulsory courses of physics in high school.