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Review of the PhD thesis

Maxwell-type viscoelasticity in small and large deformations of planetary mantles
by Vojtěch Patočka

The thesis submitted by Mr. Patočka represents an in-depth study of the dynamics of viscoelastic deformation in the interior of terrestrial bodies. Over a wide range of geologically-relevant timescales, the cold upper part of planetary lithospheres responds elastically to surface and internal loads, while the deeper and hotter mantle exhibits a ductile behavior and deforms as a viscous fluid, leading to an effective viscoelastic deformation of the mantle-lithosphere system. Although it has long been recognized that elasticity and viscoelasticity are essential to describe fundamental geophysical processes such as plate bending under volcanic loads or the Earth's glacial isostatic adjustment (GIA), much less attention has been devoted to the effects of elasticity on the large-scale convective dynamics of the mantle. With this work, Mr. Patočka succeeded in presenting a comprehensive description of viscoelasticity from its theoretical foundations, through its role in controlling the small deformations associated with the GIA over thousands of years, to its effects on the large deformations associated with mantle convection over millions or even billions of years.

The thesis is well written and clearly structured. After an introduction discussing the motivations and goals of the work, a detailed and lucid analysis of the theoretical foundations of Maxwell viscoelasticity is presented in Chapter 1. This chapter clearly reveals that Mr. Patočka spent a great deal of time and effort to master the (not simple) principles at the base of the numerical models he discusses later. Even without being an expert in the theory of continuum mechanics, I could fully appreciate the need to construct a more general time-derivative of the stress tensor satisfying the principle of objectivity in order to deal with large viscoelastic deformations. The extension of the classical material time derivative is obtained following two approaches, a traditional one based on mechanical considerations and a more modern one based on the entropy principle. The limited number of references that appear in this chapter makes the reader think that the author himself elaborated a significant part of the material presented. Even if this was not the case, the chapter would still be a very valuable and useful work of synthesis; perhaps it could have been useful to spell out more clearly whether and where there are significant novelties in the treatment of the theory.

The main results of the thesis consist of two independent investigations related to the GIA (Chapter 2) and mantle convection (Chapters 3 and 4) that are based on the use of two different numerical codes. For the first, the author employed an Eulerian spectral code in a spherical shell geometry. This code solves the conservation equations for the small deformations of a Maxwell-type viscoelastic medium induced by surface loading, coupled with the (non-linear) Liouville equation controlling the rotational response induced by the deformation. Mr. Patočka investigated in detail the largely overlooked problem of the energetic consistency of the GIA formulation for a rotating planet. He showed that linearizing the Liouville equation, a widely employed approximation in the GIA-community, violates energy conservation. On the contrary, he proved that energy is conserved when the fully nonlinear Liouville equation is solved. Although the linearization barely affects the solution for the displacement field, it may lead to a significant error in the evolution of the polar motion and should thus be used with care. The results of this work were published in Geophysical Journal International.

As far as mantle convection is concerned, Mr. Patočka implemented the effects of elasticity into an existing and well-developed code already able to treat viscous and viscoplastic flows. With this addition, this code is now probably the first within the geodynamics community that is able to deal with visco-elasto-plastic convection at a global scale in various geometries. Mr. Patočka applied this quite unique tool to two important problems. First, in the framework of simple models of stagnant-lid convection, he simulated the convective cooling of a mantle with a Maxwell viscoelastic rheology from an initial hot state (Chapter 3). He demonstrated that the elastic lithosphere can preserve a "memory" of the initial bending stresses induced by the underlying convection for several billions of years. This is an important result with possibly far-reaching consequences. It suggests that the stress state of the stagnant lid may contain information on the thermal evolution of the mantle, and that the way the elastic lithosphere deforms in response to surface or internal loads is affected by its own previous deformation history. These results are contained in a second paper that was also published in Geophysical Journal International.

In addition, Mr. Patočka addressed the problem of the transition from stagnant lid to plate tectonics using visco-elasto-plastic convection models (Chapter 4). In order to develop lithosphere-scale shear zones that ultimately lead to surface deformation, these models generally require the use of an unrealistically low yield stress with respect to the well-constrained (and high) strength of the lithosphere. Searching for mechanisms that can affect such critical yield stress (positively or negatively) is an active area of current research. Mr. Patočka obtained the somewhat negative - yet significant - result that the effects of elasticity are basically irrelevant with respect to the maximum critical yield stress that allows the initiation of surface deformation. In the same context, he also analyzed the effects of the free-surface, which, in a previous study, was suggested to be able to significantly increase the critical yield stress. In contrast, he showed that these effects are also minor and that the previously reported claim that the free surface facilitates the initiation of surface deformation was due to a peculiar choice of both initial and boundary conditions that tended to promote stress accumulation and facilitate plate failure.

The thesis is concluded by a summary of the work and a discussion of its possible developments. The author suggests to apply his models of viscoelastic stagnant lid convection to carry out realistic 3D simulations of specific bodies, of Mars in particular. Given the relatively large amount of data available, this is certainly a promising application as the evolution of the martian lithosphere might indeed help to better constrain the overall thermochemical evolution of the mantle. He further proposes that the effects of elasticity on the development of surface mobilization may become important if combined with more sophisticated descriptions of the brittle and ductile behaviour of the lithosphere. In contrast to the simple pseudo-plastic flow models used in the thesis, more complex lithospheric rheologies can generate narrow shear zones where elastic stresses accumulated in the lithosphere could be released, thus promoting more deformation. This also represents a promising development that could help bridging the gap between regional- and planetary-scale deformation models.

I believe that this thesis and the accompanying publications are an important contribution to the literature on mantle dynamics. Although numerical models of GIA and viscoelastic mantle convection share a common theoretical ground, they are generally used by different research communities to address different problems. I found it very impressive for a PhD student to be at ease with both modelling approaches and their mathematical foundations, and to be able to produce significant research results in both fields. This work demonstrates that Vojtěch Patočka is a creative and promising scientist and I fully recommend it to by accepted as a PhD thesis.

A series of specific comments and questions for the author is listed below:

1. In Section 1.2, a geometrical argument is introduced to propose the lower convected time derivative as a preferable choice for Maxwell viscoelasticity. Yet for the numerical simulations presented in Chapters 3 and 4, the Jaumann time derivative is employed as in most studies of viscoelastic convection. What motivated this choice? Should one expect significant differences in the simulation results if different convected derivatives were used? Is there any experimental and/or theoretical approach that could be used to select a "most suitable" derivative, at least for a specific problem?

2. The numerical techniques employed in GIA and mantle convection modelling are substantially different. Spectral techniques solving for the displacement field and limited to small deformations are typically adopted in the first case, while finite (volume, element, or difference) techniques solving for displacement rates and allowing for large deformations are used in the second. Since "small deformations" are obviously a subset of large ones, shouldn't one try to unify the two approaches? To what extent could the small deformations of GIA problems be reproduced with a viscoelastic flow model? For example, what would be necessary to reproduce the results of Chapter 2 with the finite volume convection code?

3. For the implementation of viscoelasticity (Sect. 3.2.3), the author discusses about the possibility to choose two time steps (as proposed by Moresi et al., 2003): an "elastic" one for the stress advection and the standard one for the advection of temperature and tracers. The author concludes that this is actually not necessary and that choosing the time stepping according to the CFL criterion is sufficient. Also, the "numerical viscosity" $Z\eta$ depends on the amplitude of the time step. Given the non-linearity of the problem, one may think that the choice of Δt could actually affect the solution to some extent. Did the author conduct convergence tests that prove the application of the CFL criterion to be sufficiently accurate? It is mentioned that the Courant number is set to 0.5. Is this a requirement of the advection scheme? What motivated this choice?

4. The advection of the stress tensor is obtained using either the grid-based donor-cell method or via tracers, with the second option employed in the simulations of thermal convection. However, to reproduce the viscoelastic convection benchmark of Harder (1991), the author reported the need to use up to 1000 tracers per cell. According to this figure, for high-resolution or 3D simulations, the total number of tracers could quickly become prohibitively large. How many tracers were used in the thermal convection runs (Sect. 3.4.1)? Again, did the author carry out convergence tests to verify whether the number of tracers was sufficient?

5. Two stagnant-lid models are described in Sect. 3.4.1 based on Earth-like and Mars-like parameters. However, what controls the outcome of the corresponding simulations is probably the choice of the reference viscosity and the resulting convective vigor. Since these simulations are quite simplified, it might have been more appropriate to perform a parameter study simply based on different Rayleigh and Deborah numbers. Furthermore, these simulations are meant to describe the elastic response of the lithosphere in a mantle "that cools down from an initially hot state". However, these are actually steady-state (or statistically steady-state) simulations with basal heating and without cooling (no decaying heat sources nor core cooling is prescribed). The lithosphere thickens or shrinks just because convection adjusts the thermal boundary layers to the effective Rayleigh number of the system, which should happen quite rapidly. Therefore I found it somewhat surprising that the initial thickness of the upper thermal boundary layer has a very large influence on the stress pattern in the lithosphere even after long simulation times. In particular, no detailed explanation is provided as to why no significant difference in the lithospheric stress field is observed between viscous and viscoelastic models

initialized with a thick boundary layer of 300 km (Figure 3.6b). Can the author comment on these observations?

6. The importance of the “memory effect” in viscoelastic convection is shown to depend crucially on the absolute viscosity of the lithosphere or, in other words, on its Maxwell time. The author is clearly well-aware of this fact, which is mentioned several times in the thesis. Yet the actual viscosity of the lithosphere is difficult to determine and, as far as numerical models are concerned, it is often defined simply by a cut-off preventing it from growing to the huge values predicted by the Arrhenius law. Does the author have some ideas regarding how to solve or make progress on this issue? Could the interaction with experimentalists help in this sense?

Sincerely,