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Opponent review on the Habilitation Thesis of Mgr. David Krejčiřík, Ph.D.

The (cumulative) habilitation thesis submitted by Mgr. David Krejčiřík, Ph.D. collects contributions of Mgr. David Krejčiřík, Ph.D. and his co-authors in the field of mathematical analysis and mathematical physics. It is divided into three key groups:

1. Toy models: 8 papers
2. Waveguides: 8 papers (1 preprint on ArXiv, 2 accepted paper)
3. Pseudospectra: 5 papers (1 preprint on ArXiv, 1 accepted paper)

Except for the papers [36] (5 pages) and [38] (2 pages), all others are co-authored. All papers (except the two preprints) have appeared in prestigious international journals; all of them are peer-reviewed and at least two of them appeared in very top level journals (Trans. Amer. Math. Soc. and Phys. Rev. D).

On 25 pages (Introduction and the first two chapters), Mgr. David Krejčiřík, Ph.D. describes the results achieved in the above papers and their mutual connections.

The thesis is aiming at the investigation of spectral properties of non-self-adjoint operators. This is strongly motivated by new concepts in quantum mechanics where observables are represented by non-self-adjoint operators. Here two concepts are the driving force: Quasi-Hermitian quantum mechanics and $\mathscr{P}\mathscr{T}$-symmetric quantum mechanics. The focus is on similarity to self-adjoint and normal operators and an explicit description of the so-called metric operator for explicit solvable models in 1-d (toy problems) and, subsequently, in 2-d and arbitrary dimension (waveguide). Moreover, this investigation is amended by a subtle use of pseudospectra and pseudomodes in the description of non-self-adjoint features (pseudospectra).

1. **Toy models.** The results in this group are motivated by the idea to provide a solvable model for Quasi-Hermitian quantum mechanics. In essence the provided toy model is just the second derivative on a bounded interval. But with a very (non-self-adjoint) boundary condition of Robin-type. This model is at the same time $\mathscr{P}\mathscr{T}$-symmetric. The main contribution of Mgr. David Krejčiřík, Ph.D. (and his co-authors) is to give an explicit description of the metric operator, i.e., the transformation which turns the non-self-adjoint model via similarity into a self-adjoint one. In a sequel of papers different formulas for the metric operator were developed and the model was extended to curved manifolds (which is an interesting result for its own) and to operator matrices of Pauli type. This kind of ideas were also utilized for the investigations of the Laplacian on finite metric graphs. Solvable models are seldom in quantum mechanics and especially when it comes to non-self-adjoint problems. It is an extremely valuable contribution to provide such an interesting model class as toy problems.

2. **Waveguides.** Of course it is very natural to enlarge the above toy problems to higher dimensions. This was done by Mgr. David Krejčiřík, Ph.D. (and his co-authors) for a two-dimensional infinite strip. Moreover, this operator was studied when the waveguide tends to zero. In addition, numerical simulations were used and higher dimensions were considered. Here the spectrum is approximated via a perturbation scheme. Especially the obtained eigenvalue asymptotics are very impressive. Moreover, other models were considered like the Laplacian with complex delta interaction or electromagnetic Schrödinger operators with complex potentials. Papers in this group concentrate on the description of spectral properties.

3. **Pseudospectra.** The last significant contribution of Mgr. David Krejčiřík, Ph.D. is connected with pseudospectra. This concept was used to rigorously describe non-self-adjoint features of Quasi-Hermitian and $\mathscr{P}\mathscr{T}$-symmetric operators. In [61] it was shown that the eigenfunctions of the of the imaginary cubic oscillator – a very popular and intensively investigated operator in the $\mathscr{P}\mathscr{T}$-community – are complete but do not form a Riesz basis. More precisely, Mgr. David Krejčiřík, Ph.D. (and his co-authors) showed that the corresponding metric operator is bounded with an unbounded inverse. This implies that the corresponding operator is not quasi-self-adjoint. Consequently, the model is not relevant quantum-mechanically – a very remarkable result. This was done by using a very simple argument involving trivial and non-trivial pseudospectra. The developed technique was then subsequently applied to Schrödinger operators with an imaginary sign potential on the real line or to Schrödinger operators with complex potential.

Mgr. David Krejčiřík, Ph.D. is for sure able to conduct independent scientific research in the field of mathematical analysis and mathematical physics. This clearly emerges from the first chapters of the submitted Habilitation thesis and the papers he submitted. In the first two chapters he gives a concise and very understandable presentation of his results and the relation to the existing literature. This is also reflected in many references.

Therefore, I explicitly recommend the appointment of Mgr. David Krejčiřík, Ph.D. as an associate professor. I am sure he will be an excellent choice for such kind of position.