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Report on the habilitation thesis manuscript entitled
Mathematical aspects of quantum mechanics with non-self-adjoint operators ,
submitted by Dr David Krejčířík in view to obtain the title “docent” at
Charles University, Prague

The content of the thesis presented by Dr David Krejčířík concerns various features and properties of non-self-adjoint Hamiltonian operators, as they might appear in quantum mechanics (essentially) and in other domains of physics, e.g., classical electrodynamics.

The almost 500 page manuscript is impressive by the number of articles (19) or chapter of a book, published over the last 11 years, and independent of the 2012 Krejčířík's PhD thesis.

After analysing the works and achievements presented in the manuscript, one has to agree with the author when he claims that

his principal objective of the thesis is to contribute to the new area of physics by providing a mathematically rigorous approach for a correct implementation of the interesting idea and by resolving some of the puzzles with help of standard as well as unconventional methods of modern operator theory.

As a matter of fact, the constant leitmotiv is indeed to set on a firm basis a certain number of results or intuitions developed by various authors since the advent of the “non-hermitian” wave in quantum formalism, more or less situated in 1992 with the Scholtz, Geyer, and Hahne paper. Of course, non self-adjoint operators were studied at the early stage of quantum physics, and the subtle distinction between symmetric and self-adjoint unbounded operators is or should be part of the learning of any student in functional analysis and its application to Schrödinger operator and other observables of the quantum formalism.

The range of questions examined by Dr. Krejčířík about non-self-adjointness was first centered around the spectral properties of those Hamiltonian operators which present the so-called \mathcal{PT} -symmetry, the reality or not of some part of the spectrum being of physical interest in view of possible measurement. Moreover the role played by boundary conditions is crucial in these studies. Reality of the spectrum of non self-adjoint operators may result from similarity transforms to self-adjoint operators, and the questions of their existence and explicit formulations is highly non-trivial when one deals with unbounded operators. It is evident that Dr. Krejčířík has been developing over the last twelve years a strong competence on these topics, actually not only on them, but also on the very promising notion of pseudospectrum. Moreover, the set of potentials and boundary condition, whether toy or more

realistic, considered in the presented works, is amazingly large and various. One feels a constant concern about explicit examples in order to give substance to the underlying abstract formalism.

The 482 pages manuscript presented by the applicant is divided into one introductory part with presentation of results, and the set of 15 reprints, 3 papers to appear, 2 preprints, and 1 chapter in a book. The introductory part is a clear, concise, and well documented description of the publications which form the content of the manuscript. Of course, it would be useless to describe in a detailed way the content of each one of those publications, and the fact that most of them have been already published in high level international journals in mathematics or in mathematical physics proves their scientific validity. I really appreciated the pedagogical skills of the applicant in explaining in a so synthetical and clear way the content of his achievements, namely (by fully endorsing his claim) :

- new models for which the similarity transforms are found in a closed form
- proof of absence of Riesz basis property for the popular imaginary cubic oscillator and other models
- analysis of quantum graphs with non-self-adjoint boundary conditions together with a new classification
- non-semi-classical approach for obtaining pseudonodes of Schrödinger operators with complex potentials.

These original results follow from a strong and constant collaboration with P. Siegl and others specialists and form the content of an impressive (I insist) list of publications in international peer reviewed journals. In this regard, I note the strong positive influence of the applicant for motivating young researchers on all these topics.

Since 19 of these papers have been already published or accepted, and since his contribution to each one of these works is clearly central, I estimate that Dr David Krejčířík has reached that high international level of competence and expertise on the studied subjects which allows him to obtain the title "docent" at Charles University.

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