Report on Premysl Jedlicka's Habilitation Thesis

"Commutative automorphic loops"

The applicant presents 6 papers in his thesis on the results obtained in the area of commutative automorphic loops. 3 of presented papers are joint work with coauthors Michael Kinyon and Petr Vojtechovsky, who are considered leading researchers of loop theory.

A loop – a quasigroup with neutral element – is called automorphic loop, if every inner mapping is an automorphism. This class includes for example groups and commutative Moufang loops.

The study of automorphic loops was initiated in the '50-es by Bruck and Paige, who described basic properties of this class, namely the power-associativity of loops, and they analized the connection of middle, right and left nuclei.

The most important part of the dissertation is "The structure of commutative automorpfic loops". The author together with M. Kinyon and P. Vojtechovsky constructing a new loop operation – namely a Bruck loop – managed to prove the Lagrange and Cauchy theorems for commutative automorphic loops. Their main results in this paper are that every commutative automorphic loop is solvable, and the decomposition theorem that every finite commutative automorphic loop is a direct product of a subloop of odd order in which every element has order a power of 2.

The consequences of these nice results are that a finite simple commutative automorphic loop is either a cyclic group of odd prime order or it has exponent 2. They showed even that a commutative automorphic loop of exponent 2 has order a power of 2.

The structural examinations opened the question of the existence of a nonassociative simple finite automorphic loop. By before mentioned result if there exists a nonassociative finite simple commutative automorphic loop, then it has exponent 2. This problem is open so far.

The candidate with his coauthors managed to construct many commutative automorphic loops of exponent 2, even they characterized all commutative loops with middle nucleus of index 2, then every commutative automorphic loop with the same property. Central extension of commutative automorphic loops are described too.

The third very important part of the dissertation with P. Vojtechovsky and M. Kinyon, the study of central nilpotency of finite commutative automorphic p-loops with an odd prime p. First they proved that a finite commutative automorphic loop has order a power of p, if and only if its elements has order a power of p. Then with the associated Bruck loop, using the properties of this loop class discovered by Glauberman, they were successful in proving that really

every finite commutative automorphic p-loop is centrally nilpotent with arbitrary odd prime. This result can not be extended to $p = 2$, because of the existence of a loop of order 8 of such kind with trivial center. The construction of this loop also belongs to our applicant.

The dissertation contains a paper about the study of commutative automorphic loops of order pq, using a construction introduced by Ales Drapal. I have to mention too the study in connection with semidirect extensions of middle nuclei of commutative automorphic loops by abelian groups, then extensions of order 3 and 5.

The applicant's statements are new, matematically precise, deep and nice. He has been working with well-known leading researchers. In some cases the authors needed computer aid for the proof. Some of the new results were presented in lectures on international conferences.

A proof for the importance and the relevance of these examinations are that the obtained results are published in international journals of high standard, – Journal of Algebra, Transactions of American Mathematical Society, Communications in Algebra, further the very high number of citations specially to those papers that concern the structure and the construction of commutative automorphic loops.

I recommend definitely the habilitation process of Premysl Jedlicka, further I recommend that the applicant should be appointed as an associate professor.

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