Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Yield curve dynamics: Co-movements of latent global and Czech yield curves

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Declaration of Authorship

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Abstract

This thesis focuses on yield curve modelling. It estimates unobserved “global” yield curve factors which drive changes in real individual yield curves. Yield curves of USD, GBP, JPY and EUR are considered and global factors are able to explain a substantial part of their variances. The method is built on a Nelson-Siegel model which is implemented in a state-space form to be able to extract the unobserved yield factors. The estimated global yield factors are further used in a time-varying regression to explain the evolution of the Czech yield curve. The results show that the impact of the global factors is stronger during the years of the interventions of the Czech National Bank and thus suggests that the interventions help to transmit the global low interest rates to the Czech economy.

JEL Classification	C51, C58, E43
Keywords	Yield curve modelling, Czech yield curve, Nelson-Siegel model, FX interventions, latent factors

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Abstrakt

Tato diplomová práce se věnuje modelování výnosových křivek. Odhaduje nepozorovatelné faktory „světové" výnosové křivky, které dokáží vysvětlit změny ve výnosových křivkách různých zemí. Pro její odhad byly použity výnosové křivky USD, GBP, JPY a EUR s výsledkem, že globální faktory dokáží vysvětlit velkou část jejich pohybů. Metoda je postavena na modelu Nelson-Siegel a je použita ve formě stavové rovnice, aby dokázala odhadnout nepozorovatelné faktory výnosových křivek. Odhadnuté globální faktory jsou následně použity pro vysvětlení pohybů české výnosové křivky. Jejich vliv na české úrokové sazby je odhadován jako regrese s pohyblivými koeficienty. Její výsledky ukazují, že vliv globálních faktorů je vyšší během intervencí České národní banky a tedy naznačuje, že intervence pomohly přenést globální nízké úrokové sazby do českého hospodářství.

Klasifikace	C51, C58, E43
Klíčová slova	Modelování výnosových křivek, česká výnosová křivka, Nelson-Siegel model, devizové intervence, nepozorované veličiny

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[1] The source of all the tables and figures: The author's calculations

Acronyms

AR Autoregression
Bp Basis point (0.01%)
C Number of currencies in global model (4)
CEE Central and Eastern Europe
CNB Czech National bank (the central bank for the Czech Republic)
CZK Czech Koruna
ECB European Central Bank
EU European Union
EUR Euro currency
FX Foreign exchange
G-7 Group of 7 advanced countries (Canada, France, Germany, Italy, Japan, United Kingdom, USA)
GBP British Pound sterling
GDP Gross domestic product
GYC Global Yield Curve
IBOR Interbank Offer Rates
IRS Interest rate swaps
JPY Japanese Yen
LHS Left-hand side
Max. Maximum
Min. Minimum
n Number of tenors in the models (14)
N-S Nelson-Siegel model
OLS Ordinary least squares
QE Quantitative easing (monetary expansion by a central bank)
RHS Right-hand side
Std. dev. Standard deviation
Std. err. Standard error
UIRP Uncovered interest rate parity
UK United Kingdom
US(A) United States (of America)

USD US Dollar

VAR Variance

YC Yield Curve

Master's Thesis Proposal

Author:	Bc. Jaromír Šimáně
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Proposed Topic:

Yield curve dynamics: Co-movements of latent global and Czech yield curves

Motivtion:

Thorough understanding of yield curve dynamics is important for financial institutions' as well as for sovereign governments' policymaking. Moreover, the shape of a yield curve express market expectation about future economic growth and inflation as is documented in Diebold et al. (2006).

The wide range of studies (e.g. Kaminska et al. (2011) or Zhu and Rahman (2009) among others) shows us that the evolution of yield curves of different countries are linked. In-depth insight of this co-movements can help us to better diversify the interest rate risk; or to evaluate the governmental policy (whether the change of yield curve is imported from global changes or whether it is the result of improved/worsened governmental policy).

The aim of the thesis is to estimate the evolution of unobserved global yield curve through dynamic Nelson-Siegel approach described in Diebold et al. (2008); then estimate the evolution of Nelson-Siegel parameters of the Czech yield curve. The following step will be to find out whether the global yield curve significantly influence the Czech yields and, if so, estimate how this influence evolved over time.

Regarding the influence, focus will be made on certain dates that could have changed the magnitude of the influence. These are e.g. the announcement of US and European quantitative easings or Czech currency intervention in November 2013.

The method described above can help us to understand to what extent the Czech yield curve is affected by domestic and to what extent by global factors. Moreover it can show us which part of YC is affected more and which less. The current findings (e.g.: Kaminska et al. (2011) or Hoffmaister et al. (2010)) tell us that the co-movements of long yields (duration about 10 years) are stronger than co-movements of short yields (duration below one year). This thesis will show us whether it has also been the case of the Czech YC.

Several other studies estimated the global yield curve using method similar to Diebold et al. (2006). E.g.: Bae and Kim (2011), Diebold et al. (2008) and Zhu and Rahman (2009). They all proved the existence of global yield curve.

Considering central Europe, only a few studies focused on this region; Hoffmaister et al. (2010) and Sopov, Seidler (2010) are among exceptions. Sopov and Seidler use the same method as I will use but focus solely on the dynamics of regional /Central European/ yield curves. Hoffmaister et al. (2010) also decompose the dynamics of central European yield curves to theirs Nelson-Siegel parameters, however, they do not estimate the latent (global nor regional) factors as I will do. Instead of latent GYC they use the EUR yields and focus on relationship between macroeconomic variables and yields.

Hypotheses:

1. Hypothesis #1: Global yield curve exists and can partially explain variance of individual yield curves
2. Hypothesis #2: Czech yield curve is positively influenced by the global yield curve
3. Hypothesis #3: The influence is increasing in time and is stronger after CNB's 2013 intervention

Methodology:

I will estimate the dynamics of unobserved "global yield curve" (GYC) from the evolution of major sovereign yield curves. I need the model to be estimable and parsimonious, so I will employ the method by Nelson and Siegel (1987), and will simplify yield curves to just 3 parameters – their levels (corresponds to duration of some 10 years), their slopes (corresponds to the difference between durations of 10y and 3 months) and their curvatures (the duration in between).

I will describe the model by state space equations that enables to create the evolution of latent GYC from the observable sovereign yield curves. The model will not only be able to estimate the evolution of all 3 parameters of GYC but also the load of particular sovereign yield curves to GYC.
I will employ the method of Kalman filter to estimate the state space models. The above described method will mostly tracks Diebold et al. (2008)).

Subsequently, I will separately estimate the evolution of 3 Nelson-Siegel parameters for the Czech yield curve during the same period as GYC, and then will examine what portion of variance of the Czech yield curve can be explained by GYC. Afterwards will investigate whether this proportion is stable or evolving over time. I will specifically focus on certain event that could have changed the relationship between Czech and global yield curves. This events are e.g. Czech FX intervention start of quantitative easings in Europe or USA.

Considering data, I do not want to deal with sovereign credit risk (default risk), so will use sovereign bond yields of countries that are considered to be practically risk free. I will used credit default swap data or independent ratings to indicate whether the country bonds are thought to be virtually without risk of default. To estimate the GYC, I should use yield curve of countries that have developed financial markets and have a strong impact on world financial markets. The candidates are e.g: the USA, Germany, the UK and Japan.

For estimation to be feasible, the data frequency will be at least on a monthly basis (as in Diebold and Li. (2006)), however, if computationally feasible, the weakly or daily data would be preferable.

Expected Contribution:

I will update older estimations of global yield curve dynamics (e.g. Diebold et al. (2008), Bae,Kim (2011) or Zhu and Rahman (2009)) with newest data. Besides, I will find out to what degree the Czech yield curve is affected by domestic factors and to what degree by global yields using method by Diebold et al. (2008). Also, will focus on whether the relation between the Czech and global yield curve is stable or evolving in time. I will focus on specific occasions made by central banks and find out how these events affected global and Czech yield curve dynamics.

Outline:

1. Motivation: to what extent is Czech yield curve affected by domestic factors and to what extent is it affected by global changes? What was the impact of CNB FX intervention on the relationship?

2. Other studies. I will describe other studies about the yield curves co-movements that use either the same or different method that I will use.

3. Data: I will explain the characteristics of raw data and, possibly, how I will have adjusted raw data before inputting them to the model.

4. Methodology and theory: I will explain why different yield curves should move together and why they should not. Then I will describe Nelson Siegel approach; how latent global yield curve will be extracted from national yield curves; and briefly present the Kalman filter. Also I will show how I will deal with the special events from central banks.

5. Results: I will present the results, discuss them, and check their robustness.

6. Conclusion: I will summarize the research, discuss the potential policy implications and point out where the further research can continue.

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Author **Supervisor**

1 Introduction

Thorough understanding of yield curve movements is important for financial institutions' as well as for sovereign governments' policymaking. Moreover, the shape of a yield curve expresses a market expectation of future economic growth and inflation as is documented in Diebold et al. (2006). The wide range of studies (e.g. Kaminska et al. (2011) or Zhu and Rahman (2009) among others) shows that the evolution of yield curves of different countries are linked. In-depth insights of these co-movements can help to better diversify interest rate risk or to evaluate governmental policy. These insights can be won by analysing whether the changes of yield curves are imported from global changes or whether they are the result of improved/worsened governmental policy (the focus here lies on the former).

The objective of this thesis is to estimate latent global yield curve factors, which are able to explain a substantial part of the variance of individual currencies' yields. The estimation of global factors closely follows the method developed by Diebold et al (2008). The estimated global yield factors ("global model") are further used to explain the evolution of the Czech yield curve (in the "Czech model"). The impact of the global factors is modelled as time-variant and is thus able to show an evolvolution over time. Special focus lies on the years when the Czech National Bank intervened in the exchange rate with the EUR. The increased strength of the global factors in explaining the Czech yields is consequently interpreted in the light of the theory of the "impossible trinity". The complex, global model is also compared with a simpler, one-currency model ("EUR model"). The Czech model is used as an example to cmpare the strength of the global model.

The thesis is structured as follows: Chapter 2 summarises the relevant literature, Chapter 3 presents the Nelson-Siegel model of the yield curve structure. It subsequently presents a "global" model which is used for an estimation of latent global yield curve factors and a "Czech" model that estimates the impact of the global yield curve on the Czech one. How the model is calibrated for estimation is described in Chapter 4. The next Chapter is describing the input data, which are used for the model estimation together with a preliminary analysis. Chapter 6 shows the results of the entire model together with their tests, and also an alternative model where only a EUR yield curve is used instead of the global one. The last part of chapter 6 summaries the results and Chapter 7 concludes.

2 Relevant literature

This thesis follows the method of parsimonious modelling of yield curves defined by Nelson and Siegel (1987), which was further developed by Diebold and Li (2006) and Diebold et al. (2008). The aim of this thesis is to estimate a multi-currency latent yield curve and then to focus on one currency in the region of Central and Eastern Europe (CEE) - the Czech Koruna. Consequently, this literature review is structured as follows: firstly, it covers papers which take the same (or similar) approach(es), then multi-currency models, and finally CEE countries.

Nelson and Siegel (1987) derived a method on yield curve modelling; the method is parsimonious and requires only 3 parameters to model a yield curve for all maturities. The model can describe "shapes generally associated with yield curves: monotonic, humped, and S-shaped" (Nelson and Siegel 1987, p.473). When they applied the method to US data (1981-1983), the model explained 96% of variation with a median deviation of 7 basis points.

Two decades later, Diebold and Li (2006) used the model of Nelson and Siegel (1987) and slightly adjusted it to improve estimation and interpretation of parameters. They interpreted the 3 Nelson-Siegel parameters as "level", "slope" and "curvature" and estimate them for every month in the period 1985-2005 using US bond data. The results were used for forecasting the yield curve with the result that: "our models produce one-year-ahead forecasts that are noticeably more accurate than standard benchmarks" (Diebold and Li, 2006, p.2). This result was confirmed in Koopman et al. (2007) where the forecast using Diebold's and Li's (2006) method outperformed more sophisticated models.[2]

Diebold et al. (2006) used the method derived in Diebold and Li (2006) and improved the yield curve modelling by adding macroeconomic variables. Their contribution, which is also used in this thesis, is that the evolution of Nelson-Siegel parameters is not estimated step-by-step by OLS but rather at once in a state-space model. This is better because: "The two-step procedure […] suffers from the fact that the parameter estimation and signal extraction uncertainty associated with the first step is not acknowledged in the second step." (Diebold et al. 2006, p.313).

[2] The overview can be found in Koopman et al. (2007, p29, table 6)

A further contribution of Diebold et al. (2006) stems from the fact that they related Nelson-Siegel (NS) parameters to macroeconomic variables and helped us to understand NS parameters more intuitively. Their result, using US data, is that the Level correlates with price inflation (43%) and Slope with capacity utilisation (39%). In vector autoregressive modelling they showed that: “the effects of the yield curve on the macro variables are less important than the effects of the macro variables on the yield curve.” (Diebold et al. 2006, p.326).

Broader results of relationships between macro variables and yield curve factors can be found for example in Koopman and Wel (2011) or Hautsch and Ou (2012), which are described below. Other papers also studying this phenomenon are described later in the contex of multi-currency studies.

Albeit Koopman and Wel (2011) did not use the Nelson-Siegel model they also decomposed the yield curve into 3 factors - Level, Slope and Curvature and reached results that were similar to those of the Nelson-Siegel model. Using US data of yield curves and 110 macro-variables, they found out that Level and Curvature correlate mainly with macroeconomic variables related to employment and housing sales; whereas Slope mainly correlates with inventories, orders and prices.

Hautsh and Ou (2012) continued the work of Diebold and Li (2006) and augmented the method. They not only estimate Level, Slope and Curvature but also their volatilities. Using US bond data, 1964-2003 they found out that capacity utilisation significantly correlates with all factors and also with volatilities of Level and Curvature factors. Regarding the Slope, their results are in line with Diebold et al. (2006), however, they are not regarding Level. Inflation does not significantly correlate with Level, just volatility of Level does.

Multi-currency studies

Diebold et al. (2008) build on the method derived in Diebold and Li (2006). They use the state-space structure derived in Diebold et al. (2006) and elaborate it further to estimate the evolution of latent global yield curve factors. They use yield curves from four big economies with high credit rating (USA, UK, Japan and Germany) from the period 1985-2005. Using a Kalman filter, they estimated the evolution of Nelson-Siegel parameters of a global yield curve. The resulting global parameters can explain more than half of the variance of national parameters. When they compared them with global economic factors, they observed that global Level highly correlates (75%) with the average inflation of G-7 countries, while global Slope slightly (27%) correlates with average G-7 growth.

The method of Diebold et al (2008) is very influential and several papers build on their method. Zhu and Rahman (2009) for example use the method and adjusted it to an “arbitrage-free” model for higher theoretical consistency. They focus on market integration between the UK, US, Germany and Japan using the Markov-switching method. Their result is that the integration was rather stable during the observed period.

Another paper referring to the work of Diebold et al (2008) was written by Bae and Kim (2011). They estimated the global yield curve (YC) from individual yield curves of big advanced economies and an “East-Asian” regional YC of Japan, South Korea, Hong-Kong and Singapore. They found out that even when controlling for regional YC, the Japanese, Hong-Kong and Singapore YC are mainly driven by the global YC (regarding the Level factor). A similar observation holds true for the global Slope factor.

Byrne et al. (2015) use the method derived from Diebold et al. (2008) and estimate global yield factors from data of seven advanced economies from the period 1994-2014 (so they are among few who use data from after the 2007/8 financial crisis). The resulting factors are able to explain “more than half of the variation in the bond yields of seven advanced markets” (Byrne et al. 2015, p.1). When comparing with macro factors, they found out that inflation is the most important factor driving the changes of the yield curve.

The study of Kaminska et al (2011) is among those, which do not use the parsimonious model of Nelson-Siegel but a “No-arbitrage affine term structure” model instead. Despite using a different method they still found two global factors driving yield curves of UK, US and the Euro area currencies and they related these factors to economic variables. The result is that global inflation and global economic activity are related to global YC factors; while monetary policy rates are linked to domestic YC factors.

CEE countries

The rest of the literature review focuses on the yield curve modelling of Central European countries. Both presented papers are written in the tradition of Diebold and Li (2006). Hoffmaister et al (2010) estimated global yield factors (but differently than Diebold et al, 2008) and euro area yield factors and then observed their impact to the Czech, Hungarian and Polish YC factors. When using data before and after ascension of the selected countries to the EU (2000-2008) they found out that the relationship is high: “Shifts in the euro yield curve are transmitted both to interest rates and inflation expectations in the CEE countries and transmission is stronger after 2004.” (Hoffmaister et al, 2010, p.1). A special result was found for the Czech Republic - the correlation of its yield curve with the euro curve is “consistently larger than in Hungary and Poland.” (Hoffmaister et al, 2010, p.26).

Sopov and Seidler (2010) used the method of Diebold et al (2008) and estimated regional latent yield curve factors from the Czech, Polish, Slovak and Hungarian YCs. Using data from 2003-2008 they confirmed the somewhat special behaviour of the Czech YC (as mentioned above) within the CEE region when they found out that it does not load on regional factors (neither level nor slope). This is in contrast to other CEE countries which load significantly to yield curve factors.

3 The model

Co-movement of yield curves

In the world of free capital flows, there is a tendency for yields, of the same risk, to be equal in different countries. As is also documented in Kaminska et al (2011, p.3): if a yield in country A is higher than in country B and investment in both countries carries the same risk, people will rather invest in country A until they compress the local yield to the Level of country B; and thus make both yields equal.

This, however, is not what exactly happens in the real world due to several reasons. Methodologically, it is impossible to exactly assess the risk of each investment and thus to be perfectly informed. Also, "home bias[3]" exists as an investment in different countries is connected with exchange rate risks and also with the transactional costs (e.g.: money exchange and transfers).

The fact that complete co-movements do not usually occur can be documented by testing (and often rejecting, e.g. Fama (1984)) the Uncovered Interest Rate Parity (UIRP). The theory UIRP predicts that if a foreign interest rate is higher than a domestic one, the foreign currency should depreciate (during maturity of that interest rate) and make the investment in both countries equally profitable. However, "it is well documented that currencies in high interest rate countries have tended to appreciate on average" (Kaminska et al, 2011, p.3).

Nelson-Siegel model

Nelson-Siegel (1987) propose a model that can fit a whole yield curve using just 3+1 parameters. I use a variation of the Nelson-Siegel model (further as N-S) that is described in Diebold and Li (2006). Their variation is better than the original model in two ways – as Diebold and Li (2006, p.6) explain: firstly, it allows to interpret resulting β-coefficients as Level, Slope and Curvature and secondly, it eases the estimation -

[3] Defined in Barr and Priestley (2004)

shapes of the original Nelson-Siegel coefficients are more similar to each other and can thus produce multicolinearity (Diebold and Li, 2006, p.6). The variation of the N-S model can be described by equation (X1), which is a variation of Diebold and Li (2006, p.8), where it is in its dynamic form.

$$y_t(\tau) = \; l_t + s_t\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + c_t\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{X1}$$

$y_t(\tau)$ is a yield given maturity τ in period t; l_t, s_t and c_t are parameters for Level, Slope and Curvature respectively; and λ_t is a parameter influencing where the loading for Curvature reaches its maximum. To make the model more parsimonious, λ_t is modelled as a static parameter throughout this thesis and thus it is used without time subscript in the next equations.

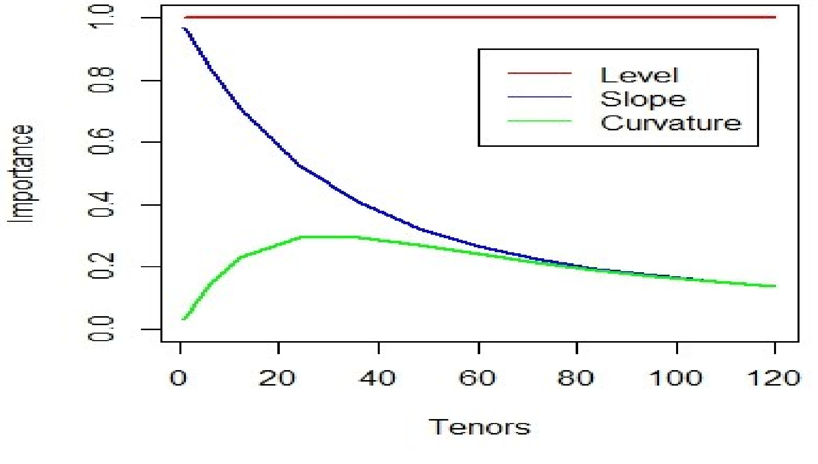


Figure 1 illustrates the importance of factor loadings with respect to maturity. Constant lambda = 0.0609 is used.

The model can be used statically to fit a certain yield curve or dynamically, as presented in this thesis, to estimate the evolution of N-S-parameters over time.

Figure 1 helps to better understand the importance of each factor loading with respect to maturities. It can be seen that Level loads uniformly to the yield curve and, as it is the only factor loading which does not decay to zero in longer maturities, it can be interpreted as a long-term yield (as in Diebold and Li, 2006, p.5). Preliminary results using OLS (see data part) show that the estimated value of Level is very similar to an average of 10-year yields for all currencies.

Slope loads the most on short yields and then its impact exponentially decays to zero. The difference between Level and Slope can be interpreted as short-term yield (as in Diebold and Li, 2006, p.5). It can be seen, in the data part that the difference between estimated Level and Slope is for all currencies very similar to an average 3-monts yield; for all currencies.

Curvature loads the most on medium maturities and is zero for both short and long maturities. Where exactly Curvature reaches its peak is influenced by the Lambda parameter. When Lambda = 0.0609 is used as Diebold and Li (2006) propose, Curvature loading reaches its maximum for a 30-months tenor. Other studies fix Lambda similarly. Bae and Kim (2011) use Lambda = 0.0609; Diebold et al (2006) use 0.077. Koopman et al (2007) let Lambda to be time-varying. Their results for Lambda in different models range from 0.06 and 0.12.

In this thesis I mainly used post-2007/8-crisis data (the dataset ranges from the beginning of 2005 to autumn 2017), which shows a different behaviour than data used in older studies. I calculated an average throughout all dates for every currency's yield curve and also, an average of these averages. Table 1 shows the optimal lambdas for each currency's yield curve average and the optimal lambda for the pooled dataset (average of currency's averages). I rounded the results for the pooled dataset and chose lambda for the entire model to be 0.01. It can be seen in Annex A1 how this suits all the currencies.

	USD	GBP	JPY	EUR	CZK	Pooled dataset
Optimal lambda	0.009	0.016	0.002	0.010	0.015	0.009

Table 1 shows which values of the lambda coefficient fit the best to the yield curves of selected currencies and also to the average yield curve of currencies used in the global model (USD, GBP, JPY, EUR)

Nelson-Siegel model in State Space form

A series of N-S-parameters can be estimated, step-by-step, e.g. by a least square estimator. However, as presented hereunder, a different and stronger method can be used. Diebold et al. (2006, p.312) suggest imposing a vector-autoregressive structure describing the evolution of N-S-parameters. The structure is as follows:

$$\begin{pmatrix} l_t - \mu_L \\ s_t - \mu_S \\ c_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l_{t-1} - \mu_L \\ s_{t-1} - \mu_S \\ c_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(l) \\ \eta_t(s) \\ \eta_t(c) \end{pmatrix} \tag{X2}$$

where μ_L, μ_S and μ_C are mean values of N-S-parameters, a_{11}, $a_{12} \dots a_{33}$ are autoregressive parameters and $\eta_t(.)$ are updates (shocks) to the N-S parameters.

When the vector auto-regression structure, as described in equation (X2), is used, then the model (X1) can be estimated for all observations at once. To maintain this, the following equation, as in Diebold et al (2006, p.313) is used:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ 1 & \vdots & \vdots \\ 1 & \vdots & \vdots \\ 1 & \vdots & \vdots \\ 1 & \frac{1-e^{-\tau_n\lambda}}{\tau_n\lambda} & \frac{1-e^{-\tau_n\lambda}}{\tau_n\lambda} - e^{-\tau_n\lambda} \end{pmatrix} \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix} \tag{X3}$$

where $\varepsilon_t(\tau_i)$ are model residuals in time *t* of tenor τ_i, i = 1,2…n

Equation (X3) is estimated together with (X2) by a Kalman filter; equation (X3) is then called a measurement equation and (X2) a state equation of the Kalman filter.

When all N-S-factors are estimated together using the Kalman filter, the results should be superior to those obtained by OLS-step-by-step estimation. The reason is that the filter employs residual-information from one yield-curve estimation to another (Diebold et al, 2006, p.313). In contrast, the estimation using OLS treats estimations individually and thus does not employ any information from one estimation to another.

To enable proper estimation, the residuals $\varepsilon_t(.)$ and the updates $\eta_t(.)$ should be independent of each other (as in Diebold et al, 2006, p.313).

Global Nelson-Siegel model

Diebold et al (2008) further develop the state space model of Diebold et al (2006) and make a model that is able to estimate a latent Global Yield Curve (GYC) from "local" yield curves. The shape of the GYC can be described by equation X4 below (variation of Diebold et al, 2008, p. 352),

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \qquad \text{(X4)}$$

where $Y_t(\tau)$ is a latent global yield given maturity τ, in period t and L_t, S_t as well as C_t are global yield factors for Level, Slope and Curvature respectively.

It can be seen that the equation (X4) is just a variation of equation (X1) where global N-S-factors are used instead of local factors.

The estimation – an introduction

The whole model is built in two steps: the first one is a global model and the other one is a Czech model.

The global model estimates global yield factors, not in one single point in time, but evolving over time. Only two global factors are used in the model, namely global Level and global Slope. As the global yield curve is latent and cannot be directly observed, global factors are estimated from a real dataset of four currency yield curves. The whole global model is estimated in one step for every period of time.

The global model assumes that the yield curve of each currency depends on both global and idiosyncratic factors. The global factors are evolving over time but the loadings on them are assumed to be constant. The currency-specific factors can vary over time.

The whole global model needs to estimate:

- *2* autoregressive parameters for global Level and Slope
- *2C* autoregressive parameters for all currency's idiosyncratic factors
- *2C* for variances of their updates
- *2C* loadings to global factors (loadings to the global Level and Slope for every currency)
- *nC* variances for the measurement equation for all *n* tenors of all *C* currencies

Those sums to 2 + 6C + nC parameters and, as C=4 currencies and n=14 tenors are considered, then 82 parameters are to be estimated.

The Czech model estimates how global yield curve factors affect the Czech yield curve and how this impact evolves over time. It thus uses the evolution of global factors estimated in the global model as its input and then it estimates how the Czech yields loads to global factors. Contrary to the global model, the loadings on global factors have more flexibility and can move over time. The model thus allows us to see how the impact of the global yield curve on the Czech one evolves over time.

The Czech model, like its global counterpart, considers idiosyncratic yield curve factors that are able to evolve over time. Overall the Czech model uses two sets of input variables – the Czech data itself and the estimated global factors. Both inputs are used in a single estimation using a Kalman filter.

The Czech model needs to estimate 2 autoregressive parameters for loadings to global factors and another *2* for the variances of their updates. Then also 2 autoregressive parameters are needed for CZK idiosyncratic factors and 2 for their variances. Additionally, n=14 variances are estimated for each tenor so in total 22 parameters need to be estimated.

The estimation – detailed description

It can be seen in table 3, showing the results of the principal component analysis, that around 99% of the variance of the data can be explained by just two components. Following these results and keeping in mind parsimoniousness, I just used two components – global Level and global Slope. The same parsimony was used e.g. in Diebold et al. (2008).

The general model thus shrinks to:

$$Y_t(\tau) = L_t + S_t\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) \tag{X5}$$

And is estimated in a way that is a variation of Diebold et al (2008, p 355):

$$\begin{pmatrix} y_{USD,t}(\tau_1) \\ y_{USD,t}(\tau_2) \\ \vdots \\ y_{GBP,t}(\tau_1) \\ \vdots \\ y_{JPY,t}(\tau_1) \\ \vdots \\ y_{EUR,t}(\tau_1) \\ \vdots \\ y_{EUR,t}(\tau_n) \end{pmatrix} = B \cdot \begin{pmatrix} L_t \\ S_t \end{pmatrix} + A \cdot \begin{pmatrix} \varepsilon^l_{USD,t} \\ \varepsilon^s_{USD,t} \\ \varepsilon^l_{GBP,t} \\ \varepsilon^s_{GBP,t} \\ \varepsilon^l_{JPY,t} \\ \varepsilon^s_{JPY,t} \\ \varepsilon^l_{EUR,t} \\ \varepsilon^s_{EUR,t} \end{pmatrix} + \begin{pmatrix} \upsilon_{USD,t}(\tau_1) \\ \upsilon_{USD,t}(\tau_2) \\ \vdots \\ \upsilon_{GBP,t}(\tau_1) \\ \vdots \\ \upsilon_{JPY,t}(\tau_1) \\ \vdots \\ \upsilon_{EUR,t}(\tau_1) \\ \vdots \\ \upsilon_{EUR,t}(\tau_n) \end{pmatrix} \tag{X6}$$

where $y_{USD,t}(\tau_i)$ is USD interest rate of maturity τ_i in time *t* for *i=1,2…n.* USD data are followed by GBP, JPY and EUR data. There is one row for each tenor of each currency's yield and thus the dependent vector has dimension of 4*n* rows.

A and *B* are auxiliary matrices defined as in Diebold et al (2008, p.355):

$$A = \begin{pmatrix} 1 & \left(\frac{1 - e^{-\tau_1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) & 0 & \cdots & 0 \\ 1 & \left(\frac{1 - e^{-\tau_2 \cdot \lambda}}{\tau_2 \cdot \lambda}\right) & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & \cdots\left(\frac{1 - e^{-\tau_n \cdot \lambda}}{\tau_n \cdot \lambda}\right) \end{pmatrix}$$

The dimension of matrix A is $Cn \times C = 56 \times 4$

And dimension of matrix B is $Cn \times 2 = 56 \times 2$

$$B = \begin{pmatrix} \beta^l_{USD} & \beta^s_{USD} \cdot \left(\frac{1-e^{-\tau_1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \vdots & \vdots \\ \beta^l_{GBP} & \beta^s_{GBP} \cdot \left(\frac{1-e^{-\tau_1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \vdots & \vdots \\ \beta^l_{JPY} & \beta^s_{JPY} \cdot \left(\frac{1-e^{-1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \vdots & \vdots \\ \beta^l_{EUR} & \beta^s_{EUR} \cdot \left(\frac{1-e^{-\tau_1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \vdots & \vdots \\ \beta^l_{EUR} & \beta^s_{EUR} \cdot \left(\frac{1-e^{-\tau_n \cdot \lambda}}{\tau_n \cdot \lambda}\right) \end{pmatrix}$$

Where C stands for the number of currencies, thus 4 and n for the number of maturities, thus 14.

The following assumption of residuals and shocks of the global model are the same or a variation of Diebold et al (2008, p.352)). The variance of the measurement equation is assumed to be different for each currency and for each tenor but stable over time, thus *4n* variances need to be estimated. The covariance of residuals is assumed to be 0 and the variance to be:

$$VAR\left(\upsilon_{currency,t}(\tau_i)\right) = \left(\sigma^{\tau_i}_{currency}\right)^2 ;\ i = 1,2 \dots n \qquad \text{(X7)}$$

The evolution of global factors (Level and Slope) follows an autoregressive process

$$\begin{pmatrix} L_t \\ S_t \end{pmatrix} = \begin{pmatrix} \phi_{l,l} & \phi_{l,s} \\ \phi_{s,l} & \phi_{s,s} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} \psi^l_t \\ \psi^s_t \end{pmatrix} \qquad \text{(X8)}$$

Where $\phi_{l,s}$ and $\phi_{s,l}$are thought to be 0 and ψ^l_t and ψ^s_t are “shocks” independent to each other. Their variances are estimated in the model and assumed to be constant over time.

$$VAR\left(\psi^f_t\right) = \left(\sigma^f_{GLO}\right)^2; \qquad f = l,\ s \qquad \text{(X9)}$$

The model also includes 2 idiosyncratic local factors for each of the four counties, which are written as $\varepsilon^{l}_{currency,t}$ and $\varepsilon^{s}_{currency,t}$ in the model, where currency stands for USD, GBP, JPY or EUR. They follow an autoregressive process, are independent to each other and can be described by the structure hereunder:

$$\varepsilon^{f}_{currency,t} = \phi^{f}_{\varepsilon,currency} \cdot \varepsilon^{f}_{currency,t-1} + \psi^{\varepsilon,f}_{currency,t} \tag{X10}$$

$$VAR(\psi^{\varepsilon,f}_{currency,t}) = \left(\sigma^{\varepsilon,f}_{currency}\right)^{2} \tag{X11}$$

where *f* stands for factor and can be either *l* or *s;* $\phi^{f}_{\varepsilon,currency}$ is an autoregressive coefficient and ψ^{f}_{t} are disturbances. Variances of ψ^{f}_{t} are estimated for each idiosyncratic process and are assumed to be constant over time.

$\beta^{l}_{\text{Currency}}$ is a loading to global Level and $\beta^{s}_{\text{Currency}}$ is a loading to global Slope. They are constant over time in the model.

Identification

$\beta^{l}_{Currency}$ or $\beta^{s}_{Currency}$ can be of *any* value *T,* and a global factor (in particular time) can be of *any* value R, unless the multiple of both, $T \cdot R$ is correct. $\beta^{l}_{Currency}$ could then be identified as k-times bigger $(k \cdot \beta^{l,***}_{Currency})$, and factor L_t as k-times smaller $(\frac{1}{k} \cdot L^{***}_{t})$, where *k* is *any* real number and *stars* $(***)$ stand for some optimal value.

To overcome this, the variances of global factor updates have some predefined variance that is constant over time. The value of 0.01 is chosen and this selection is explained in the "model calibration" part of this thesis.

Additionally, loadings to USD (of both Level and Slope) are assumed to be positive, otherwise, global factors could be of an opposite sign than all the observed yields. The settings of the predefined variance of the factor updates and the positive load of USD to global factors are the same as in Diebold et al (2008, p.352).

The impact of the global yield curve on the Czech yield curve

Global factors, L_t and S_t estimated via model (X6) are used as an input to calculate the impact of the evolution of the global yield curve on the Czech one. The model is a variation of (X6) with the differences that dependent vector consists of data of just one currency. The other differences are that global factors are exogenous to the model and that the loadings to global factors can *vary* over time.

The model looks as follows:

$$\begin{pmatrix} y_{CZK,t}(\tau_1) \\ y_{CZK,t}(\tau_2) \\ \vdots \\ y_{CZK,t}(\tau_n) \end{pmatrix} = B_{CZK,t} \cdot \begin{pmatrix} L_t \\ S_t \end{pmatrix} + A_{CZK} \cdot \begin{pmatrix} \varepsilon^l_{CZK,t} \\ \varepsilon^s_{CZK,t} \end{pmatrix} + \begin{pmatrix} \upsilon_{CZK,t}(\tau_1) \\ \upsilon_{CZK,t}(\tau_2) \\ \vdots \\ \upsilon_{CZK,t}(\tau_n) \end{pmatrix} \quad \text{(X12)}$$

The variance of disturbances, $\upsilon_{CZK,t}(\tau_i)$ is thought to be constant over time and different for each tenor, thus *n* coefficients needs to be estimated. Covariance is assumed to be 0 and variance to be:

$$VAR(\upsilon_{CZK,t}(\tau_i)) = \sigma^2_{CZK\,(\tau_i)}$$

$y_{CZK,t}(\tau_i)$ are input data of CZK yields for all maturities $\tau_i, i = 1,2 \dots n$, in time *t*. Auxiliary matrices A_{CZK} and $B_{CZK,t}$ are defined as:

$$A_{CZK} = \begin{pmatrix} 1 & \left(\frac{1-e^{-\tau_1\cdot\lambda}}{\tau_1\cdot\lambda}\right) \\ 1 & \left(\frac{1-e^{-2\cdot\lambda}}{\tau_2\cdot\lambda}\right) \\ \vdots & \vdots \\ 1 & \left(\frac{1-e^{-120\cdot\lambda}}{\tau_n\cdot\lambda}\right) \end{pmatrix}, \quad B_{CZK,t} = \begin{pmatrix} \beta^l_{CZK,t} & \beta^s_{CZK,t} \cdot \left(\frac{1-e^{-\tau_1\cdot\lambda}}{\tau_1\cdot\lambda}\right) \\ \beta^l_{CZK,t} & \beta^s_{CZK,t} \cdot \left(\frac{1-e^{-\tau_2\cdot\lambda}}{\tau_1\cdot\lambda}\right) \\ \vdots & \vdots \\ \vdots & \vdots \\ \beta^l_{CZK,t} & \beta^s_{CZK,t} \cdot \left(\frac{1-e^{-\tau_n\cdot\lambda}}{\tau_n\cdot\lambda}\right) \end{pmatrix} \quad \text{(X13)}$$

And $\varepsilon^{l}_{CZK,t}$ and $\varepsilon^{s}_{CZK,t}$ follow an autoregressive process:

$$\begin{pmatrix}\varepsilon^{l}_{CZK,t}\\ \varepsilon^{s}_{CZK,t}\end{pmatrix} = \begin{pmatrix}\phi^{l,l}_{\varepsilon,CZK} & \phi^{l,s}_{\varepsilon,CZK}\\ \phi^{s,l}_{\varepsilon,CZK} & \phi^{s,s}_{\varepsilon,CZK}\end{pmatrix}\begin{pmatrix}\varepsilon^{l}_{CZK,t-1}\\ \varepsilon^{s}_{CZK,t-1}\end{pmatrix} + \begin{pmatrix}\psi^{\varepsilon,l}_{CZK,t}\\ \psi^{\varepsilon,s}_{CZK,t}\end{pmatrix}$$

where $\phi^{l,s}_{CZK}$ and $\phi^{s,l}_{CZK}$ are thought to be 0 and their shocks $\psi^{\varepsilon,l}_{CZK,t}$ and $\psi^{\varepsilon,s}_{CZK,t}$ to be independent to each other and thus their covariance to be 0. Variances of $\psi^{l}_{CZK,t}$ and $\psi^{s}_{CZK,t}$ are estimated in the model and assumed to be constant over time.

$$VAR\left(\psi^{\varepsilon,f}_{CZK,t}\right) = \left(\sigma^{f}_{\varepsilon,CZK}\right)^{2}, where\ f = l, s$$

Finally, also loadings to global parameters follow an autoregressive process:

$$\begin{pmatrix}\beta^{l}_{CZK,t}\\ \beta^{s}_{CZK,t}\end{pmatrix} = \begin{pmatrix}\phi^{l,l}_{CZK} & \phi^{l,s}_{CZK}\\ \phi^{s,l}_{CZK} & \phi^{s,s}_{CZK}\end{pmatrix}\begin{pmatrix}\beta^{l}_{CZK,t-1}\\ \beta^{s}_{CZK,t-1}\end{pmatrix} + \begin{pmatrix}\psi^{l}_{CZK,t}\\ \psi^{s}_{CZK,t}\end{pmatrix}$$

Where $\psi^{l}_{CZK,t}$ and $\psi^{s}_{CZK,t}$ are shocks to the CZK model loadings that are independent to each other and have a constant variance over time

$$VAR\left(\psi^{f}_{CZK,t}\right) = \left(\sigma^{f}_{CZK}\right)^{2}, where\ f = l, s$$

And $\phi^{l,s}_{CZK}$ and $\phi^{s,l}_{CZK}$ are modelled as 0.

The impact of EUR yield curve to the CZK yield curve

Model X12 estimates how the global yield curve impacted the CZK yield curve over time. However, the global yield curve is not directly observable and its estimation is complex and rests upon several assumptions (like which currencies to consider among others).

It would be interesting to compare model X12 with a much simpler model where only the EUR yield curve is considered instead of a latent global yield curve. The EUR curve is a nice proxy for the global curve in this example because the Eurozone is by far the biggest business partner for the Czech small open economy and, their business cycles are well connected[4]. Also, the Czech Koruna was under a currency peg with the *EUR* currency between 2013 and 2017. Those two arguments make the EUR yield curve as a great candidate to explain the CZK yield curve's evolution. Model X12 thus changes to

$$\begin{pmatrix} y_{CZK,t}(\tau_1) \\ y_{CZK,t}(\tau_2) \\ \vdots \\ y_{CZK,t}(\tau_n) \end{pmatrix} = B_{CZK2,t} \cdot \begin{pmatrix} L_{EUR,t} \\ S_{EUR,t} \end{pmatrix} + A_{CZK} \cdot \begin{pmatrix} \varepsilon^l_{CZK2,t} \\ \varepsilon^s_{CZK2,t} \end{pmatrix} + \begin{pmatrix} \upsilon_{CZK2,t}(\tau_1) \\ \upsilon_{CZK2,t}(\tau_2) \\ \vdots \\ \upsilon_{CZK2,t}(\tau_n) \end{pmatrix} \quad \text{(X14)}$$

Where $B_{CZK2,t}$, $\varepsilon^l_{CZK2,t}$and $\varepsilon^s_{CZK2,t}$ are equivalents to $B_{CZK,t}$, $\varepsilon^l_{CZK,t}$, $\varepsilon^s_{CZK,t}$ respectively; subscript "2" just reminds us that they are used in different equations and thus the estimated values will be different; A_{CZK} is entirely the same as in X12.

The EUR yield factors, $L_{EUR,t}$ and $S_{EUR,t}$ are estimated as

$$\begin{pmatrix} y_{EUR,t}(\tau_1) \\ y_{EUR,t}(\tau_2) \\ \vdots \\ y_{EUR,t}(\tau_n) \end{pmatrix} = A_{EUR} \cdot \begin{pmatrix} L_{EUR,t} \\ S_{EUR,t} \end{pmatrix} + \begin{pmatrix} \upsilon_{EUR2,t}(\tau_1) \\ \upsilon_{EUR2,t}(\tau_2) \\ \vdots \\ \upsilon_{EUR2,t}(\tau_n) \end{pmatrix} \quad \text{(X15)}$$

where $A_{EUR} \equiv A_{CZK}$, and the EUR yield factors, $L_{EUR,t}$ and $S_{EUR,t}$ follow auto-regressions which is a variation of X8.

[4] The correlation of GDP growth of the Eurozone and the Czech Republic was 80% from 1Q/2005 to 2Q/2017. The calculation is based on growth rates over 4 consequtive quarters to avoid seasonal effects. GDPs in current prices are considered. Source of data: Bloomberg terminal, author's calculation.

4 Model calibration

I estimated the latent global factors of the model X6 using the method of a Kalman filter. This required choosing a set of initial values which includes:

- 10 autoregressive coefficients of transition equation - $\phi_{f,f}$ in equation X8 and $\phi^{f}_{currency}$ in equation X10
- 10 variances of their updates - $\left(\sigma^{f}_{GLO}\right)^2$ in the equation X9 and $\left(\sigma^{\varepsilon,f}_{currency}\right)^2$
- 56 initial variances of each tenor i and of each currency $(\upsilon_{Currency,t}(\tau_i)$ in equation X6
- 4 initial loadings to global Level and 4 to global Slope ($\beta^{f}_{currency}$).

Moreover, 20 initial values of the distributions of the *first* observations of the state variables needed to be chosen:

- 2 (1 mean and 1 variance) for of $L_{t=1}$
- 2 for $S_{t=1}$
- 8 means and 8 variances for currency-specific factors, $\varepsilon^{f}_{currency,t=1}$.

All interest rates of all currencies I used in this thesis show a high level of serial correlation. It is then reasonable to expect that also global Level and Slope as well as all idiosyncratic (currency-specific) levels and slopes of each currency will also be highly serially correlated. I then chose initial values as:

$$\phi_{f,f} = \phi^{f}_{currency} = 1 \text{ for all currencies and both factors}$$

To check whether the model is robust and does not depend much on the initial autoregressive coefficients I estimated the model with initial autoregressive coefficients equal to 0.5. The results are similar to those where initial coefficients were set to 1 – the biggest difference is 0.0002 for all the autoregressive coefficients. The figures of estimated global factors under the assumption of $\phi_{f,f} = \phi^{f}_{currency} = 0.5$ can be found in the Annex, in figure A2[5].

[5] The model was not further tested for other sets of assumptions for $\phi_{f,f}$ and $\phi^{f}_{currency}$.

Initial variances of updates of currency-specific factors were set to

$$\left(\sigma_{currency}^{\varepsilon,f}\right)^2 = 1 \text{ for all factors and currencies}$$

They were tested for different initial variances of 0.25, which had also no substantial impact on the results. This was *not* true for variances of shocks of global factors. These are not separately identifiable and need to be properly set because they have a large impact on the overall magnitude of global factors. For example, when variances of global factors are set to 1 then global Level peaks just below 50 and loadings to global factors are then of a very low magnitude. To increase the comparability of global factors with the initial dataset of interest rates, I set variances of their shocks to be:

$$\left(\sigma_{GLO}^{f}\right)^2 \equiv 0.01$$

This setting resulted in the magnitude of global Level peaking around 5 and thus being comparable with the input data. One can visually compare the results in figure 2 below. They seem to be largely similar in shape but different in magnitude[6].

It is important to note that the setting of $\left(\sigma_{GLO}^{f}\right)^2$ is of a *different* kind than settings of other initial values. $\left(\sigma_{GLO}^{f}\right)^2 \equiv 0.01$ is a *restriction* of the model and its value does not change during the maximum likelihood estimation of the model. This setting is a variation of Diebold et al (2008) but they restricted the variance to be of a unit size.

[6] Another difference lies in the initial shapes (years 2005 – 2009) because the model needed many observations to settle down when initial values were set far from optimal

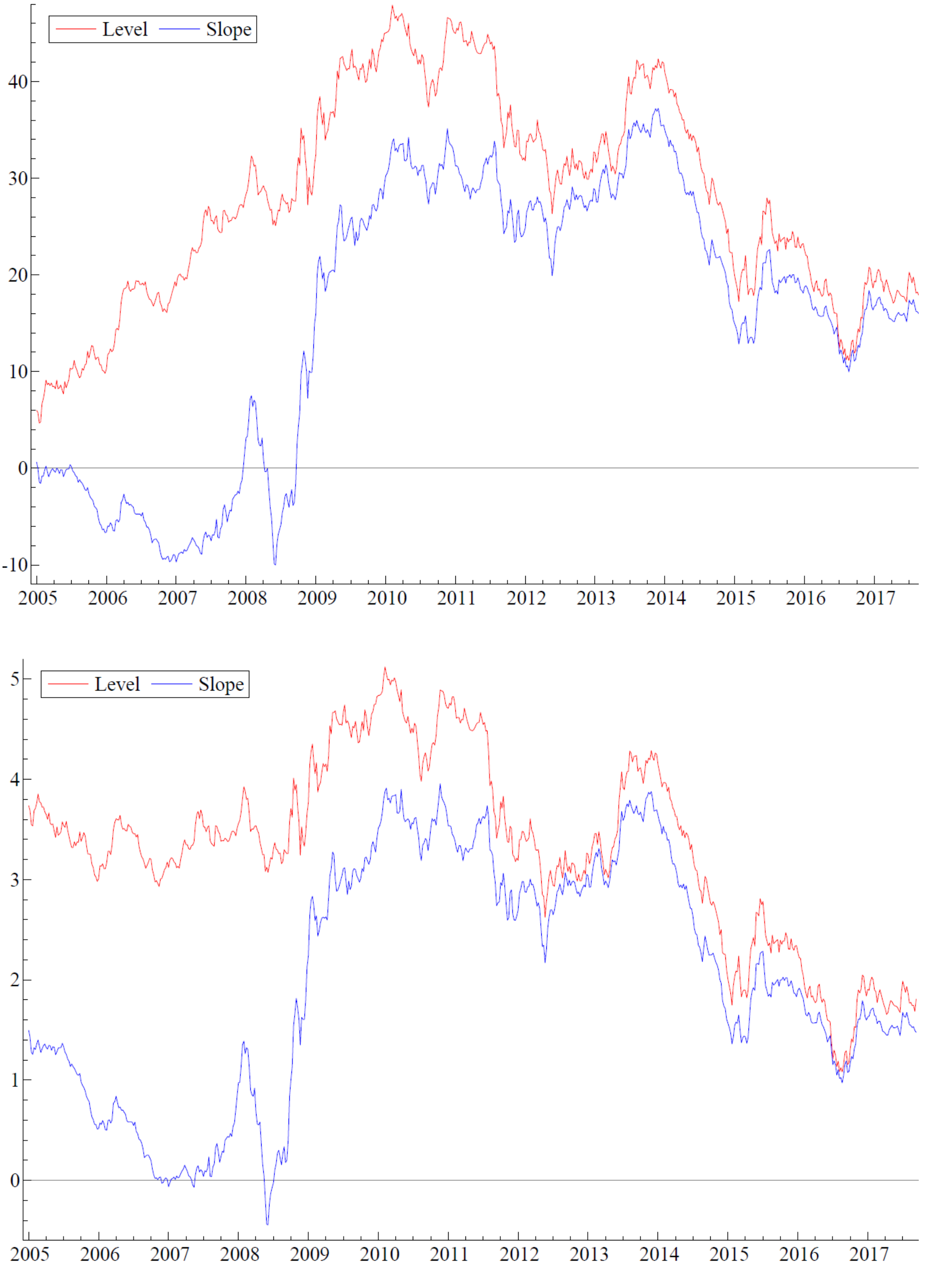


Figure 2: Top: global Level and global Slope with variance of shocks = 1 have a high magnitude making them difficult to be compared with the initial dataset. **Bottom:** global Level and global Slope with variance of shocks = 0.01 show very similar movements as in the top part, especially when the model settles down after several observations

All the initial variances in measurement equation X6 were set to:

$$\left(\sigma_{currency}^{\tau_i}\right)^2 = 1, \quad \text{for all currencies and all tenors } \tau_i$$

Thus 14 variances for each tenor of each of the 4 currencies - so a total of 56 initial variances - were set for the measurement equation. The model was also estimated with the initial variances of 0.25 and this change had no substantial impact on the results.

Loadings to global Level and Slope were all set as

$$\beta_{currency}^{f} = 1, \text{ for all currencies and both factors}$$

Thus 8 initial loadings were set. The model was also estimated as

$$\beta_{currency}^{f} = 0.5 \text{ for all currencies and both factors}$$

but it did not significantly change the results.

Distributions of the initial values of global factors were set to

$$L_{t=1} \sim N(1,20) \text{ and } S_{t=1} \sim N(1,20),$$

and for currency-specific factors to

$$\varepsilon_{currency,t=1}^{f} = 0,$$

as 0 would be their ideal value if the variance of interest rate movements would be entirely explained by the global factors. Also here I controlled for robustness and changed all the initial means to 0.5 and the initial variances to 50 instead of 20. The results remain virtually unchanged.

5 Data description

In this section, the data and a preliminary analysis of the model are presented. Data of a weekly frequency from the period of 7th January 2005 to 15th September 2017 is used, so a total of 663 weeks are considered. The data source is a Bloomberg terminal from which closing values of daily trading are gathered. I use tenors ranging from 1 month to 10 years. Short tenors - 1, 2, 3 and 6 months – are obtained from Interbank Offer Rates (IBORs), while the longer tenors – every year from 1 to 10 years – are gathered from Interest rate swaps (IRS). In total 4 + 10 = 14 tenors are used.

Yield data of four currencies – from the USA, Great Britain, the Euro Area and Japan – serve to construct a global yield curve. These currencies are from countries, which have high credit ratings, so the yield should mirror just the price of money without being influenced by potential defaults. Moreover, these are big world economies intensivly trading with the rest of the world and are thus well suited to build a global yield curve.

In table 2, the average yields of selected tenors are shown together with their standard deviations, minima, maxima and autocorrelations of the first order. We can see that all yield curves are upward-sloping and are highly serially correlated. With the exception of the Japanese YC, the standard deviation decreases with maturity.

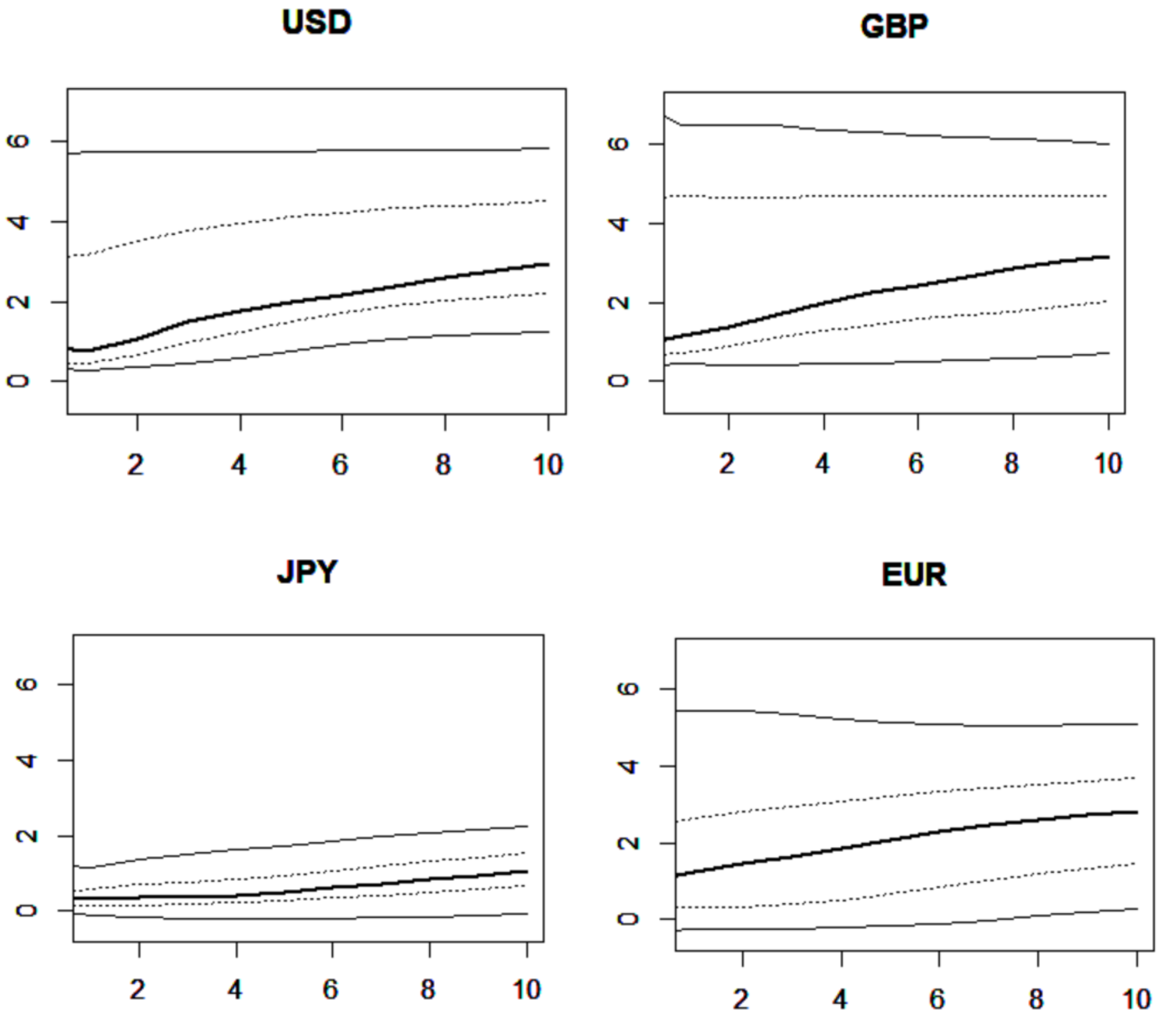
Regarding the overall level, yields are noticeably lower than in studies mentioned in the literature review of this thesis. Some yields are even negative, which is, however, not problematic for the method that is applied here.

Figure X.1 shows that the yield curves of selected countries are of a simple shape without many bumps, mostly increasing with maturities. This makes it possible to factor them by the Nelson-Siegel method. Every single yield ranges between slightly negative numbers up to 7%, showing that the yields of the considered countries are quite comparable.

Figure 4 presents the evolution of yield curves. We can see that the co-movement of CZK, EUR, USD and GBP after 2008 is pretty high – yields were highest before the crisis and then fell sharply and never returned to their pre-crisis values. The same is true for Japan, however with a much smaller magnitude.

	Tenor	Mean	Std. dev.	Min.	Max.	AR
USD	3	1.68	1.88	0.22	5.73	1.00
	12	1.78	1.84	0.25	5.75	1.00
	36	2.20	1.63	0.43	5.72	1.00
	120	3.27	1.26	1.25	5.83	1.00
GBP	3	2.14	2.19	0.28	6.89	1.00
	12	2.25	2.06	0.43	6.49	1.00
	36	2.56	1.84	0.41	6.46	1.00
	120	3.28	1.42	0.69	6.02	1.00
JPY	3	0.28	0.29	-0.07	1.08	1.00
	12	0.38	0.31	-0.11	1.16	1.00
	36	0.48	0.40	-0.21	1.51	1.00
	120	1.09	0.56	-0.09	2.23	1.00
EUR	3	1.43	1.64	-0.33	5.38	1.00
	12	1.62	1.63	-0.26	5.41	1.00
	36	1.82	1.58	-0.25	5.33	1.00
	120	2.64	1.38	0.27	5.07	1.00
CZK	3	1.49	1.18	0.28	4.52	1.00
	12	1.50	1.23	0.13	4.64	1.00
	36	1.88	1.28	0.17	4.76	1.00
	120	2.55	1.27	0.43	4.84	1.00

Table 2: Descriptive statistics of input data, selected tenors: 3, 12, 36 and 120 months.



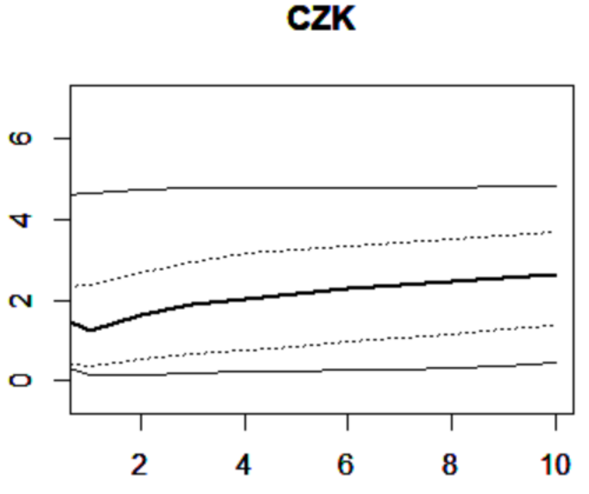
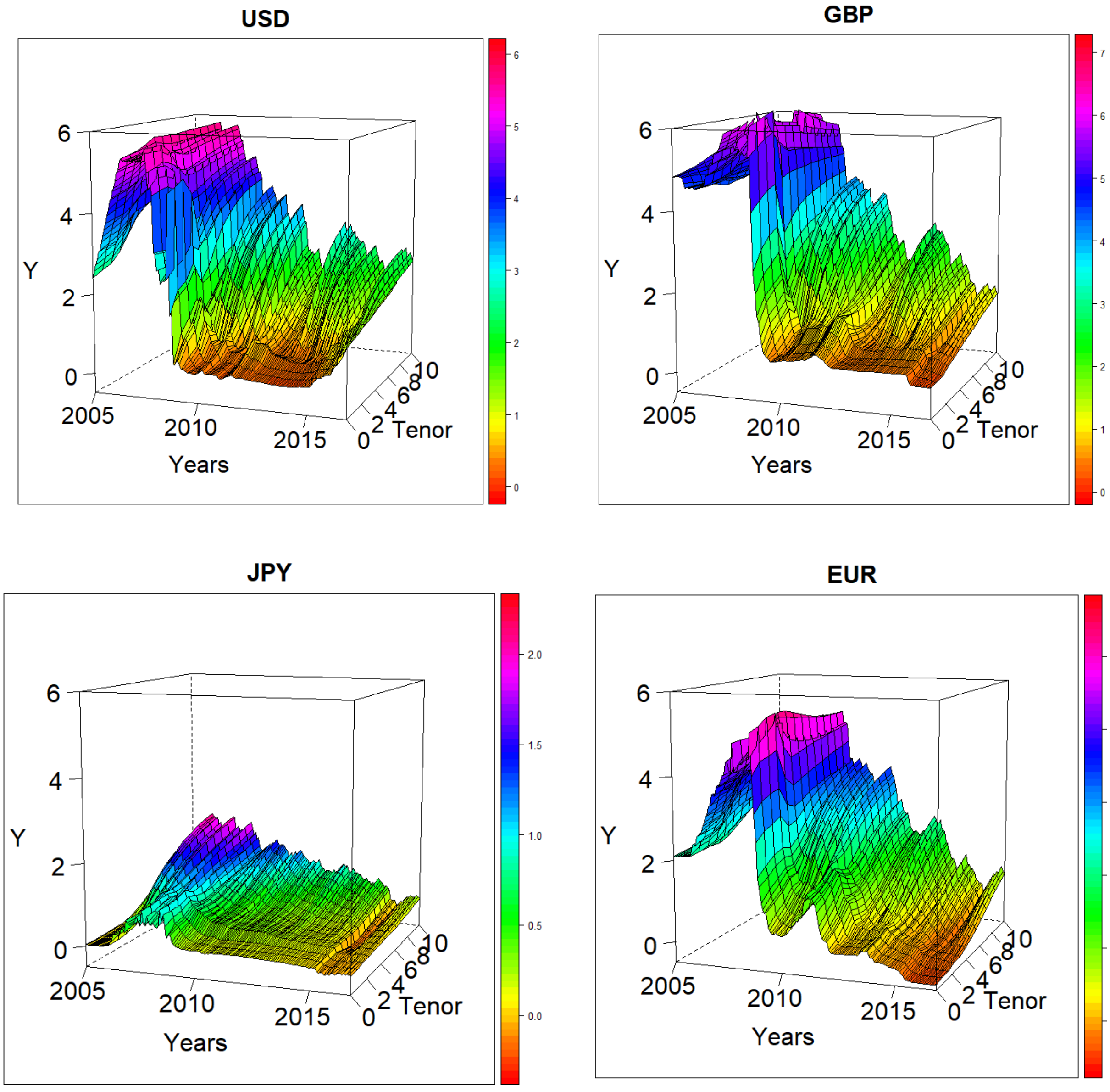


Figure 3: The medians of yield curves of selected currencies (thick line) together with minima, maxima, 25th and 75th percentiles. Yield data are of a weekly frequency from 1/2005 to 9/2017. Yields are shown in percentages (vertical axes) and tenors in years (horizontal axes).



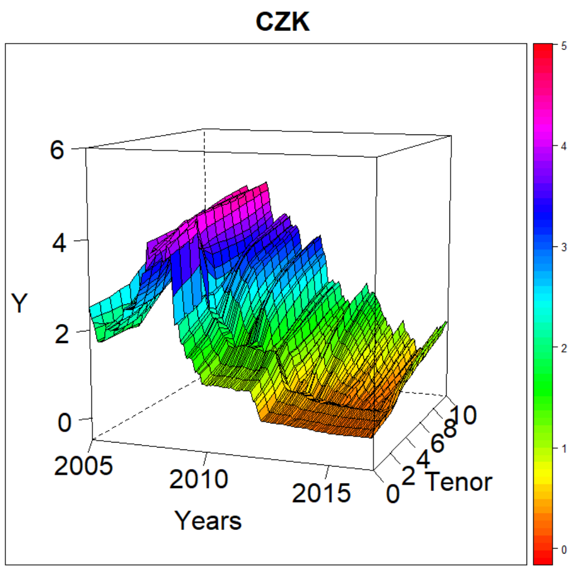


Figure 4: Yield curves per currency in time. The box covering the graph is of the same size for all currencies. The scale of the colouring is different for each currency, as indicated in the colour-yield legends on the right hand side of each figure. The yields are shown in percentages. Yield data are of a weekly frequency from 1/2005 to 9/2017; every fourth observation was used to improve the legibility.

Preliminary analysis

Table 3 presents the results of a principal component analysis of the series of yield curves for each currency. It tells us that the first component of each YC can explain most of the variance and, in combination with the second component, can explain more than 98.7% of the variance. Thus it is likely that a two-factor model is sufficient to describe the data.

		Proportion of variance	Cumulative Proportion
USD	First	95.5%	95.5%
	Second	4.0%	99.5%
GBP	First	95.7%	95.7%
	Second	4.0%	99.7%
JPY	First	88.9%	88.9%
	Second	9.8%	98.7%
EUR	First	96.5%	96.5%
	Second	3.3%	99.8%
CZK	First	96.2%	96.2%
	Second	3.4%	99.6%

Table 3: Principal component analysis of the yield curves; data were centred and scaled before the analysis.

Then, I proceeded to estimate the evolutions of the first two factors, Level and Slope, for each currency. I obtained these by least square estimation of all yield curves of all weeks – as in Diebold and Li (2006) – with a constant Lambda of 0.0609. The descriptive statistics of estimated Levels and Slopes can be seen in table 4.

		Mean	Std. dev	Min	Max	AR(1)
USD	Level	3.25	1.31	1.19	5.88	0.99
	Slope	1.85	1.23	-1.06	4.60	0.99
GBP	Level	3.30	1.50	0.61	6.25	1.00
	Slope	1.35	1.44	-1.84	4.60	0.99
JPY	Level	0.97	0.60	-0.20	2.43	1.00
	Slope	0.81	0.51	-0.14	2.51	0.99
EUR	Level	2.59	1.47	0.09	5.22	1.00
	Slope	1.36	0.91	-0.95	3.61	0.99
CZK	Level	2.54	1.35	0.33	5.02	1.00
	Slope	1.27	0.74	-1.33	2.81	0.99

Table 4: Descriptive statistics of Nelson-Siegel coefficients computed for every week for each currency by OLS

It is worth noting that the Level coefficient of each currency is virtually the same as the 10-year yield. Interestingly, the variance of Level is higher than the variance of Slope in each currency. This result is non-intuitive because table 2 shows that the variances of long yields are lower than of shorter yields.

	USD	GBP	JPY	EUR	CZK
USD	1.00				
GBP	0.95	1.00			
JPY	0.86	0.89	1.00		
EUR	0.85	0.95	0.90	1.00	
CZK	0.84	0.91	0.92	0.95	1.00

Table 5a: Correlation table of computed Level coefficients among countries. Level computed by OLS.

	USD	GBP	JPY	EUR	CZK
USD	1.00				
GBP	0.87	1.00			
JPY	-0.22	-0.18	1.00		
EUR	0.72	0.86	0.18	1.00	
CZK	0.32	0.50	0.59	0.75	1.00

Table 5b: Correlation table of computed Slope coefficients among countries. Slope computed by OLS.

Whether the Level and Slope coefficient really co-move can be seen from table 5a and 5b. It shows that the correlation of Level factors is really high, which is even true for the Japanese YC that behaves a bit differently in the analysis above. Japanese Level correlated by more than 85% with the Level of every other yield curve.

Co-movements of individual Slopes are lower and, in case of JPY, even negative. This result is similar as in Bae and Kim (2011) when they say that: "the Japanese yield slope is comparatively divorced from the global slope" (Bae and Kim, 2011, p.729).

The Czech Slope is the most correlated with its neighbour and biggest trade partner, the Euro area. Surprisingly, the correlation with the Japanese Slope is the second biggest, indicating that including the Japanese yield curve into the model could improve its fit, despite the different behaviour of Japanese yields in other statistics.

6 Model estimation

Raw data were directly used in the model, including dates where some values were missing because an interest rate swap was not traded or interbank offer rates (IBORs) were not set by corresponding quoting banks, for example due to bank holidays. The units of source data are percentages.

The model was then estimated in one-step, in an environment of OxMetrics by Doornik, J.A. (2007). The estimation was designed as a Kalman filter and the best fit was found numerically, by a maximum likelihood estimator using a Broyden–Fletcher–Goldfarb–Shanno algorithm (BFGS). Employing the BFGS algorithm made the model estimation quite quick – it was estimated within several hours on a standard computer even when data of higher frequency were used (weekly in this model compared to monthly in Diebold et al, (2008) or in Sopov and Seidler (2010).

This approach of using data directly and estimating the evolution of global factors in just one run differs from e.g. Diebold et al (2008). They employed a two-step approach where level and slope were estimated separately for each currency and then latent global factors were estimated out of the individual countries' results. The advantage of the one-step estimation used in this thesis is that less information is lost during the estimation.

Global yield curve estimation

The estimated global factors are both highly serially correlated, by the coefficient 0.999[7]. Standard errors are quite small, 0.001 for global Level and 0.002 for global Slope implying robust results. Variance of shocks were not estimated within the model but defined to be 0.01 (further explained in the "Model calibration" part).

---

[7] The fact that autocorrelation coefficients are very close to a unit root does not imply any problem for the used method

$$L_t = \phi_{l,l} \cdot L_{t-1} + \psi_t^l \qquad \phi_{l,l} = 0.999\ (s.e. = 0.001) \qquad VAR(\psi_t^l) = 0.01$$

$$S_t = \phi_{s,s} \cdot S_{t-1} + \psi_t^s \qquad \phi_{s,s} = 0.999\ (s.e. = 0.002) \qquad VAR(\psi_t^s) = 0.01$$

While the results above were estimated by a Kalman filter, a Kalman smoother was used to derive the whole sequences of global Level and global Slope that are depicted in figure 5. It implies that "long"[8] global rates were quite high before the crisis of 2007/8, even increased just after the crisis and then showed a decreasing trend until these days (dataset ends on 15th September 2017).

On the other hand, global Slope was quite high[9] before the crisis and even positive just before it. This is perfectly in line with Chauvet and Zeynep (2012) who studied economic cycles from 1971 to 2007 and found out that economic slowdown usually occurs after the flattening of the yield curve (high slope in this model). Global Slope then increases substantially during and after the crisis and subsequently global Level and global Slope move together until these days implying very low global "short" rates.

A summary analysis of both global factors can be found in table 6:

	Mean	Variance
Global Level	3.32	0.84
Global Slope	-2.01	1.42

Table 6: Mean and variance of global Level and Global Slope

Figures 6 and 7 compare global factors with the input data and show a high level of co-movements. Figure 6 compares the evolution of estimated global Level with 10-year interest rates of USD, GBP, JPY and EUR. The correlation is semi-strong ranging from 40% to 64% (the exact figures can be found in table 7). Figure 6 therefore suggests that the model is suitable for the input data – we can see many moments where global Level

[8] The term "long" rates is used for rates of long tenors (like 10 years and more) whereas "short" rates refers to rates of short tenors in financial jargon.

[9] Global Slope is depicted with opposite sign in the figure to ease the comparison with global Level

and interest rates strongly co-moved. Moreover, the magnitude of co-movement is higher for USD, GBP and EUR while it is lower for JPY.

Those moments include for example:

- V-shape evolutions in the second half of 2010,
- A decrease in 2011
- An increase in 2013
- A strong decrease in 2014
- and another V-shape in 2016

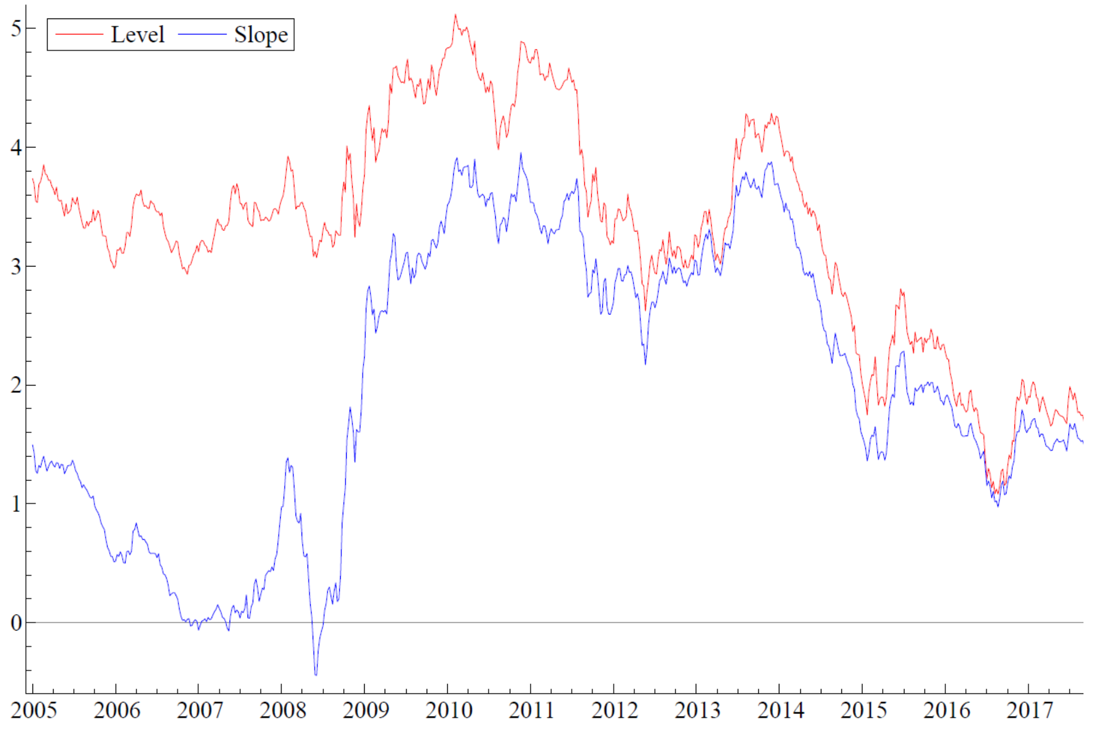


Figure 5 shows the evolution of estimated global Level (in red) and global Slope (in blue). Please notice that **global Slope is depicted with opposite sign**.

Those moments then show that USD, GBP, EUR and global Level moved strongly together - and also that JPY co-moved with them but with a smaller magnitude. The fact that rates moved together, but with different volatility, illustrates well how the model works. It is designed including some latent factors (global Level and Slope) on which individual interest rates load in different magnitudes. Those magnitudes are estimated via the model and are denoted as $\beta^{i}_{Currency}$. Their estimates are shown in table 8.

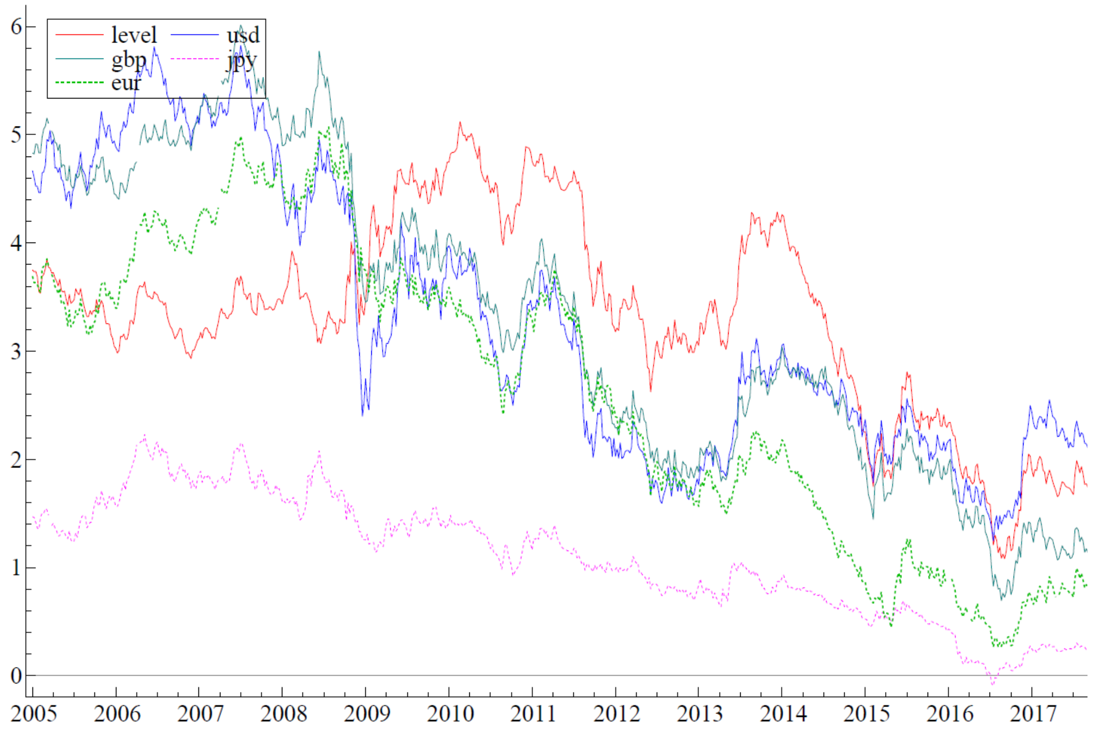


Figure 6 depicts the evolution of global Level (in red) with the evolution of 10-year interest rate swaps for each currency. Global Level has periods when it moves in opposite direction to the individual rates, but overall matches well their direction. The correlation with the interest rates is between 40% and 64%.

Figure 7 compares the evolution of the estimated global Slope with slopes of USD, GBP, EUR and JPY. The slope of individual yield curves is defined as a simple difference between 10-year and 3-month rates. The cross-correlations between global Slope, USD, GBP and EUR are quite strong, ranging from 75% to 90%, whereas the correlation of JPY and global Slope is negative at 20%. These results suggest that the short tenors of the JPY yield curve are independent to global moves, whereas those of USD, GBP and EUR are strongly interlinked with global factors.

	Level	USD	GBP	JPY	EUR
Level	1.00				
USD	0.40	1.00			
GBP	0.56	0.95	1.00		
JPY	0.61	0.91	0.96	1.00	
EUR	0.64	0.87	0.96	0.96	1.00

	Slope	USD	GBP	JPY	EUR
Slope	1.00				
USD	0.83	1.00			
GBP	0.90	0.88	1.00		
JPY	-0.20	-0.10	-0.12	1.00	
EUR	0.79	0.75	0.88	0.19	1.00

Table 7 (Top) shows the correlation between global Level and 10-year interest rates and also correlations between rates of selected currencies among each other.
The bottom table shows the correlation between global Slope (opposite sign) and the slope of currencies' yield curves and also correlations of selected currencies between each other. The currency slope is defined as the difference between yields of 10-year and 3-month.

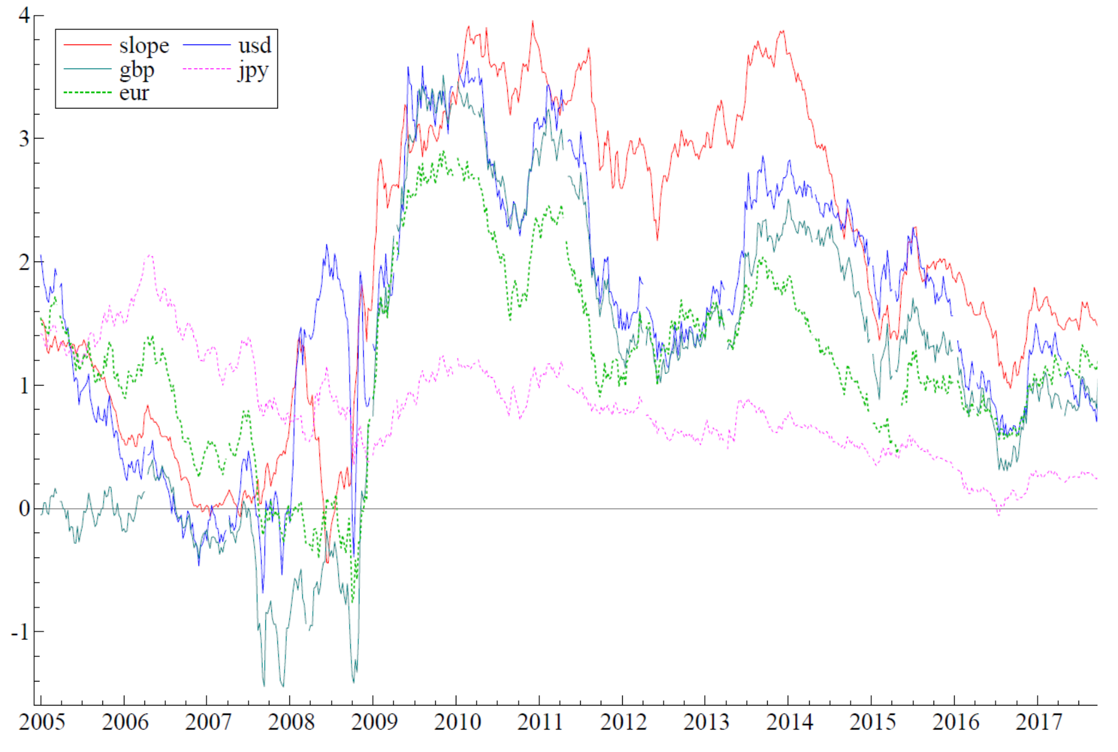


Figure 7: depicts the evolution of global Slope (in red, opposite sign) with the evolution of each currency's slopes defined as the differences between 10-year interest rate swaps and 3-month interbank offer rates (IBORs). Global Slope is highly correlated with the slopes of USD, GBP and EUR (80% to 90%) while the correlation with JPY is negative -20%.

Loading of individual yield curves to global factors:

	Loadings to Global Level		Loadings to Global Slope	
	Estimate	Std. err	Estimate	Std. err
USD	1.60***	0.09	1.58***	0.07
GBP	1.56***	0.05	1.72***	0.08
JPY	0.42***	0.05	0.26***	0.06
EUR	1.35***	0.05	1.48***	0.07

Table 8: shows the resulting coefficients of how yield curves of each currency load to global Level and global Slope.
*** stands for a confidence level higher than 99.9%

Table 8 shows $\beta^{l}_{Currency}$ and $\beta^{s}_{Currency}$ of each currency's yield curve. It could be surprising that the loadings are so high even when it can be seen from figure 6 that the global Level is similar in magnitude to the long rates. Figure 7 shows that a similar observation is true for global Slope. An example of how the 10-year USD rate (120 months) is loaded in the model will help to clarify how the model works:

Fitting into equation X6:

$$y_{USD,t}(\tau) = \beta^{l}_{USD} \cdot L_t + \beta^{s}_{USD} \cdot \left(\frac{1-e^{-\tau\cdot\lambda}}{\tau\cdot\lambda}\right) \cdot S_t + \varepsilon^{l}_{USD,t} + \varepsilon^{s}_{USD,t} \cdot \left(\frac{1-e^{-\tau\cdot\lambda}}{\tau\cdot\lambda}\right) + \upsilon_{USD,t}(\tau)$$

$$y_{USD,t}(120) \cong 1.60 \cdot L_t + 1.58 \cdot \left(\frac{1-e^{-120\cdot 0.01}}{120\cdot 0.01}\right) \cdot S_t + \varepsilon^{l}_{USD,t} + \varepsilon^{s}_{USD,t} \cdot \left(\frac{1-e^{-120\cdot 0.01}}{120\cdot 0.01}\right) + \upsilon_{USD,t}(\tau)$$

$$y_{USD,t}(120) \cong 1.60 \cdot L_t + 1.58 \cdot 0.58 \cdot S_t + \varepsilon^{l}_{USD,t} + \varepsilon^{s}_{USD,t} \cdot 0.58 + \upsilon_{USD,t}(\tau)$$

$$y_{USD,t}(120) \cong 1.60 \cdot L_t + 0.93 \cdot S_t + \varepsilon^{l}_{USD,t} + \varepsilon^{s}_{USD,t} \cdot 0.58 + \upsilon_{USD,t}(\tau)$$

shows that the 10-year USD rate loads 160% to global Level but also by 93% to global Slope. This high impact of global Slope can also explain why all the 10-year interest rates are falling in figure 6 while global Level is increasing. It is because global Slope increases substantially during that period.

Idiosyncratic factors

The estimates of autocorrelation coefficients of the currency-specific factors and standard deviations of their shocks $(\sigma^{\varepsilon,f}_{currency})$ are listed in table 9:

Currency	Factor	AR-Mean	AR-Std. error	Shocks-std. dev [bps]
USD	Level	0.993	0.005	8.8
	Slope	0.997	0.003	10.4
GBP	Level	0.994	0.004	4.1
	Slope	0.995	0.003	6.8
JPY	Level	0.997	0.002	7.2
	Slope	0.997	0.002	7.5
EUR	Level	0.993	0.004	5.5
	Slope	0.996	0.004	7.5

Table 9: estimates of autocorrelations' coefficients of currency-specific factors

Currency-specific factors are, as well as the global factors, highly serially correlated and the variances of their shocks are well comparable with variances of shocks to global factors, which are both 0.01.

It can be seen in figure 8 how they evolved over time:

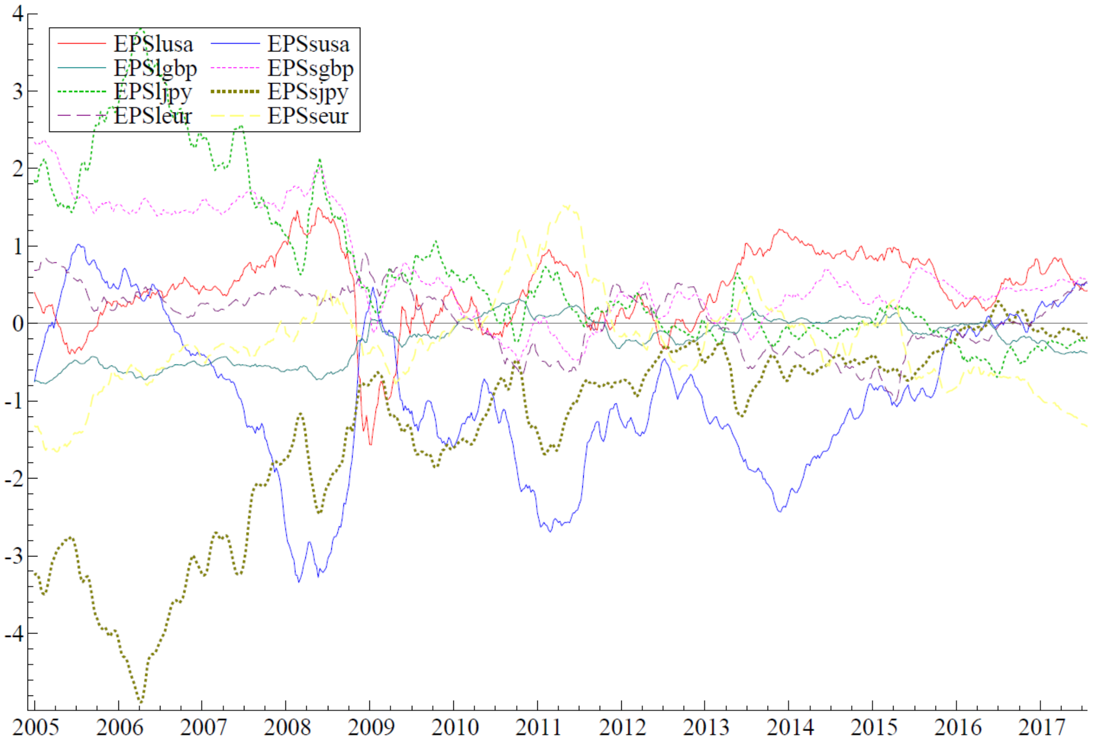


Figure 8: evolution of currency-specific factors over time

and their summary analysis in shown in table 10.

Currency	Factor	Mean	Variance
USD	Level	0.43	0.25
	Slope	-0.94	1.02
GBP	Level	-0.22	0.08
	Slope	0.68	0.47
JPY	Level	0.69	1.13
	Slope	-1.37	1.55
EUR	Level	0.07	0.16
	Slope	-0.27	0.39

Table 10: means and variances of currency-specific factors.

Variance decomposition

There are two sets of factors in the "global model", global factors and currency-specific (idiosyncratic) factors. I decomposed the variance of the input data into one part that is explained by global factors (global Level and Slope together) and another one that is explained by currency-specific factors. One could expect the sum of variances of global factors, currency-specific factors and residuals to be equal to 100%. Unfortunately, this is not the case since there exist correlations between all the parts, which drive the sum to be distinct from 0. The results are shown in table 11.

	Global factors	Currency factors	Model residuals
USD	81%	11%	4%
GBP	94%	21%	3%
JPY	59%	50%	2%
EUR	92%	24%	3%

Table 11: the decomposition of variance into different components shows that variances of USD, GBP and EUR curves are well explained by global factors whereas JPY is explained somewhat less.

Analysis of residuals

Global yield curve factors are estimated via the equation X6 and it is useful to remind here that there are no constants defined for each row (tenor and currency) of input data. This is because the estimation would not be stable and the results would potentially be unfeasible to obtain. This construction implies that the sum of residuals for each row is not guaranteed to be zero and indeed, the sum is distinct from 0.

Table 12 shows the means of residuals for each currency and each tenor. The LHS shows that the model fits well on medium and long tenors but very poorly on short tenors. The RHS of table 12 shows the variance of the residuals and confirms that the model fits tenors longer than 2 years quite well, whereas the fit of short tenors is poor. The problem would very likely be overcome by including time-variant lambda and curvatures for all currency-specific factors.

MEAN [bps]				
	USD	GBP	JPY	EUR
1	-10.4	-17.5	-1.0	-4.2
2	-4.6	-12.0	1.8	2.2
3	0.7	-6.4	4.2	8.8
6	12.5	3.6	10.1	17.7
12	-6.7	-5.9	6.0	13.2
24	-10.2	-6.7	0.4	4.5
36	-7.2	-3.5	-2.7	1.1
48	-3.3	-1.2	-4.1	-0.1
60	-0.3	-0.1	-3.9	-0.3
72	1.4	0.2	-2.4	-0.1
84	1.5	0.0	0.0	0.0
96	0.0	-0.8	3.0	0.0
108	-2.5	-2.2	6.3	-0.3
120	-5.5	-4.1	9.5	-1.0

STD. DEV [bps]				
	USD	GBP	JPY	EUR
1	60	57	10	48
2	57	56	8	46
3	55	55	7	44
6	49	53	9	40
12	40	46	5	29
24	24	25	3	16
36	11	11	4	8
48	4	4	4	3
60	2	1	4	1
72	2	1	2	1
84	2	*	*	1
96	*	2	3	*
108	2	4	6	2
120	5	7	9	4

Table 12 LHS: shows the simple averages of global model residuals for each tenor and each currency. **RHS** shows the standard deviations of the residuals. *stands for variances lower than 0.5

The lambda coefficient that defines the shape of the global yield curve was selected to be just one, $\lambda = 0.01$, for all currency-specific and global factors and for all the dates between 2005 and 2017. As it was selected just based on the averages it does not fit well to the big swings of short terms rates that occurred (e.g. during the 2007/8 financial crisis, EURIBOR 1m fell from 5.12% on 10/10/2008 to just 0.82%

15/5/2009). Allowing lambda to evolve over time would improve the fit, but on the other hand, would decrease the parsimony. Thus the best answer for future modelling should either be to incorporate time-variant lambda or to not include short term rates in the model and just focus on medium and long tenors - because their fit in the model is overall very good.

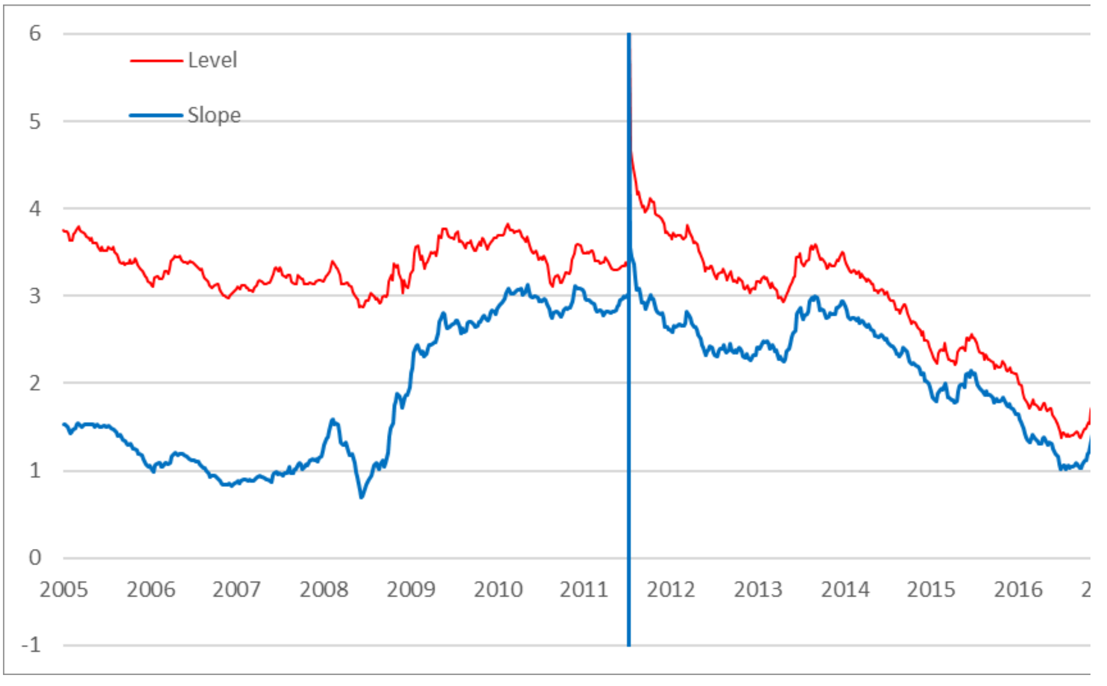
Table 13 analyses the relation between shocks (updates of the state-equations) and shows us that the assumption of independent shocks is not fulfilled. The correlation of shocks to global Level and to global Slope is very high as can also be seen in figure 5 where global factors move closely together. The model would thus be improved if it would also incorporate cross dependencies in the matrices of updates (shocks).

		Global		USD		GBP		JPY		EUR
		Level	Slope	Level	Slope	Level	Slope	Level	Slope	Level
Global Slope		-0.79								
USD	Level	0.11	0.03							
	Slope	-0.19	0.14	-0.33						
GBP	Level	0.24	-0.25	-0.42	0.00					
	Slope	-0.11	0.31	0.09	-0.38	-0.33				
JPY	Level	0.05	0.17	0.13	0.03	-0.31	0.20			
	Slope	-0.30	0.02	-0.20	0.11	0.21	-0.27	-0.79		
EUR	Level	0.16	-0.22	-0.33	0.16	-0.57	0.19	-0.02	-0.01	
	Slope	-0.21	0.21	0.22	-0.30	0.16	-0.56	0.02	0.03	-0.48

Table 13: correlation of shocks to the state equations. Correlation coefficients higher than 0.4 (in absolute value) are highlighted.

Robustness

I made a test to control whether the model is stable with regards to input data. For this test I divided the dataset into two parts, one ranging from the beginning of 2005 to the middle of 2011 and another one that continues where the first one ends up to 15th September 2017.



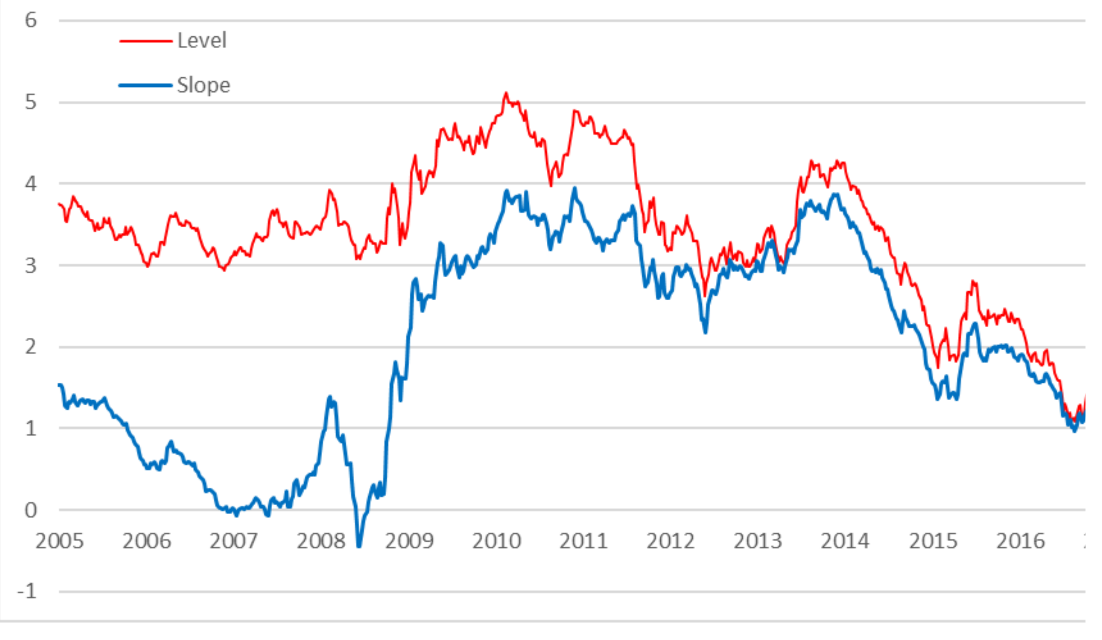


Figure 9: estimated global factors. **The top part** shows the result of model X6 estimated separately from 2 datasets. The vertical line around the middle shows where one set of data ends end and where the other one begins. The initial values were copied from the results of a "full dataset" estimation to both subset-estimations and std. dev of the shocks were halved to improve the comparison.
The bottom part repeats figure 5 to ease the comparison with model X6 estimated via the full dataset (2005 – September 2017)
Please notice that the Slope is depicted with opposite sign.

The top part of figure 9 shows the results of both parts stacked together with the vertical line in the middle to show where one part ends and the other one begins. The results of the estimation based on the full dataset are depicted below it in the same figure to ease the visual comparison of both estimates.

The two parts of figure 9 look similar but are not the same and thus show that the results are not entirely stable. The detailed comparison is shown in table 14, which shows that the correlation of both subsets with the full data set is rather strong, especially in the second part. However, the absolute values of global factors are quite different.

	Dataset	1/2015 – 7/2011	7/2011 – 9/2017	1/2015 – 9/2017
Correlation with full dataset	Level	0.71	0.93	1.00
	Slope	1.00	0.92	1.00
Mean	Level	3.36	2.69	3.03
	Slope	1.81	2.11	1.96
Variance	Level	0.05	0.66	0.46
	Slope	0.69	0.38	0.56
Min	Level	2.87	1.38	0.70
	Slope	0.70	1.02	0.70
Max	Level	3.82	4.67	4.67
	Slope	3.13	3.55	3.55

Table 14: summary analysis of the model estimated via the first part of the dataset (first column), the second part of the dataset (second column) and the full dataset (third column).

The impact of global yield curve to the Czech yield curve

I used the results of the global model described above and estimated with them the model X12, which is a regression where some coefficients are time-variant and part of the residuals is modelled as an auto-regression process. Those time-variant coefficients are further referred to as “loadings”, the global factors are explanatory variables and Czech yields are the dependent variable in this model.

The aim of the model X12 is to estimate how the Czech yields load on global factors over time. However, the load on global Slope appeared to be very unstable in a stability test[10] so it was finally estimated as time-invariant. $B_{CZK,t}$ from X13 thus changes to that defined in X16:

$$B_{CZK,t} = \begin{pmatrix} \beta^{l}_{CZK,t} & \beta^{s}_{CZK} \cdot \left(\frac{1-e^{-\tau_1 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \beta^{l}_{CZK,t} & \beta^{s}_{CZK} \cdot \left(\frac{1-e^{-\tau_2 \cdot \lambda}}{\tau_1 \cdot \lambda}\right) \\ \vdots & \vdots \\ \vdots & \vdots \\ \beta^{l}_{CZK,t} & \beta^{s}_{CZK} \cdot \left(\frac{1-e^{-\tau_n \cdot \lambda}}{\tau_n \cdot \lambda}\right) \end{pmatrix} \quad \text{(X16)}$$

The estimated loadings to global Factors are depicted in figure 10 together with important dates from recent economic history. The load to global Slope is shown there as a constant line, $\beta^{S}_{CZK} = 1.01$. The load to global Level is on average $\beta^{L}_{CZK} = 1.03$ and it was decreasing from 2005 to its lowest value that occurred during the 2007/8-financial crisis.

Afterwards, the load was increasing until the end of the sample with sharp growth just before the start of exchange-rate interventions[11] of the Czech National Bank (further as CNB) and it increased further after the European Central Bank (further as ECB) started its program of asset purchases ("quantitative easing", "QE").

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[10] More details are provided in the Annex, in figure A3

[11] On 7th November 2013, CNB devaluated the Czech Koruna and pledged the exange rate with the Euro to be at least 27 CZK/EUR. The rate was around 25.8 CZK/EUR one day before.

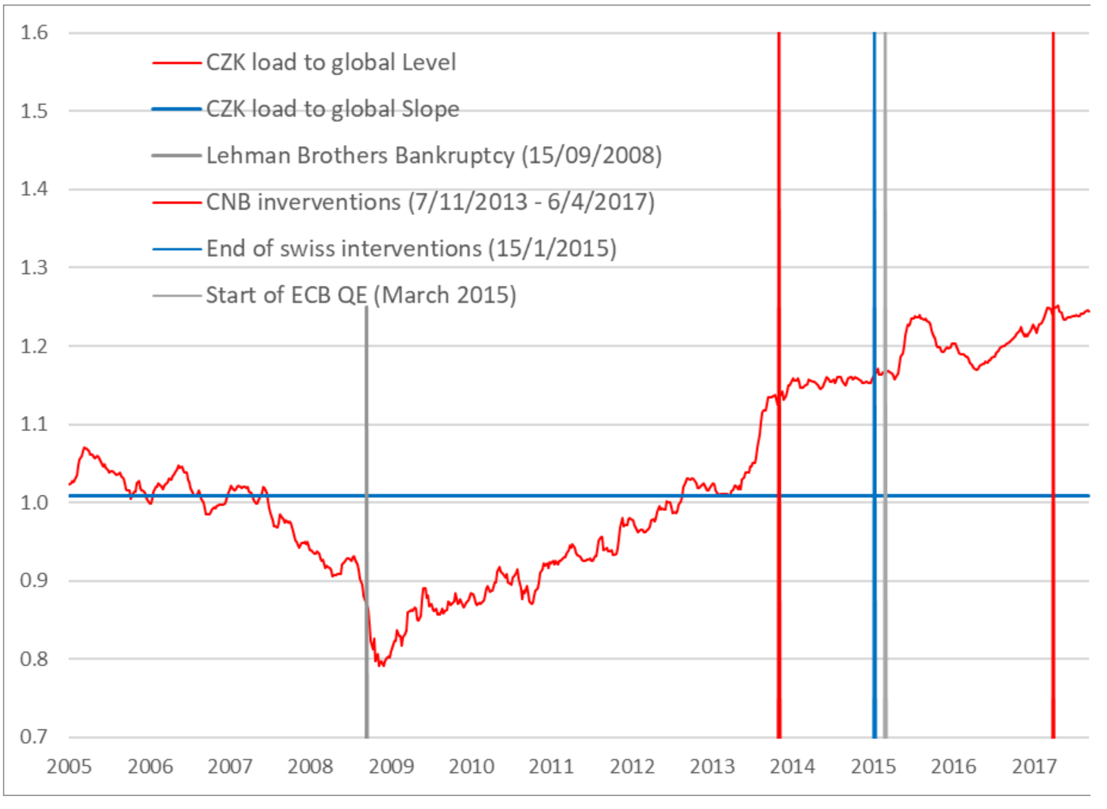

Figure 10 shows how CZK data loads to global factors. Load to global Slope was unstable in a stability test and was modelled as time-invariant. Additionally, several macroeconomic dates are depicted.

Summing up, it seems from figure 10 that the impact of global factors increased recently. This result is supported by figure 11 which shows the variance of residuals (as defined in X17), and tells us that the variance of residuals decreased substantially after 2013 (the year when the CNB started to intervene). A detailed analysis is presented in the Annex, in tables A2 and A3.

$$\hat{v}'_{CZK,t}(\tau_i) = y_{CZK,t}(\tau_i) - \hat{B}_{CZK,t} \cdot \begin{pmatrix} \hat{L}_t \\ \hat{S}_t \end{pmatrix} \tag{X17}$$

(The calculation of residuals is done solely from global factors)

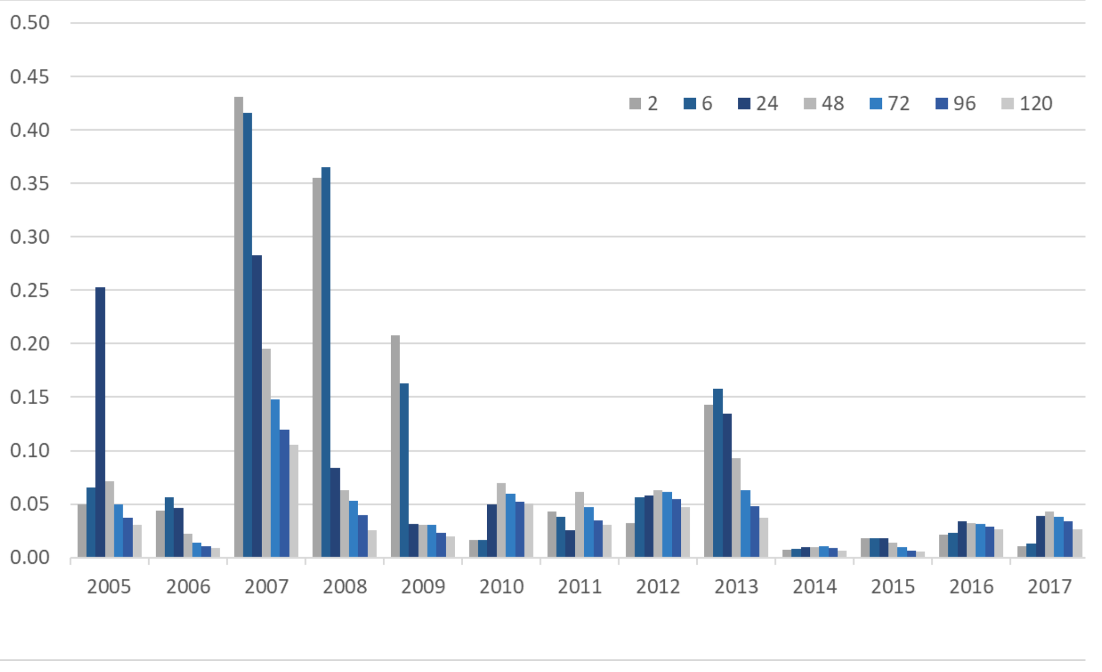

Figure 11: Variance of CZK yields residuals as defined in X17; per year and per selected tenor of 2 and 6 months as well as 24 months (2 years), 48 months (4 years), 72 months (6 years), 96 months (8 years) and 120 months (10 years). Details in Annex A2 and A3.

A detailed analysis of these results shows that the serial correlation of the loading coefficient to global Level is very strong,

$$\phi_{CZK}^{l,l} = 1.000 \ (std.err: 0.001)$$

and that variance of its shocks (updates) is $VAR(\psi_{CZK,t}^{l}) = 10^{-4}$.

The idiosyncratic factors the of the CZK yield curve, $\varepsilon_{CZK,t}^{l}$ and $\varepsilon_{CZK,t}^{s}$have the following estimates of their autocorrelation coefficients, $\phi_{\varepsilon,CZK}^{l,l}$ and $\phi_{\varepsilon,CZK}^{s,s}$:

	Mean	Std. error	Shock variances
$\phi_{\varepsilon,CZK}^{l,l}$	0.997	0.002	$(\sigma_{\varepsilon,CZK}^{l})^2 = 0.004$
$\phi_{\varepsilon,CZK}^{s,s}$	0.991	0.005	$(\sigma_{\varepsilon,CZK}^{s})^2 = 0.015$

Table 15: estimates of autocorrelations' coefficients of CZK-specific factors

And their means and variances are listed in table 16:

	Mean	VAR
$\varepsilon^{l}_{CZK,t}$	0.68	0.92
$\varepsilon^{s}_{CZK,t}$	-0.48	0.87

Table 16: summary analysis of CZK specific factors

The variance of residuals shows again, as in the global model, promising results for the medium and long tenors while it depicts very negative averages and large variances for short tenors (table 17):

Tenor	1	2	3	6	12	24	36
Mean	-0.19	-0.14	-0.07	0.04	-0.12	-0.03	-0.01
Variance	0.27	0.24	0.22	0.19	0.11	0.03	0.00

Tenor	48	60	72	84	96	108	120
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Variance	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 17 shows means and variances of residuals for every tenor of the CZK model

Overall we can say that around 61% of the variation of the Czech yield curve is explained by the global factors. This is somewhat less than for USD, EUR and GBP but similar to JPY that loads by 59% on the global factors.

	Global factors	CZK factors	Model residuals
CZK	61%	51%	4%

Table 18: variance decomposition of CZK input data into different components.

The impact of the EUR yield curve on the CZK yield curve

To assess whether the complex global model is needed to explain the evolution of the Czech yields, I built a much simpler model, which uses solely EUR yields and is described in X14. The evolution of estimated EUR Level and EUR Slope is depicted in figure 12 and resembles, in broad terms, the evolution of global factors. It presumes that the impact of EUR yields and of global yields on Czech yields are similar and the detailed results below confirm this conjecture.

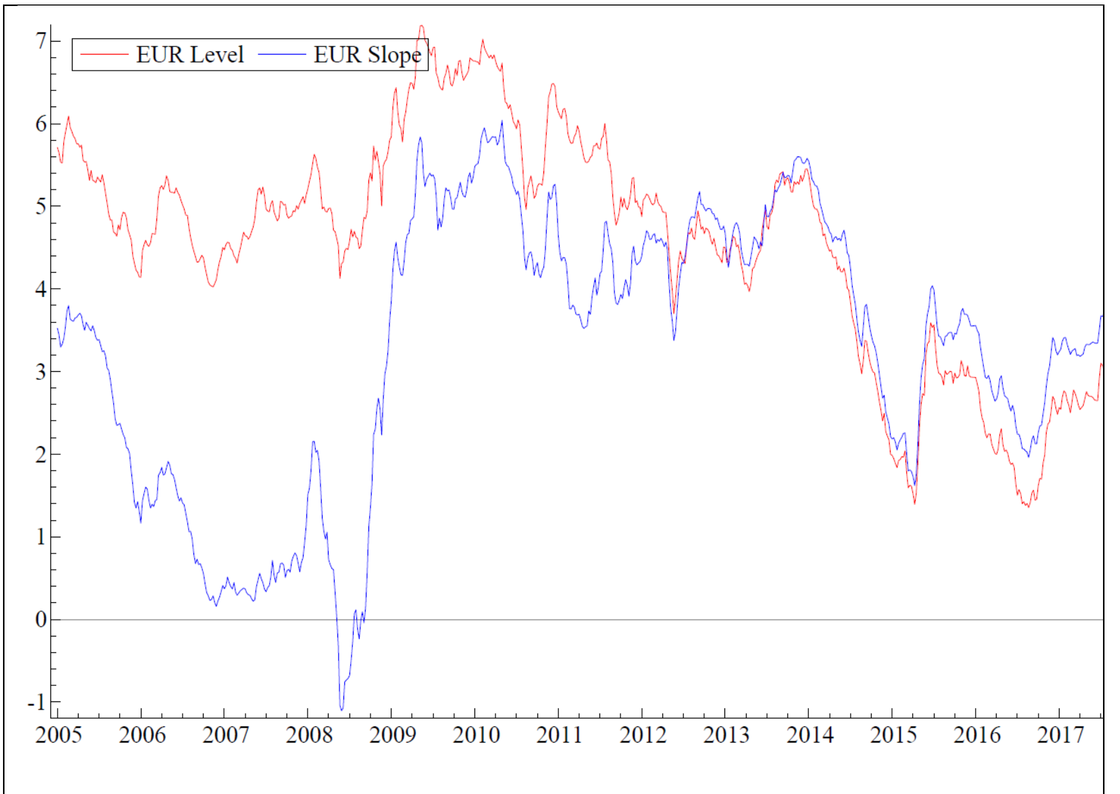

Figure 12: evolution of estimated EUR yield factors. Please notice that **EUR Slope is depicted with opposite sign**.

Table 19 suggests that using EUR factors is more precise than using global factors because 73% of the variance explained in the EUR model is higher than 61% of variance explained in the global model.

	EUR factors	CZK factors	Model residuals
CZK	73%	50%	4%

Table 19: variance decomposition of CZK input data into different components (EUR model)

Figure 13, which shows how CZK yields load to EUR factors, is an equivalent of figure 10. Both figures are very similar to each other and differ only in magnitude. The load to global Level is also sharply decreasing during the peak of the 2007/8 financial crisis and is increasing afterward with a sharp increase just before the CNB started its interventions. The average load to EUR Level is 0.81 while its counterpart for Slope is stable at 0.64.

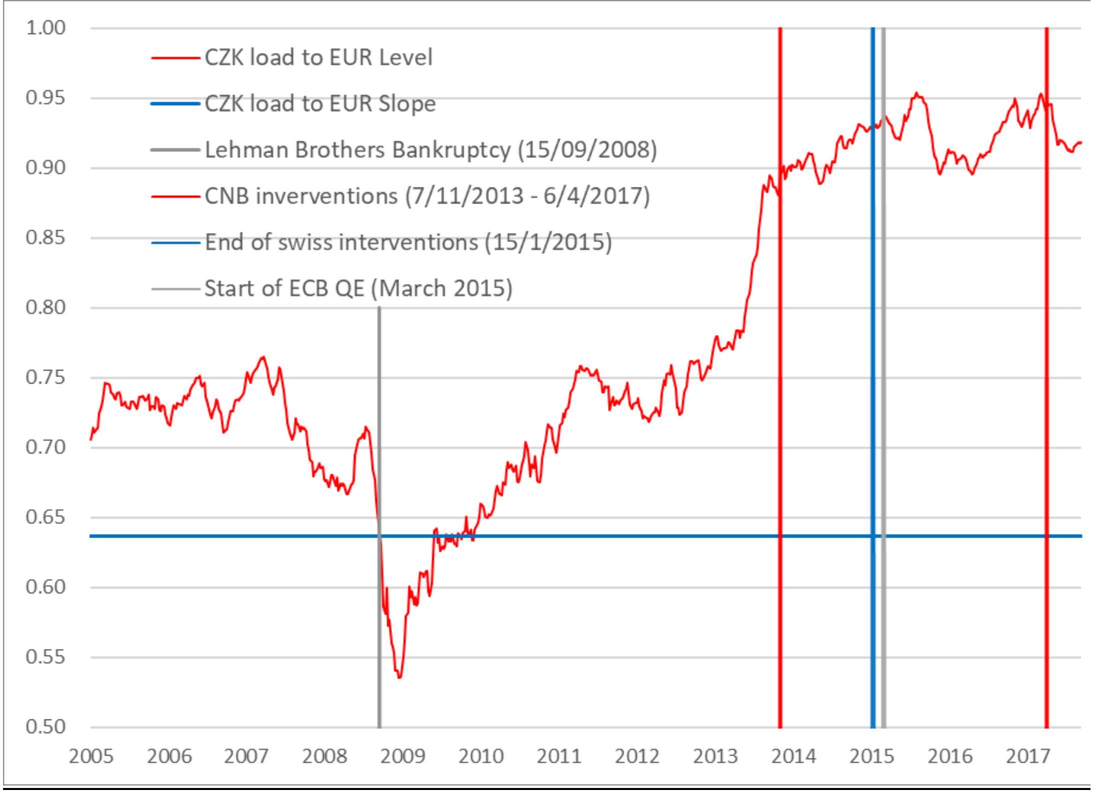

Figure 13 shows how CZK data load to EUR yield factors, to ease the comparison with figure 10, the CZK load to EUR Slope is modelled as time-invariant. Additionally, relevant macroeconomic dates are depicted.

The CZK load to EUR Level is well above its average after the CNB interventions. These results correspond to figure 14 where the variances of residuals from X18 are depicted per year and selected tenors. The result closely resembles the one shown in figure 11: they both show that CZK rates are well explained by global/EUR factors in 2006, then the fits are poor during the 2007/8-financial crisis and very good after the CNB interventions. This thesis does not show a direct link between the interventions and the fact that the CZK rates are well explained by global/EUR factors; it only observes that those two things happened in a similar timeframe. A detailed analysis of the variance of residuals from X03 is described in the Annex, in tables A2 and A3.

$$\hat{v}'_{CZK2,t}(\tau_i) = y_{CZK,t}(\tau_i) - \hat{B}_{CZK2,t} \cdot \begin{pmatrix} \hat{L}_{EUR,t} \\ \hat{S}_{EUR,t} \end{pmatrix} \quad \text{(X18)}$$

(The calculation of residuals is done solely from EUR factors)

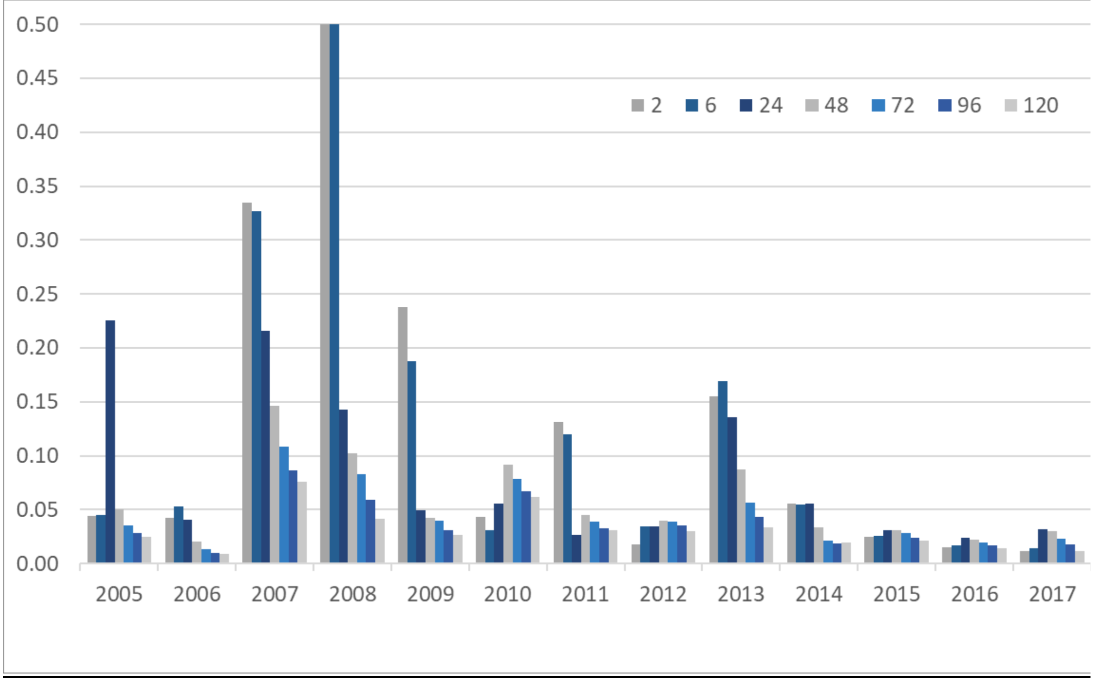

Figure 14: Variance of CZK yield curve residuals as defined in X18; per year and per selected tenors of 2 and 6 months as well as 24 months (2 years), 48 months (4 years), 72 months (6 years), 96 months (8 years) and 120 months (10 years). Details in Annex A4 and A5.

Overall, the EUR yield factors seems to be more suitable in explaining the Czech yield curve as the model is more parsimonious and the variance of residuals, that are not explained by global/EUR factors, is lower for the EUR model. The details are shows in table 20:

	Variance		
Year	Global	EUR	Difference
2005	0.22	0.13	0.09
2006	0.29	0.23	0.06
2007	0.43	0.34	0.09
2008	0.15	0.31	-0.16
2009	0.11	0.23	-0.12
2010	0.07	0.12	-0.05
2011	0.07	0.13	-0.06
2012	0.07	0.05	0.02
2013	0.10	0.13	-0.03
2014	0.01	0.07	-0.06
2015	0.02	0.04	-0.02
2016	0.04	0.03	0.01
2017	0.04	0.02	0.02
All years	0.66	0.42	0.24

Table 20: compares the variance of residuals that are not explained by global/EUR factors for the global (X17) and the EUR (X18) model.
The last row is the overall variance for all years together (***not*** the sum of the figures above it)

Summary of the results

This chapter shows the estimates of latent global yield curve factors - global Level and global Slope. Some interesting results like the flattening of the global yield curve that occurred before the 2007/8 financial crisis was also shown.

The global Level is compared with 10-year yields of selected currencies, showing high correlations with GBP, JPY and EUR, whilst the correlation with USD is somewhat smaller. The global Slope is compared with the differences between 10-year and 3-month yields and shows strong correlations with USD, GBP and the EUR while no correlation with the JPY is found. Variance decomposition between global and country-specific factors shows that the global factors are able to explain a substantial part of the variance of the selected currencies.

The model residuals show promising results for medium and long tenors. However, the fit for short tenors is poor and indicates a disadvantage of the global model. The model stability was tested by splitting the input data into 2 parts and estimating these separately. The result is that the model is quite robust but not entirely because the estimated factors are not all the same when the data are split.

Then I estimated how Czech yields are influenced by the global factors (global Level and global Slope) – whether the evolution of the Czech rates is rather influenced by domestic or by global developments and how this dependence changed over time. For this analysis, I employed a regression model where the estimated global factors are used as independent variables and Czech rates as dependent variables. In this regression I allowed the correlation coefficient ("loading") to global Level to vary over time. The analysis shows that this loading was the smallest during the 2007/8 financial crisis and pretty high during the interventions of the Czech National Bank (CNB). Also the variance of residuals that are *not* explained by the global factors is smaller during the years of the CNB interventions. This suggests that the interventions helped to transmit the global low interest rates to the Czech economy.

To check whether the complex "global model" is needed to estimate how the Czech yields are influenced by non-Czech developments, I repeated the estimation with a "EUR model". In this model only EUR yields are considered instead of global ones and its results seem to be stronger than those of the global model because it is i) more parsimonious and ii) explains a higher proportion of the variance.

The results suggest that the CNB interventions not only served as FX interventions but also as interest rate interventions because it seems that the low yields of the Eurozone were closely transferred to the Czech economy. I can relate this conjecture to the theory of the "impossible trinity" which states that a sovereign state can have only two out of following three macroecomic objectives: i) fixed exchange rate, ii) free capital flows or iii) an independent monetary policy. More can be found in Obstfeld et al (2005).

The Czech Republic has a free flow of capital and its exchange rate with the EUR fluctuated in a narrow band of 27.0 and 27.8 during the entire period[12]. Then, following the impossible trinity, its monetary policy should have been more dependent on the outside world. The results of the Czech model confirms it.

[12] Source: Bloomberg terminal, based on monthly data

7 Conclusion

This thesis follows an approach of Diebold et al (2008) and estimates an evolution of two latent “global” yield curve factors - global Level and global Slope. Yield curves of four currencies are used as input data for the estimation, namely USD, GBP, JPY and the EUR. The result is that global factors are able to explain a substantial share of the variance of medium and long tenors whereas the model is poor in explaining the variance of short tenors.

The estimated global factors are further used to explain the evolution of the yield curve of one currency that is not used in the estimation of global factors – the yield curve of the Czech Koruna. A regression with a time-varying coefficient was built and its results show that the Czech yield curve loads well on both global factors. It also shows that the importance of global Level was the lowest during the 2007/8 crisis and the greatest in recent years when the Czech National bank was intervening on the foreign exchange market. Additionally, the variance of the Czech yields that is left unexplained by the global factors is the lowest in the years of the interventions.

Subsequently the regression was repeated but global yield factors were exchanged with EUR yield factors. The results are similar as with the global factors and the use of the EUR factors seems to be superior to the global factors because the unexplained variance is lower than with the global factors.

The contribution of this thesis is that it uses a recent dataset starting in 2005 and ending in autumn 2017 and thus covers the period before and also many years after the 2007/8 financial crisis. Moreover, it covers all the years when the Czech National bank intervened in the foreign exchange market and is thus able to show what happened during these interventions with the yield curves. This thesis does not provide any direct link between the interventions and the change in the yield curve’s co-movements, but shows that those two things happened in the same time-frame.

Futher, I would suggest to repeat the approach but to use a different currency, which is less interlinked with the Eurozone than the Czech Koruna. Then, the impact of the global yield curve for such a currency should be superiod to the impact of a single-currency yield curve. Moreover, I would suggest imposing a covariance structure for the shocks to the global factors. They appear to be highly correlated and the model in this thesis did not employ their close relationship. Additionally, in further research, the global model should be modelled in a better way to improve the fit for the short tenors. The possible solution is to add curvature factors and/or time-varying lambda coeffient.

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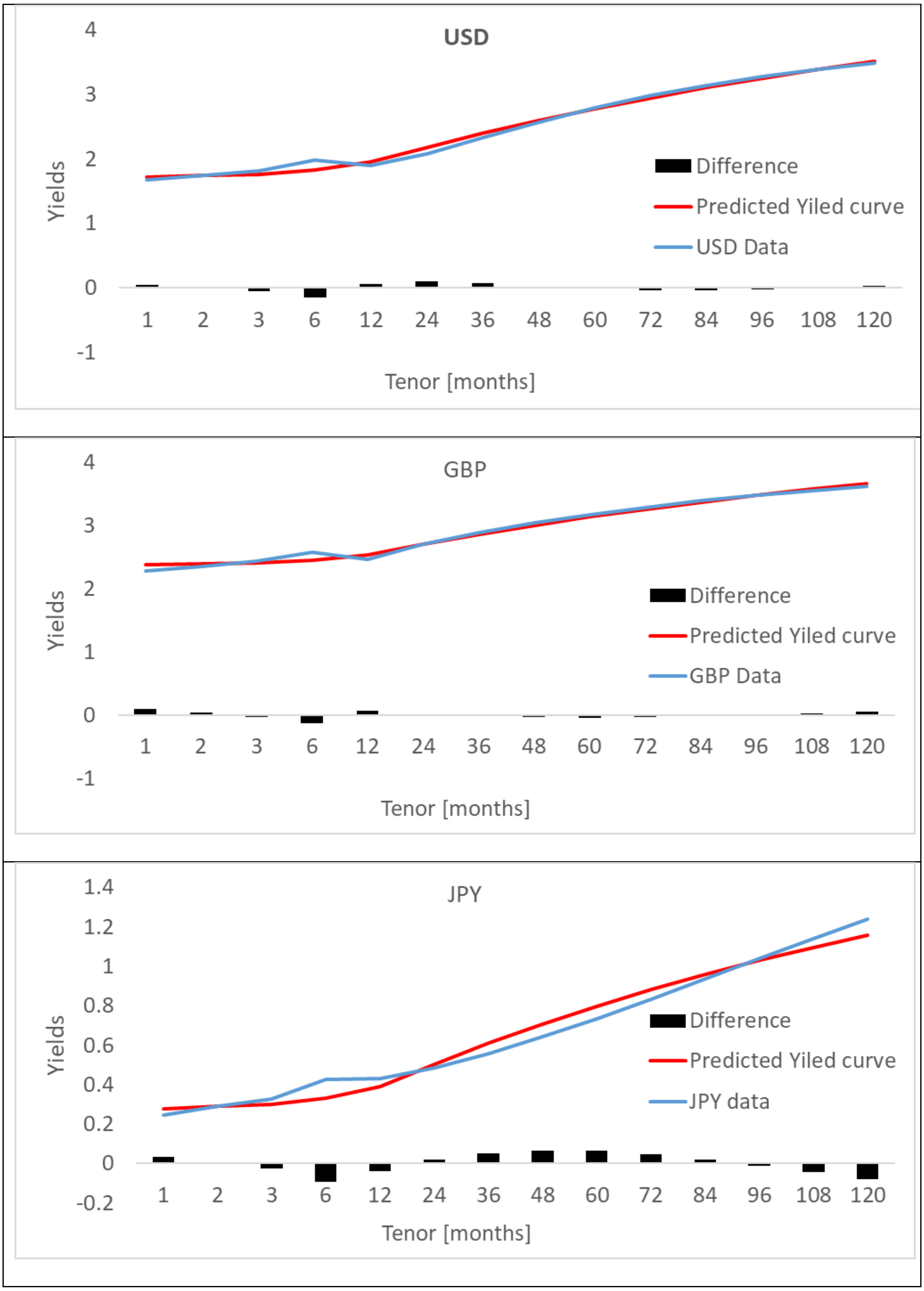
Annex


USD
Yields
4
3
2
1
0
-1
1 2 3 6 12 24 36 48 60 72 84 96 108 120
Tenor [months]
Difference
Predicted Yiled curve
USD Data
GBP
Yields
4
3
2
1
0
-1
1 2 3 6 12 24 36 48 60 72 84 96 108 120
Tenor [months]
Difference
Predicted Yiled curve
GBP Data
JPY
Yields
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1 2 3 6 12 24 36 48 60 72 84 96 108 120
Tenor [months]
Difference
Predicted Yiled curve
JPY data

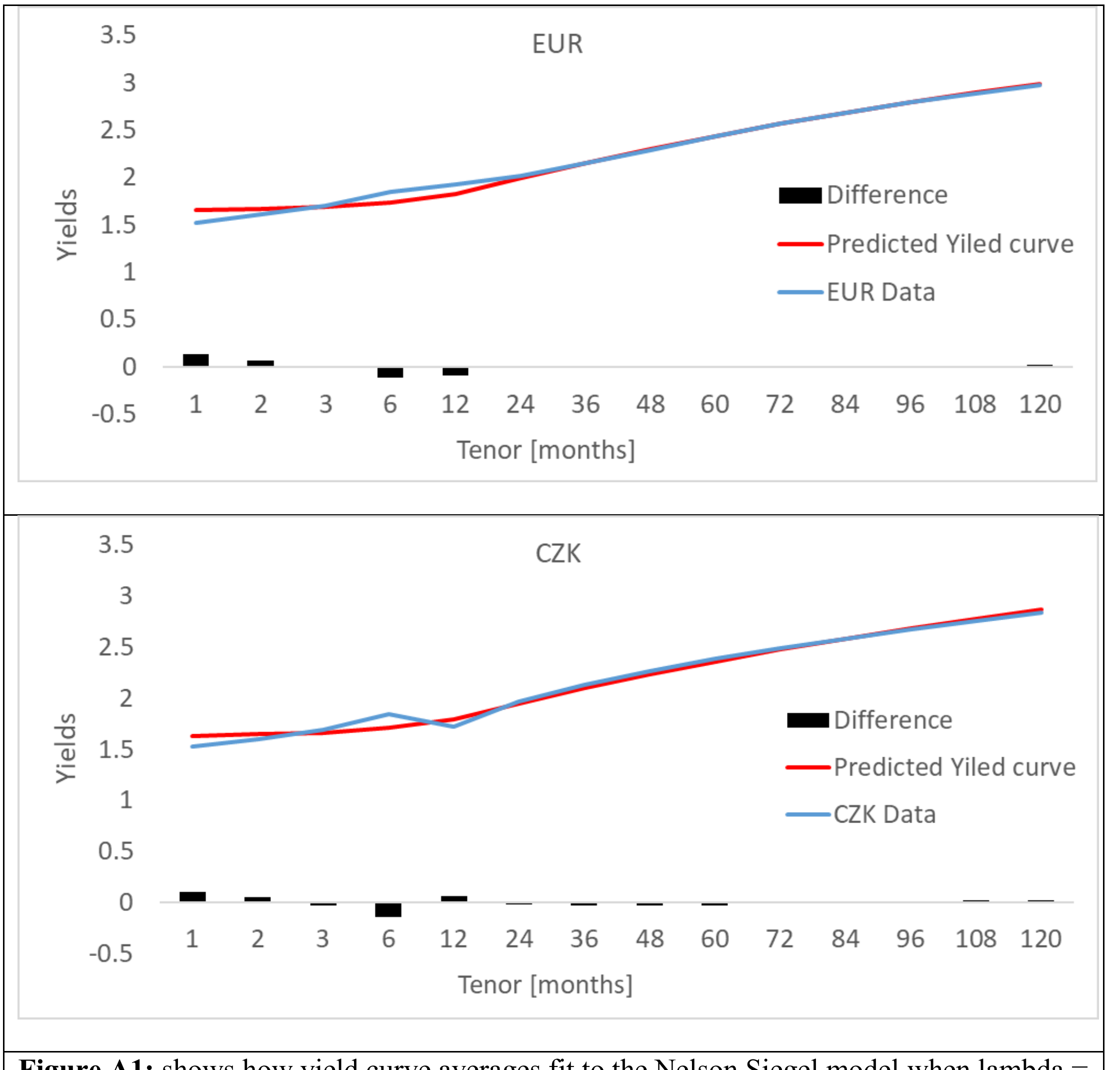


Figure A1: shows how yield curve averages fit to the Nelson Siegel model when lambda = 0.01 is used. The N=S model is based only on Level and Slope.

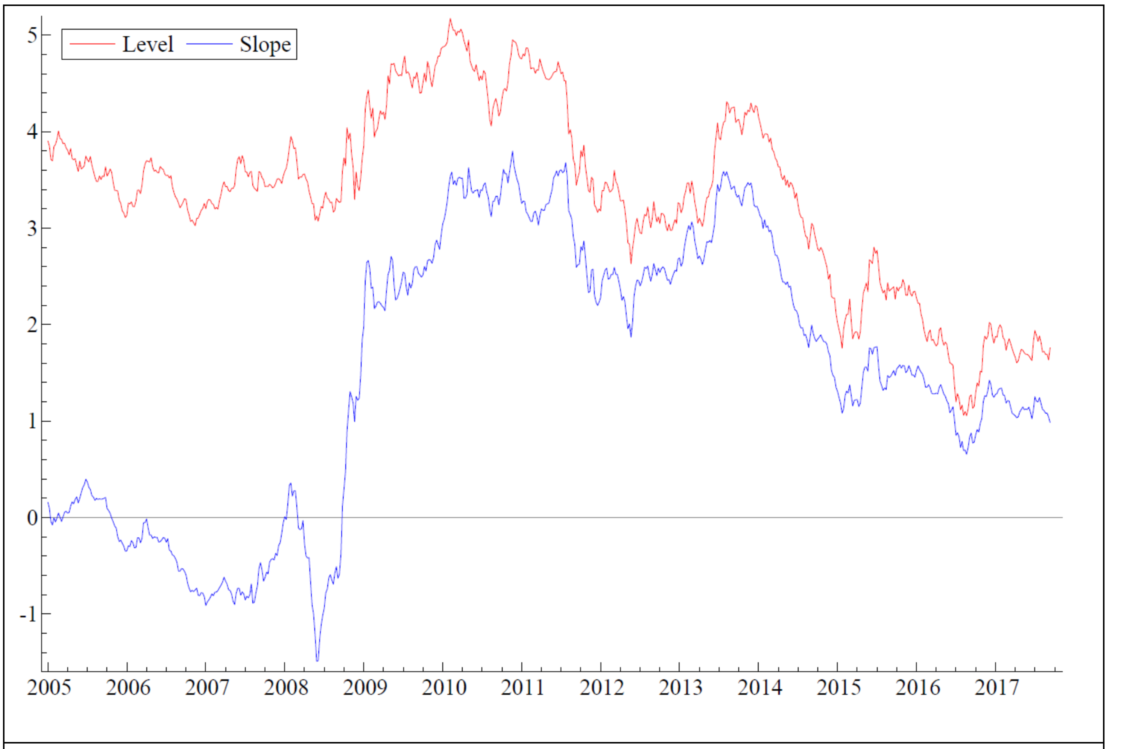


Figure A2: shows global Level and global Slope when the initial values of all auto-regression coefficients in the state equation were set to be 0.5 instead of 1.

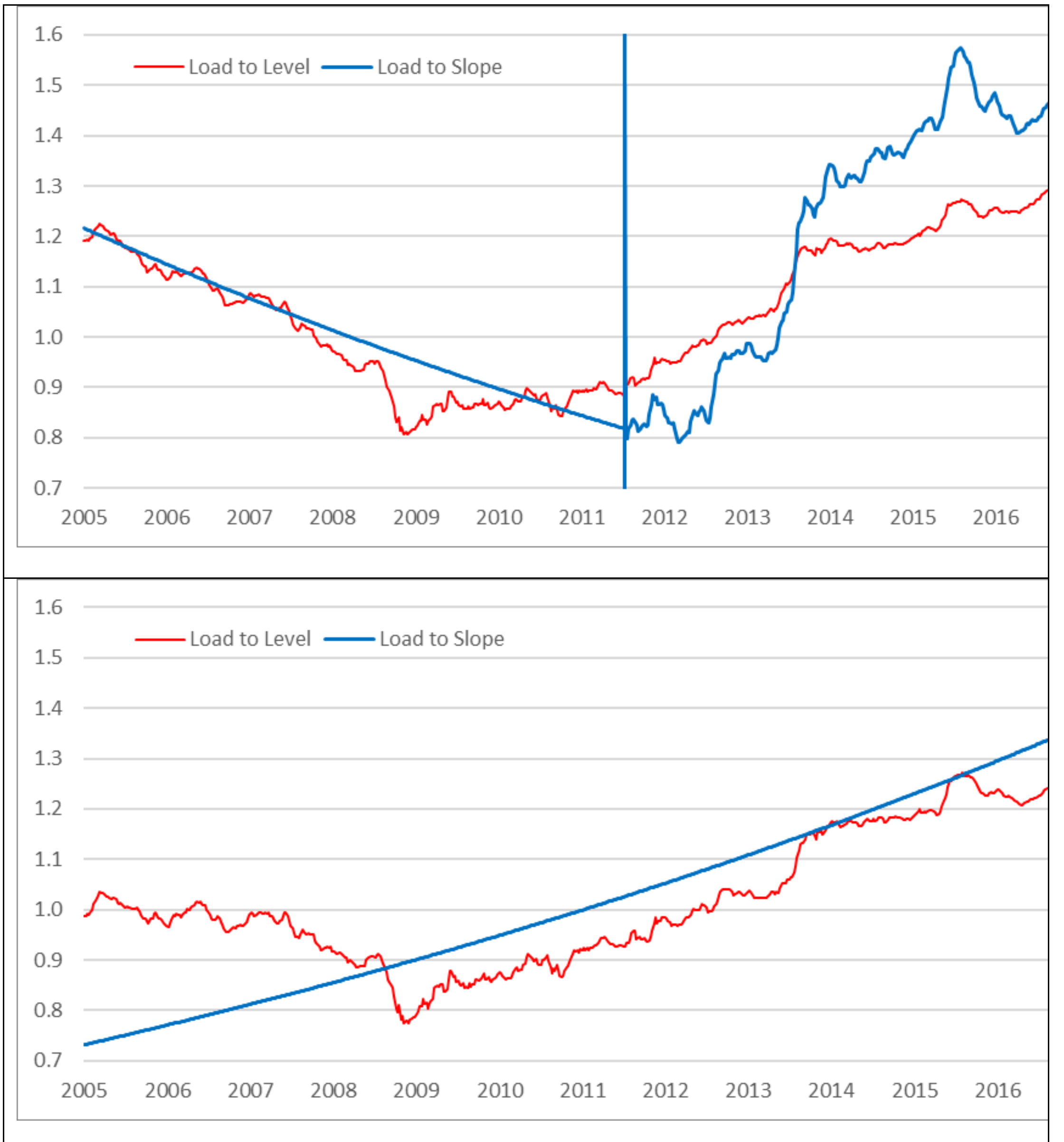


Figure A3 shows how CZK data loads to global Factors. The top part shows the result of an estimation from 2 separate datasets and the vertical line around the middle shows where one set of data ends end the other one begins. The bottom part shows the estimation of the full dataset. Overall, the picture shows high instability of the load to global Level.

Tenor	1	2	3	6	12	24	36
2005	0.05	0.05	0.05	0.07	0.10	0.25	0.09
2006	0.04	0.04	0.05	0.06	0.07	0.05	0.03
2007	0.40	0.43	0.44	0.42	0.37	0.28	0.23
2008	0.36	0.36	0.35	0.37	0.14	0.08	0.07
2009	0.20	0.21	0.20	0.16	0.10	0.03	0.03
2010	0.02	0.02	0.02	0.02	0.03	0.05	0.06
2011	0.04	0.04	0.04	0.04	0.01	0.03	0.05

Tenor	48	60	72	84	96	108	120
2005	0.07	0.06	0.05	0.04	0.04	0.03	0.03
2006	0.02	0.02	0.01	0.01	0.01	0.01	0.01
2007	0.20	0.17	0.15	0.13	0.12	0.11	0.11
2008	0.06	0.06	0.05	0.05	0.04	0.03	0.03
2009	0.03	0.03	0.03	0.03	0.02	0.02	0.02
2010	0.07	0.07	0.06	0.05	0.05	0.05	0.05
2011	0.06	0.06	0.05	0.04	0.04	0.03	0.03

Tenor	1	2	3	6	12	24	36
2012	0.03	0.03	0.04	0.06	0.04	0.06	0.06
2013	0.13	0.14	0.15	0.16	0.14	0.13	0.12
2014	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2015	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2016	0.02	0.02	0.02	0.02	0.03	0.03	0.03
2017	0.01	0.01	0.01	0.01	0.04	0.04	0.04

Tenor	48	60	72	84	96	108	120
2012	0.06	0.06	0.06	0.06	0.05	0.05	0.05
2013	0.09	0.07	0.06	0.05	0.05	0.04	0.04
2014	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2015	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2016	0.03	0.03	0.03	0.03	0.03	0.03	0.03
2017	0.04	0.04	0.04	0.04	0.03	0.03	0.03

Table A1: residuals of CZK yields which are not explained by the global factors, per year and tenor

Tenor	1	2	3	6	12	24	36
All years	0.57	0.62	0.66	0.73	0.59	0.62	0.66

Tenor	48	60	72	84	96	108	120
All years	0.68	0.67	0.66	0.65	0.64	0.64	0.64

Year	Variance
2005	0.22
2006	0.29
2007	0.43
2008	0.15
2009	0.11
2010	0.07
2011	0.07
2012	0.07
2013	0.10
2014	0.01
2015	0.02
2016	0.04
2017	0.04
All years	**0.66**

Table A2: residuals of CZK yields which are not explained by the global factors, the **Top** part shows variances of each tenor (years are pooled) and the **bottom** part shows variaces of each year (tenors are pooled).

Tenor	1	2	3	6	12	24	36
2005	0.05	0.04	0.04	0.04	0.06	0.23	0.06
2006	0.04	0.04	0.05	0.05	0.06	0.04	0.03
2007	0.30	0.34	0.34	0.33	0.28	0.22	0.18
2008	0.51	0.51	0.50	0.52	0.24	0.14	0.12
2009	0.23	0.24	0.23	0.19	0.14	0.05	0.04
2010	0.04	0.04	0.04	0.03	0.04	0.06	0.08
2011	0.13	0.13	0.13	0.12	0.04	0.03	0.04

Tenor	48	60	72	84	96	108	120
2005	0.05	0.04	0.04	0.03	0.03	0.03	0.02
2006	0.02	0.02	0.01	0.01	0.01	0.01	0.01
2007	0.15	0.12	0.11	0.10	0.09	0.08	0.08
2008	0.10	0.09	0.08	0.07	0.06	0.05	0.04
2009	0.04	0.04	0.04	0.03	0.03	0.03	0.03
2010	0.09	0.09	0.08	0.07	0.07	0.06	0.06
2011	0.04	0.04	0.04	0.04	0.03	0.03	0.03

Tenor	1	2	3	6	12	24	36
2012	0.01	0.02	0.02	0.03	0.02	0.03	0.04
2013	0.15	0.16	0.16	0.17	0.15	0.14	0.11
2014	0.06	0.06	0.06	0.05	0.06	0.06	0.05
2015	0.02	0.03	0.02	0.03	0.03	0.03	0.03
2016	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2017	0.01	0.01	0.01	0.01	0.04	0.03	0.03

Tenor	48	60	72	84	96	108	120
2012	0.04	0.04	0.04	0.04	0.04	0.03	0.03
2013	0.09	0.07	0.06	0.05	0.04	0.04	0.03
2014	0.03	0.02	0.02	0.02	0.02	0.02	0.02
2015	0.03	0.03	0.03	0.03	0.02	0.02	0.02
2016	0.02	0.02	0.02	0.02	0.02	0.02	0.01
2017	0.03	0.03	0.02	0.02	0.02	0.01	0.01

Table A3: residuals of CZK yields which are not explained by the EUR factors, per year and tenor

Tenor	1	2	3	6	12	24	36
All years	0.41	0.45	0.47	0.48	0.40	0.41	0.44

Tenor	48	60	72	84	96	108	120
All years	0.46	0.46	0.45	0.45	0.45	0.45	0.45

Year	Variance
2005	0.13
2006	0.23
2007	0.34
2008	0.31
2009	0.23
2010	0.12
2011	0.13
2012	0.05
2013	0.13
2014	0.07
2015	0.04
2016	0.03
2017	0.02
All years	**0.42**

Table A4: residuals of CZK yields which are not explained by the EUR factors, the **Top** part shows variances of each tenor (years are pooled) and the **bottom** part shows variaces of each year (tenors are pooled).