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A Competition on Market with Protection

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Abstract

The aim of this thesis is to model a market with protection, where protection providers can compete for their clients by price competition, but also by military forces. Besides the analytical approach, we examine an agent-based computational model which simulate market with protection. We found out that ability of protection providers to take into account the long-term impact of their actions is necessary condition for emergence of tax competition, and we also described how various factors can influence behavior of protection providers.

Abstrakt

Cílem této bakalářské práce je namodelovat trh s poskytováním bezpečnosti, kde poskytovatelé ochrany mohou soutěžit o své klienty cenovou konkurencí, ale také vojenskými silami. Kromě standardního analytického přístupu prozkoumáme chování agent-based modelu který simuluje trh s obranou. Zjistili jsme, že schopnost poskytovatelů obrany brát do úvahy dlouhodobé dopady jejich akcí je nezbytnou podmínkou pro vznik cenové konkurence mezi nimi. Kromě toho jsme prozkoumali, jak různé faktory ovlivňují chování těchto poskytovatelů bezpečnosti.

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Declaration of Authorship

1. The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.
2. The author hereby declares that all the sources and literature used have been properly cited.
3. The author hereby declares that the thesis has not been used to obtain a different or the same degree.

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Chapter 1: Introduction

In this paper, we will examine the situations in which exogenous enforcement of property rights does not exist, and agents are thus living in some kind of anarchy. Our particular interest will be the industrial organization of security provision – the problem closely related to the theory of origin of the state. The organized protection providers as a kings, Mafia dons or groups of gunslingers can provide protection against bandits or robbers in more efficient way than the single individuals are able defend themselves. But the coercion power of guards and soldiers can be also used against their clients: thus, the former protection provider may act as a predatory organization which is able to extract even more value from the citizens than the ordinary criminals are able to. Moreover, the protection providers may fight against each other in war, with intention to capture more productive agents (or other means of production) under their control. This may lead to the tragedy of coercion, when all the benefits from collective provision of security are lost, and the welfare of agents may be even lower than in absence of organized protection providers.

In this thesis, we will focus on the price competition on market with protection. We will start from the paper of Konrad and Skaperdas (2010), which provide an analytical model of competing protection providers. We will modify this model to capture the effect of tax competition between fighting lords, in order to bring the analysis closer to the conventional microeconomic study of markets and industrial organization.

Furthermore, we will build an agent-based computational model of competing protection providers. This computer simulation will help us model some problems which would be too problematic or difficult to describe by conventional analytical tools. Namely, we will build this model as spatial, so we will be able to explicitly model the mobility of agents and work with the distance between them. Also, this approach will allow us to describe the dynamic of the whole system, not only the equilibria.

The paper is organized as follows: Chapter 2 provide the survey of literature, Chapter 3 describes the analytical model, Chapter 4 describes the agent-based computational model and Chapter 5 concludes. The mathematical background is presented in Appendix A, the description of source code of computational model is contained in Appendix B and the results of computational model is in Appendix C.

Chapter 2: Survey of Literature

The first work which try to describe the provision of security from the similar perspective as provision of any other goods was probably the essay *De la Production de la Sécurité* written by French economist Gustave de Molinari in 1849. Molinari distinguished three basic types of industrial organization – monopoly, competition and collectivism. He claimed that structure of market with protection depends on the type of industrial organization in very similar way as the structure of market with any other good. Molinari argued that market structure of competition with many protection providers may bring the highest welfare for consumers, and this structure may also remain stable, if the consumers are able to flexibly change their protection provider. However, since the protection is the most necessary good, its monopolistic provision may yield very high rent for the monopolist. Thus, the incentive of protection providers to become monopolist is very strong.

The De Molinari's work inspired many political philosophers and economist, mainly from Austrian school. These authors claimed that service of property rights enforcement can be theoretically provided by the private firms operating on free market. Works of Rothbard (1970), Hoppe (2009) or Murphy (2002) provide and insightful view into the properties of law system which would allow the competing firms providing protection minimize their war effort, and avoid the monopolization of security provision. But altought these theories of such law system (usually called anarcho-capitalism or market anarchism by their authors) are interesting, in this paper we will omit such possibilities of law institutions used to avert fighting, and we will focus on more stark setting in which the ability of agents to avoid conflict is strongly limited.

After the foundation of game theory in 1950s and formalization of economic theory, the problem of security provision could be studied in more former manners. Altought the problem of agents acting in some kind of anarchy is definitely not described by current economic literature as deeply as the standard microeconomic models with exogenously enforced property rights, we can observe increasing interest of economists on this problem in last few decades. We will now provide the survey of literature of this kind and discuss the main factors which influence the behavior of agents in anarchy.

Contest technology

The contest between fighting agents is usually modeled by so called contest functions, which determine the probability of winning for each agent. The inputs for these functions are means of warfare - guns, weapons or soldiers, or more generally, resources spend to war effort. Unlike in the case of ordinary economic production, the inputs of conflict are not combined cooperatively, but adversarially against other players. Since the contest functions represent the probability that given player will win the conflict (or, the proportion of prize which he will capture), they need to take values between 0 and 1, and add up to 1.

A wide class of contest technologies that has been examined take the following form:

$$p_i(G_i, G_{-i}) = \begin{cases} \frac{f(G_i)}{\sum_{j=1}^n f(G_j)} & \text{if } \sum_{j=1}^n f(G_j) > 0; \\ \frac{1}{n} & \text{otherwise.} \end{cases} \quad (2.1)$$

Where $f(G_i)$ is a non-negative, increasing function, G_i (G stands for guns) is an input of player i and G_{-i} is a vector of inputs of other players.

This form was axiomatized by Skaperdas (1994) from the axiom of independence on irrelevant alternatives. Hirshleifer (1989) provided a description of different types of contest functions of this form. Important property of the contest function is its “decisiveness” or the “effectiveness of conflict”. This property is related to the economics of scale of function $f(\cdot)$. Consider, for example, the following contest function (this functional form was used by Hirshleifer (1995) or by G. Tullock in his work on rent-seeking):

$$p_1(G_1, G_2) = \frac{G_1^m}{G_1^m + G_2^m}. \quad (2.2)$$

Then the parameter m is so called decisiveness parameter. As m increases, the difference between inputs of each player become more important. Thus, higher decisiveness parameter implies higher incentive for contesting parties

to spend resources on arming. Hirshleifer (1995) described that low value of parameter m can for example describe the war technology of World War I, when the entrenchment and machine guns made successful attack very difficult even for much stronger party. On the other hand, the contest technology of World War II correspond to much higher parameter m , as the mechanized infantry with combination of tanks and airplanes allow the offense to concentrate firepower more rapidly.

Time preference and social norms

As demonstrated by Axelrod (1981), in repeated games exist strong incentive for emergence of cooperation between players. Namely, the strategy tit-for-tat, which cooperate with other players and punish their defection, showed to be one of the most successful strategy in repeated prisoner's dilemma. In such games, the more the players value the future payoffs, the more likely is that they will adopt a cooperative strategy. This approach could provide a rationale for adoption of rules or institutions that call for lower spending on arming, thereby leaving more resources for production of other goods.

However, Powell (1993) and Skaperdas (2008) have found that higher valuation of future outcomes can also increase the player's incentive for arming, as in case of his victory and destroying the opposite party, he may enjoy the next rounds from the position of winner. Therefore, the impact of player's time preference on the intensity of conflict is not *a priori* clear.

Opportunity costs and comparative advantages

The opportunity costs of agents in anarchy may play crucial role in the determination of conflict's intensity. As described by Skaperdas (1992), if the ability of players to produce ordinary good is sufficiently high (compared to their ability of fighting and coercion), they will not spend resources for arming at all, since the ordinary production provide them higher utility than they can obtain by fighting. Also the comparative advantages may be important – in asymmetry setting with one player comparatively superior in fighting, the partial cooperation equilibrium may emerge, in which only one player spend resources for arming.

Similar problem was described by Hirshleifer (1991), who showed that in some cases the worse-endowed party may has stronger incentive for arming and end up as relative winner of the contest, since it has comparative disadvantage in production and comparative advantage in fighting.

Strategic position and availability of commitments

The fact that the agents fight against each others have partially roots in their inability to commit themselves for lower (or zero) war effort. The situation in which all agents don't spend resources to war effort at all is usually Pareto-optimal for them, but not stable since every agent can exploit the fact that others players left themselves defenseless. Thus, we may see the player's interaction also as a bargaining game. As described by Schelling (1980), the ability of players to credibly commit for given action play crucial role in their bargaining ability. But since in our setting of anarchy the agents usually don't have possibility of such credible commitment, the conflict becomes inevitable.

However, Hirsleifer (1995) studied the setting in which one agent is a Stackelberg leader and can perform his action before all others players (who then perform their actions simultaneously). Despite the fact that the agents who act as first obtain lower utility than the group of agents who was reacting to his action, this setting lead to lower total spending for war and was also Pareto-dominant for all players. (The Stackelberg leader can be here view as a party who make a first step in disarmament). This example can illustrate the importance and effect of availability of commitments.

Mobility of consumers

Model similar to case of our interest was described by Wilhite (2006), who also used agent-based modeling. This model allows the peasants (consumers of protection) to move from one protection provider to other, and this factor of mobility lead to lower price of protection. However, this model didn't deal with the warfare between competing protection providers and also the spatial dimension was relatively omitted. But despite that, it provide a suggestion how the mobility of consumers may lower the price of protection.

Historical cases

The service of protection and property rights enforcement is today (and was also in history) provided almost exclusively by the states. But since we intend to model the protection provider as a rational profit-maximizing agent, we will omit his inner structure. For this reason, only some real-world situation can be realistically described by our model. The problem of provision of other public good than protection, the problem of democratic convention and constitutions, or the problem of international diplomacy omitted in our setting can significantly change the behavior of agents and shape of equilibria. For this reason, we will now discuss real-world situations where our model may be reasonably close to reality.

First such situation may be the western medieval Europe between 11th - 16th centuries. The property rights enforcement was provided by feudal lords, who own piece of land, and who also can clash in war against their competitors. North (1991) describe the impact of mobility of capital on behavior of lords – as the production factors become with improving transport technology and institutional setting more mobile, the competing lords has incentive to attract this mobile capital by tax competition and quality of protection provision service.

Another example may be the regime on medieval Iceland between 10th – 13th centuries, described by Friedman (1979). The property rights enforcement was provided by Vikings chieftains called *godar* (pl. *godar*). Unlike the feudal lord, the *godar* didn't claimed any land, thus more *godar* can operate in the same territory. Ordinary Vikings were allowed to choose under who *godar* they want to belong and change their allegiance flexibly. The status of *godar* was also transferable, and can be sold to anyone who want to provide a service of property right enforcement. Despite the fact that large amount of *godar* coexisted on relatively small territory, the emergence of war between them was very rare, and the regime on Iceland was one of the most peaceful of its time. The system remains stable for more than 300 years, until the civil war between *godar* began in 13th century and the Iceland fell under the rule of Norwegian king in 1262.

The market with protection may also emerge in situation where the property rights enforcement is formally provided by monopolistic government, but this government is practically too weak or distant, so the power vacuum exist. This may be the case of American Wild West, described by Anderson (1978). The absence of government's power on west frontier of USA in 19th century led the settlers to form an armed organized groups in order to defend their property against bandits. The intensity of warfare and behavior of these groups varied a lot, as some such groups (form example, group of gunslingers called "Regulators") become a predatory organization, and others, such a cattlemen's associations or land clubs, become a relative peaceful protection providers.

Last but not least, the environment of organized crime can be also viewed as oligopolistic market with protection. As showed by Skaperdas (2001), the competition of Mafia dons for clients is very similar to case of our interest. Also the level of warfare between Mafia dons and their behavior can vary a lot, depending on the environment in which they operate.

Chapter 3: Analytical part

In this chapter, we will examine the model from the paper *The market for protection and the origin of the state* written by Konrad and Skaperdas (2010). We will modify this model with focus on a price competition between protection providers, and we will then use this model later on in Chapter 4 to build a computational agent-based model. The paper provide description of 4 linked models, which evolve from the simplest one to more complicated settings.

The first one is a model of bandits and peasants living in anarchy, where each peasant defend his output against bandits using private protection technology. More efficient protection technology implies more peasants and less bandits, higher payoffs for the peasants and also the same higher payoffs for the bandits, since the labor market is open and agents are allowed to switch their occupation. This condition of labor market clearing holds for all models in then paper, with exception of the occupation of lord.

The second model examine peasant's self-governance regime, where the peasants form a simple government funded by their voluntary contributions. This government can use collective protection technology which is more efficient than the private protection, but since its funding is based on voluntary basis, it suffer from the tragedy of commons. Thus, the number of peasants and also their payoffs are decreasing in the size of group which they formed. This implies that peasants have an incentive to form the self-governing groups as small is possible, and they do so, if we allow for the endogenous determination of group size.

Third model involve monopolistic government called Leviathan, who hire guards to protect the peasants by the same collective protection technology as in previous model, but the guards can also extract the output from peasant's. Thus, this government is financed by involuntary taxation. Leviathan is able to through his coercion power eliminate the problem of tragedy of commons, but since he is also able to confiscate peasant's outputs, he do so, and the peasants may not be better off than in regime of self-governance. If the ability of peasants to resist the Leviathan is lower than their ability to resist bandits, the total output and payoffs of peasants may be even lower than in the case of anarchy from the first model.

The last model examine the case of competing lords, who provide protection against bandits in the same way as monopolistic Leviathan, but moreover they hire warriors for fighting against other lords in order to capture more peasants under their control. Then the technology of conflict between lords is crucial

for determination of total output and agents' payoffs. More effective technology of conflict implies more deadweight losses caused by unproductive fighting. The self-governing groups of peasants is also allowed to emerge beside competing lords, but since these self-governed groups must also defend their territory against lords, it shows up that the peasants will probably have not an incentive strong enough to forming such a group.

Because we are interested in the effect of prize (respectively tax) competition, we will describe only the latest two models, as the first two are not so relevant for our problem. The description of the models is left without any larger changes from the original paper.

A monopoly protection by Leviathan

This model consist of peasants, bandits and the monopolistic protection-provider called Leviathan. The Leviathan also hire the praetorians, his elite corps which help to keep him in power, and guards, who protect peasants against bandits. The size of population of all agents is a fixed integer N .

Each peasant has one unit of resource which represent his ability to produce wealth. He can distribute it between production effort and self-protection. We will denote this self-protection activity by $x \in [0,1]$; thus, the amount of resources used by this peasant for production is $(1-x)$.

The peasant's output need to be protected, otherwise it will be stolen by bandits. Denoting N_g the number of guards hired by Leviathan, the portion of output which can each peasant protect against bandits is given by protection function:

$$p\left(x + f\left(\frac{N_g}{N_p}\right)\right) \tag{3.1}$$

where $f(\cdot)$ is a collective protection technology, and N_p is a number of peasants.

We make following assumptions about the functions above:

Assumption 1: *The collective protection technology $f: [0, 1] \rightarrow [0, 1]$ is strictly concave, twice differentiable, except possibly at one point, and strictly increasing. Moreover, $f(0) = 0$; $f(z) > z$ for all $z \in [0, 1]$.*

Assumption 2: *The protection function $p: [0, 2] \rightarrow [0, 1]$ is twice-differentiable (except possibly at one point) non-decreasing and concave in its argument, $p(0)=0$ and $p(1)=1$.*

Please note that $p(1) = 1$ implies that $p(x+f(z)) = 1$ for all $x+f(z) \geq 1$.

The bandits steal all the unprotected peasant's outputs. With the protection technology described above, the utility of each bandit is given as follows:

$$U_b = \left[1 - p \left(x + f \left(\frac{N_g}{N_p} \right) \right) \right] * (1 - x) * \frac{N_p}{N_b} \quad (3.2)$$

That means bandits take all unprotected peasant's outputs and divide it equally between themselves. More bandits implies stronger competition between them and lower profit for each bandit.

But bandits are not the only ones who want to take peasants' output: the Leviathan, who hire guards protecting peasants against bandits, can also use these guards to confiscate peasants' output.

For the purpose of describing the portion of peasants' output which is Leviathan able to gather, we will use the same protection function which peasants use to protect their output against bandits. Thus, this function will describe the ability of peasants to resist predators (that is, the bandits and also the Leviathan with his guards) for any given level of self-protection x .

In the original model from Konrad and Skaperdas (2010), the setting where this resistance function is different from protection function was also examined. It showed up that if resistance function is sufficiently lower than protection function, the peasants may be even worse off than in case without Leviathan, where no collective protection technology is available. However, the simpler setting with resistance function to be equal to protection function is sufficient for our analysis.

Thus, the Leviathan takes away from each peasants the wealth equal to

$$\left[p \left(x + f \left(\frac{N_g}{N_p} \right) \right) - p(x) \right] * (1 - x) \quad (3.3)$$

And the payoff of peasant, that is, the remaining output which is left to peasant after bandits and Leviathan take their portions, is equal to:

$$U_p = (1 - x) * p(x) \quad (3.4)$$

Each peasant thus strategically chooses x to maximize his payoff, which equals $\rho(x) \cdot (1-x)$. We also suppose a unique such choice \hat{x} . And because the guards and praetorians are hired out from the population of peasants, they will obtain the same payoff as peasants.

Thus, the whole game proceed as follows: First, the Leviathan, in addition to hiring fixed number N_{pr} of praetorians, strategically choose number of guards N_g to hire. Then, the rest of population sort itself between bandits and peasants.

If, given a Leviathan's choice of N_g , $p\left(\hat{x} + f\left(\frac{N_g}{N_p}\right)\right) < 1$ and therefore security is less than perfect, there will be a positive number of bandits:

$$N_b = N - N_{pr} - N_g - N_p \quad (3.5)$$

With the payoff of a bandits equalized to that of a peasants in this case, the number of peasants that would emerge can be implicitly derived. When security is perfect and there are no bandits, the number of peasants simply equals $N - N_{pr} - N_g$. Overall, for each choice of N_g , there will be an induced number of peasants which we denote by the function $v(N_g)$.

Leviathan's objective is to maximize his net receipts by the choice of N_g (provided these receipts are positive), while taking into consideration the effect on the number of peasants as described by $v(N_g)$.

The Leviathan's utility is given as:

$$V_L = v(N_g) * \left[p\left(\hat{x} + f\left(\frac{N_g}{N_p}\right)\right) - p(\hat{x}) \right] * (1 - \hat{x}) - (N_{pr} + N_g) * p(\hat{x}) * (1 - \hat{x}) \quad (3.6)$$

The first term in the equation represents Leviathan's gross revenue (number of peasants times tribute rate times output per peasant). The second term represent the cost of hiring guards and praetorians.

Competing lords

Instead of monopoly protection provider Leviathan, we can consider more protection providers competing for the peasants' output. We will call them lords.

Each lord, in contrast with Leviathan, need to defend his peasants against other lords. He can do that be hiring new agent type: the warriors. Warriors can defend the lord's territory and possibly gain additional territory at expense of his competitors. In this setting, peasants have very limited options: they are tied to their land and at the mercy of their lords who compete over how to divide them up. Later on in our extension, we will allow the peasants to migrate to another lord's territory, which will introduce tax competition between lords.

Let N_{wl} denote the number of warriors hired by lord l . For a given number of lords N_l and peasants N_p , the number of peasants that lord l can conquer, and thus receive tribute from, is given by contest function

$$q(N_{wl}, N_{w-l}) * N_p \quad (3.7)$$

Where $N_{w-l} = (N_{w1}, \dots, N_{w-l-1}, N_{w-l+1}, \dots, N_{wN_l})$ is a vector of warriors hired by the other lords. Also, we assume that the contest function $q(\cdot)$ satisfies the following properties:

Assumption 3: *Let*

$$q(N_{wl}, N_{w-l}) = \frac{h(N_{wl})}{\sum_{j=1}^{N_l} h(N_{wj})} \quad (3.8)$$

Where $h(\cdot)$ is a non-negative, strictly increasing, and concave function, and $\sum_{j=1}^{N_l} h(N_{wj}) > 0$. If $\sum_{j=1}^{N_l} h(N_{wj}) = 0$, then $q(N_{wl}, N_{w-l}) = \frac{1}{N_l}$.

Letting N_{pr} be the fixed number of praetorians and N_{gl} the number of guards hired by lord l , the payoff of the lord can now be written as:

$$V_l = q(N_{wl}, N_{w-l}) * N_p * \left[p \left(\hat{x} + f \left(\frac{N_{gl}}{q(N_{wl}, N_{w-l})} \right) \right) - p(\hat{x}) \right] * (1 - \hat{x}) - (N_{wl} + N_{gl} + N_{pr}) * p(\hat{x}) * (1 - \hat{x}) \quad (3.9)$$

The main difference between this lord's utility function and the utility function of Leviathan is the determination of the number of peasants: in model with Leviathan, the chosen number of guards induces the number of peasants through function $v(N_g)$, but in this model with competing lords, the number of peasants is determined by the number of warriors that the particular lord has relative to other lords.

Now suppose that the number of lords is given at $N_l > 1$. We will next define a notion of equilibrium in which, as in case of Leviathan's monopoly regime, the population of agents other than lords sort itself between the occupations that receive the "reservation" payoff of peasants. Each lord chooses guards and warriors strategically, but (contrary to the case of Leviathan) without taking into account their effect on the total number of peasants N_p . All the occupational choices and lord's strategic choices are made simultaneously and have to be consistent so that they add up to the total population.

We define a lordship regime to be numbers of peasants N_p , bandits N_b , and for each lord l guards N_{gl} and warriors N_{wl} such that:

(I) Each lord $l=1, 2, \dots, N_l$ with a payoff function described above takes N_p as given, and chooses N_{gl} and N_{wl} simultaneously with other lords so that these choices form a Nash equilibrium;

(II) $N_b = \sum_{j=1}^{N_l} N_{bj}$ where for all j $N_{bj} = q(N_{wj}, N_{w-j}) * N_p * (1 - p^j) / p(\hat{x})$ and

$$p^j = p(\hat{x} + f(\frac{N_{gj}}{q(N_{wj}, N_{w-j}) * N_p}))$$

(III) $N = \sum_{j=1}^{N_l} N_{gj} + \sum_{j=1}^{N_l} N_{wj} + N_l * N_{pr} + N_l + N_p + N_b$

Part (I) of this definition is straightforward: the lords compete for "market share" through hiring of warriors and the protection they provide to peasants, although each lord individually does not take account of his own effect on the number of peasants.

Part (II) states that the number of bandits equals the sum of the bandits in each lord's territory, and the number of bandits in each lord's territory is such that the utility of bandits and peasants is equalized. The number of bandits in territory j is inversely related to the total protection level p^j and when there is perfect security (that is, if $p^j=1$) there are no bandits in territory j .

Finally, part (III) is a „market clearing“ condition, so that the Nash equilibrium choices of warriors and guards, the induced numbers of peasants and bandits, and the fixed numbers of praetorians and lords add up to the total population N .

The proposition that follows provides information on existence, uniqueness, and characterization of the lordship regime.

Proposition 1: *Suppose that Assumptions 1-3 are satisfied. Then*

(I) Each lord's payoff, U_L is concave in N_{gl} and N_{wl} and for any given N_p a Nash equilibrium in N_{gl} and N_{wl} exists.

*(II) If the Nash equilibrium N_{wl} s and N_{gl} s are continuous functions of N_p on the interval $[0, N - N_l * (1 + N_{pr})]$, then a lordship regime exist.*

(III) Under any lordship regime, each lord provides the same level of protection.

(IV) A lordship regime is unique in the number of lords and symmetric, whereby every lord chooses the same number of guards and warriors. In such regimes, the number of peasants is strictly decreasing in the number of lords, and each lord's payoff is strictly decreasing in the number of lords.

The proof of the Proposition 1 can be found in Konrad and Skaperdas (2010). As follows from part (IV), more competing lords implies lower total output, as lords fight more intensively if their number increases. We can see that the effect of competition is quite opposite than on the ordinary market, where more intensive competition usually leads to higher satisfaction for the consumers. This follows from the fact that the means of conflict is hiring a warriors, who don't create any wealth. More intensive conflict lead to higher deadweight losses.

We will now extend this model by price competition between lords, which will bring our model closer to standard microeconomical models of competing firms.

Extension by tax competition

Our task is to model the provision of protection as a market; thus, we want to incorporate the price of protection – that is, the tax rate of each lord – to our model.

Menezes and Quiggin (2009) show that the oligopoly market can be modeled by using contest functions. This approach allow to model full gamut of equilibrium outcomes, from Bertrand (with extremely decisive price contest

function) to Cournot model. And more importantly, it will allow us to implement the tax competition into the model of fighting lords described above.

In model of Konrad and Skaperdas (2010), lords take the maximum portion of each peasant's output which they are able to. We will call this part of output a taxable income or shortly TI. That is,

$$TI = [p \left(x + f \left(\frac{N_{gl}}{N_{pl}} \right) \right) - p(x)] * (1 - x) \quad (3.10)$$

We will also introduce new variable called taxation, strategically selected by each lord. The tax rate t_l is a portion of taxable income which lord l takes from his peasants. And the number of peasants living in given lord's territory will now depend not only on number of warriors hired by lord, but also on its tax rate – lower taxes will attract more peasants to move on this territory. Thus, we redefine the contest function $q(\cdot)$ as follows:

Assumption 4: *Let*

$$q(N_{wl}, N_{w-l}, t_l, t_{-l}) = \frac{h(N_{wl}, t_l)}{\sum_{j=1}^{N_l} h(N_{wj}, t_j)} \quad (3.11)$$

Where $h(N_{wl}, t_l)$ is non-negative, strictly increasing in N_{wl} , strictly decreasing in t_l , concave in N_{wl} and there exist such \hat{t} that $q(\cdot)$ is concave in $(-t_l)$ for any $t_l < \hat{t}$. We also assume $\sum_{j=1}^{N_l} h(N_{wj}, t_j) > 0$. If $\sum_{j=1}^{N_l} h(N_{wj}, t_j) = 0$, then $q(N_{wl}, N_{w-l}, t_l, t_{-l}) = \frac{1}{N_l}$.

The proposition about existence of Nash equilibrium similar to proposition 1 can be derived from Assumption 4.

With the contest between lords described by function above, lord may have an incentive to lower his tax rate in order to attract more consumers of protection, that is, the peasants. If we want to say more about the properties of equilibrium, we need to be more specific about properties of function $h(\cdot)$. Thus, we will now impose two additional assumptions, which can be justified relatively well in our setting.

Assumption 5: *If we assume other variables being fixed, the number of peasants acquired by each lords depends only on the relative ratio of numbers*

of lord's warriors, that is, the contest function is homogenous of degree zero in vector of warriors:

$$q(s * N_{wl}, s * N_{w-l}, t_l, t_{-l}) = q(N_{wl}, N_{w-l}, t_l, t_{-l}) \quad (3.12)$$

Where $s \in R, s > 0$.

This assumption implies that if each lord multiple the number of warriors which he hire by the same times as other lords, the sizes of territories which is every lord able to defend are left unchanged. As discussed by Hirshleifer (1989), this assumptions make sense for the military conflict, at least if additional complications as incomplete information or differentiated battlefield are not present.

Assumption 6: *If we assume other variables being fixed, the number of peasants acquired by each lord depends only on the difference of tax rates. That is:*

$$q(N_{wl}, N_{w-l}, t_l + c, t_{-l} + c_{-l}) = q(N_{wl}, N_{w-l}, t_l, t_{-l}) \quad (3.13)$$

Where $c \in R$ and $c_{-l} \in R^{N_l-1}$ is a vector with each component equal to c such that $t_i + c \in [0,1] \forall i \in \{1, 2 \dots N_l\}$.

This assumptions implies that if each lord increase or decrease his tax rate by the same value as other lords, the number of peasants living in each territory remains unchanged. The reasoning which advocate that this assumption may hold in our setting comes from the way how we expect the peasants to make a decision about migration: If peasant consider moving to territory with lower taxation, he faces a transaction cost corresponding to this movement. The size of this transaction cost depends on the technology of movement and other factors, such as language barrier, peasant's social ties, the legislative restriction imposed on migration and so on. We assume these factor to be exogenously given. If the benefit from movement which may be obtained due to relocating to the territory with lower taxation exceeds the transaction cost, the peasant will decide to move. Thus, if each lord decrease or increase the tax rate by the same value, the benefit which peasant may obtain from movement will remains unchanged; this implies that the number of peasants living in each lord's territory will remains unchanged as well.

Proposition 2:

The following contest function fulfills the Assumptions 4-6:

$$q(N_{wl}, N_{w-l}, t_l, t_{-l}) = \frac{(N_{wl})^\alpha * \exp(-\beta * t_l)}{\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * t_j))} \quad (3.14)$$

Where $\alpha > 0, \alpha \in R$ and $\beta > 0, \beta \in R$ are parameters.

The proof of Proposition 2 can be found in Appendix A.

The parameters α and β are decisiveness parameters of our contest function. Parameter α describe the effectiveness of military conflict, while the parameter β describe the effectiveness of price competition, that is, the mobility of peasants.

We will now examine the properties of Nash equilibrium which emerges between competing lords if assumptions 4, 5 and 6 are fulfilled.

Firstly, the payoff of given lord is now defined as follows:

$$V_l = \frac{(N_{wl})^\alpha * \exp(-\beta t_l)}{\sum_{j=1}^{N_l} (N_{wj})^\alpha * \exp(-\beta t_j)} * N_p * \left[p \left(x^* + f \left(\frac{N_{gl}}{N_{pl}} \right) \right) - p(x^*) \right] * (1 - x^*) * t_l - (N_{wl} + N_{gl} + N_{pr}) * U_{pl} \quad (3.14)$$

Where U_{pl} is a payoff of peasants living in territory of lord l . This peasant's payoff is defined as follows:

$$U_{pl} = p(x^*) * (1 - x^*) + (1 - t) * \left[p \left(x^* + f \left(\frac{N_{gl}}{N_{pl}} \right) \right) - p(x^*) \right] * (1 - x^*) \quad (3.15)$$

The first term in this equation represent the output which peasant is able to defend by himself, while the second term represent the portion of output which could be taken be lord, but lord decide to not do it in order to compete with other lords by tax competition.

If we want to say more about properties of Nash equilibrium which emerge in our game, we would need to be more specific about functions $p(\cdot)$ and $f(\cdot)$. But unlike in the case of contest function $q(\cdot)$, author of this thesis is not aware about any way how to derive such functions from microeconomic foundations. We may select arbitrary functions $p(\cdot)$ and $f(\cdot)$, but we will then have not much information about robustness of such results, and the solution will be also computationally very demanding. For these reasons, we will restrict our description to only few limit cases, and more detailed description will be provided in Chapter 4 by using agent-based simulation.

Proposition 3: *Suppose that Assumptions 1, 2, 3, 5 and 6 are fulfilled. Then:*

(I) *For suitable $p(\cdot)$ and $f(\cdot)$, there may exist $\tilde{\beta}$ such that equilibrium tax rate $t_l = 1$ for every $\beta < \tilde{\beta}$.*

(II) *If the equilibrium tax rate t_l and number of hired guards N_{gl} are both positive, then the equilibrium number of hired warriors N_{wl}^* is also positive.*

(III) *If there exists such $\tilde{N}_{gl} > 0$ that equilibrium $N_{gl}^* > \tilde{N}_{gl}$, then $\lim_{\beta \rightarrow \infty} t_l^* = 0$.*

The proof and further discussion can be found in Appendix A.

The part (I) says that even if the peasants are mobile and therefore the contest function $q(\cdot)$ take the form described in Proposition 2, the emergence of tax competition (that is, lower equilibrium tax rates than $t_l = 1$) is only special case. That is, if the mobility of peasants (or other productive factors) is too small, the whole model will works in the same manner as original model from Konrad and Skaperdas (2010).

From part (II) follows that unlike the tax competition, the military competition between lords will emerge always, for any level of parameter of military technology α , if we rule out extreme cases where lords collect no taxes.

Part (III) says that if parameter of peasants' mobility β grows *ad infinitum*, then lords must decrease either their tax rate or numbers of guards to zero.

Conclusion of analytical part

In this chapter, we examined the analytical model from paper *The market for protection and the origin of the state*. We extended this model by tax competition between lords and derived few properties of our extended model. To describe the comparative statics, we will now examine an agent-based computational model which simulate the market with protection.

Chapter 4: Agent-based computational model

motto: *The question of science is not 'Why?'. The question of science is: 'Why not?'*

In this chapter, we will examine a spatial agent-based computer simulation programmed by the author of this thesis in order to model the market with protection described in previous chapter. We will use it to study the dynamic of lord's competition, to explicitly model the mobility of peasants and to examine the factor of time preference on behavior of agents.

Methodology of agent-based modeling

The methodology of agent-based modeling is relatively different from the analytical approach which we used in previous chapter. For this reason, we will now discuss its main properties. Our description will be primarily based on works of Axtell (2000) and Gilbert (2008).

Agent-based models consist of agents that interact within an artificial environment. While object-oriented approach is the most common way of programming the agent-based models, the agents are usually represented as an objects (that is, an instances of some class). Agent objects have states (variables which hold information about the agent, for example his wealth, age or utility) and rules of behavior.

The rules of agent's behavior is also different from the standard analytical approach: it will usually be complicated and also sometimes undesirable to model the agents as a fully rational *homo economicus* who explicitly maximize his utility. Instead of this, the agents are modeled as bounded-rational, that is, they possess some methods or behavior rules which are designed to maximize their utility, but this rules can be very simple and their ability to maximize agent's utility may be limited.

The nature of agent model make possible to easily create the agents as heterogeneous, thus, there is no need to appeal to representative agent.

After the population of agents is created, the model is run, letting the agents interact with the environment and also with other agents in the simulation. Then, if we collected the data how the model evolved, the results which we obtain are the whole dynamic history of the process, not only the stable state.

One of the advantage of agent-based models is that they allow us to explicitly work with spatial environment – the agents may be placed (and also moved) on some continuous space or grid and interact on local level, for example with agents in their neighborhood. In some economic or social problems, like in our case of market with protection, the physical space is important part of problem, but it is usually very difficult to account for mathematically (except in highly stylized or simplified ways).

On the other hand, the computational models face one big disadvantage, compared to their analytical counterparts: single run of the model does not provide any information about the robustness of the results. That is, if we run the model and obtain some result R, we don't know how much change in our model is necessary in order for R to no longer be a result. The only way how to check robustness in agent-based model is through multiple runs, systematically varying initial conditions and parameters.

Another possible problems may be so called artifacts – the systematic errors in model which arise from mistakes in code or from another technical imperfection, and which may cause spurious correlation. The only way how to avoid artifactual results is by careful programming.

Description of basic model

Our first task is to create an agent-based model which will work in similar way as the analytical model from Konrad and Skaperdas (2010). Then, we will modify this model to capture the effect of agent's mobility.

Technically, the simulation is written in Java programming language using open-source modeling toolkit Repast Symphony. The more technical description of the code can be found in Appendix B.

We will use 3 different types of agents in our model: peasants, bandits and lords. The remaining types of agents which appear in analytical model (guards, warriors and praetorians) will be modeled only as variable assigned to each lord.

The agents are placed on grid, which consists of 70 x 70 cells. This grid is wrapped around its borders, that is, our space is toroidal. While this is definitely not realistic model of real world, the toroidal space is quite often used in agent-based models, since it helps to avoid anomaly which may arise in behavior of agent who is nearly border or corner. Peasants and bandits are placed randomly, their initial coordinates are generated from uniform distribution. Lords are also placed randomly, but with the condition that the minimal distance between them is at least 10 cells. (In reality, we may expect

that lords or capital cities of their lands will not be accumulated on small area, but since our selection of their location is exogenous, we applied condition above to avoid anomaly in their behavior).

Each lord possess a variable which represent the amount of warriors he hire. These warriors are used to conquer the area of lord's influence. Namely, each grid cell in our space evaluate the influence of lord l on this particular cell by the following formula:

$$influence_l = N_{wl}/(distance\ from\ lord\ l)^a \quad (4.1)$$

Where N_{wl} is a number of warriors hired by lord l and $a \in R, a > 0$ is a parameter.

The lord who has highest influence on each particular cells will become an owner of this piece of land. From the equation above we can see that the ability of lord to capture particular cell is decreasing in his distance from this cell. As a result, the whole space will be divided between lords, and each lord's territory will be a connected space which include the lord's position. (For illustration, please see picture 1 in Appendix C.)

The parameter a describe the effectiveness of military technology. Higher a leads to more defensive technology, which implies that each lord can easily defend territory close to his position, but the expansion to territory of his competitors is difficult and require a lot of warriors. On the other hand, lower a means more offensive contest technology, where the lord with more warriors can easily conquer territory of his weaker competitors.

Besides hiring warriors, each lord also hire guards to protect peasant on his territory from bandits and to collect tribute from these peasants. The number of guards is also represented only by variable assigned to each lord.

The space contains peasants, who were randomly placed there at the start of simulation. Each peasant, as in analytical part, has one unit of resources which he divide between production and defense. Then, the peasant's utility, his unprotected output and his taxable income are defined in the same way as in the analytical part, that is:

$$U_p = p(x) * (1 - x) + (1 - t) * TI \quad (4.2)$$

Where x is a peasant's private protection,

$$\text{Unprotected output} = \left(1 - p \left(x + f \left(\frac{N_{gl}}{N_{pl}} \right) \right) \right) * (1 - x) \quad (4.3)$$

And

$$TI = \left[p \left(x + f \left(\frac{N_g}{N_p} \right) \right) - p(x) \right] * (1 - x) \quad (4.4)$$

In this model where we don't account for tax competition yet, the lord gain the whole taxable income of his peasants. The unprotected outputs of peasants are looted by bandits. Since we can explicitly work with space (unlike in the analytical part), we define the bandit's area of influence as their neighborhood (namely, we used Moore neighborhood of range 4). If the peasant is in area of influence of more bandits, his unprotected output is split to them equally.

Agent's behavior

Each type of agent – lords, peasants and bandits – possess an algorithm through which he try to maximize his utility. Since our agents are bounded rational, this algorithm is different from explicit optimization which were used in the analytical part.

Lord agent need to optimize two variables – his number of hired warriors and the level of collective protection in his territory. Since both contest technology and the technology of collective protection are concave, he do the optimization in following manner: each round, he randomly increase or decrease these variables and observe the impact on his utility (respectively on his net profit per peasant in the case of collective protection) after 1 round. If this impact is positive, lord will incline to do the same change also in next round. Namely, he will do the same change again with 80% probability, and opposite change with 20% probability. We applied this stochastic mechanism of optimization to avoid anomaly which may occurs if agent will become „trapped“ at some level of the variable which he optimize. If the impact of variable change on lord's utility will be negative, lord will incline to do the opposite change in next round.

Peasant and bandits optimize their behavior in a little bit simpler way: we use an algorithm inspired by Willhite (2006). At each round, each peasant and

bandit observe his neighborhood (defined as Moore neighborhood of range 4) and compare his utility with utility of other agents in this neighborhood. If he finds out that he has the lowest utility in his neighborhood, he will change his behavior randomly: the peasant can increase or decrease his spending on private protection, become bandit, or do nothing. The bandit can decide to become peasant or don't change his behavior. Although this way of optimization is relatively indirect, it leads to equalization of peasant's and bandit's utility, and the peasants will learn to spend that amount of their output on protection which will maximize their utility.

Dynamic of basic model

Up to now, we have created an agent-based model with intention to imitate the model of Konrad and Skaperdas (2010). The agent-based model work in very similar way as the original analytical model – more effective military technology leads to higher warfare between lords, better protection technology leads to lower number of bandits and each lord's payoff is decreasing in number of lords. Since these results are not qualitatively different from the analytical model, we will for the sake of clarity not provide any extensive statistical analysis of these results, as it would require very large amount of repeated experiments in which we would systematically vary parameters of the model. Instead of it, we will focus on the dynamical aspect of our agent-based model, as it is the main difference from the static setting of analytical model from Chapter 3.

Dynamic of labor market

Firstly, we will examine the dynamic of labor market. The Graph 1 show how the numbers of peasants and bandits evolve during time. As we can see, the initial number of bandits was set to be lower than the stable state. Thus, the competition between bandits was relatively weak and they were obtaining very high utility. As some peasants had lowest utility in their neighborhood, they changed their behavior randomly and some of them become bandits. On the other hand, the existing bandits had no incentive to change their behavior, as they were obtaining very high utility. This lead to the growth of number of bandits, which bring stronger competition between them and equalized their payoffs with payoffs of peasants.

After the initial rapid growth of number of bandits, we can observe slight and slow decrease of their number over some time. This was caused by the learning algorithm of peasants, which caused that peasants learn to spend optimal amount of their output to private protection. As this algorithm was working relatively slow, it took some time until peasants set their private protection to level which optimize their utility. As peasants utility increase

with more optimal level of private protection, some bandits had incentive to become peasants.

After the peasants set their protection to optimal level, the labor market finally arrived to the stable state, where agents incentive to change their behavior become very weak. We can still observe some fluctuation caused by the stochastic nature of agent's learning algorithm, but this fluctuation is overall relatively small.

Dynamic of peasant's spending on protection

We will now examine the effect of learning algorithm through which the peasants learn to spend optimal portion of their output to private protection. At the beginning of the simulation, the level of private protection of each peasant is randomly set as a random number from continuous uniform distribution between 0 and 1. As the simulation begins to run, peasants with protection level set far from the optimal point will find themselves to be most unsatisfied agent in their neighborhood, and they will change their behavior randomly. Some of them will become bandits, others will try to increase or decrease their level of private protection. If this change will bring them closer to optimal level of protection, their utility will increase and they will lost an incentive to further change of their behavior. If this change will bring their level of protection to be more distant from the optimal point, the agent will remains the most unsatisfied agent in his neighborhood and he will try to change his behavior also in next round.

As we can see on graph 2, after some time the more optimal level of private protection will prevails in the population of peasants. (In the simulation from which the Graph 2 comes from, the technologies of protections are such that the optimal level of private protection is 0.5). Finally, as the model arrives to the stable state after some time, the optimal level of protection will become dominant in the population of peasants, althought some minor fluctuation will still exist. This stable state is illustrated by Graph 3.

Dynamic of warfare

The evolution of average numbers of warriors hired by each lord is shown on Graph 4. It turns out that the initial number of hired warriors was higher than the stable state, and lords had therefore incentive to decrease their number of warriors to lower level, in order to reduce costs. After some time, the intensity of warfare stabilized on some relatively stable level. Similarly as in the case of peasant's and bandit's learning algorithm, the stochastic nature of lord's optimization algorithm caused some fluctuation in number of hired warriors.

Other aspects of agent-based model

Because we modeled the warriors and guards only as the variable assigned to each lord, and also because we omitted the praetorians in our agent-based model (since they will have no effect on results), the effect of number of warriors on utility of peasants is not present in agent-based model. Also, the costs of hiring guards and warriors are exogenously given in our agent-based model. This small simplification was made for technical reason in order to make the source code easier to program, and since the primary object of our interest is the tax competition between lords and effect of peasant's mobility, these simplifications should not affect the general results of our model.

Extension by mobility of agents

We will now allow the movement of agents in our simulation. Since we explicitly work with spatial dimension and distance in our agent-based model, this approach will allow us to capture the effect of mobility and tax competition in much more realistic manner than we were available to do in analytical model.

Agent's mobility

Now, we will allow peasants to move across space, seeking for areas with lower taxation. Namely, we will create peasants as heterogeneous, with different propensity to move. We can also think about them not as about peasants or farmers, but more generally as about production factors: for example, highly mobile peasants may represent traders or caravans, while low mobile peasants may represent farmers tied to their land. Nevertheless, we will still call these mobile production factors as "peasants" for simplicity.

In each round, peasant may decide to move. The probability he will do so depends on his propensity to move. If the peasant decides to move, he will prospect the grid cells in his neighborhood, and these cells will contest for the winning cell, to which the peasant will move. The probability of each grid cell to "win" this contest depends on its tax rate – the cells with lower tax rate are more likely to be the destination where the peasant will finally move.

We will impose the following rule (similar to Assumption 6 from Chapter 3) for the contest of grid cells for peasant's final destination: The probability of winning depends only on the difference in the taxes among all possible cells.

Assumption 1:

$$p^i(t) = p^i(t + c) \forall c \in R^n \text{ such that } t_i + c \in [0,1], \forall i \in N \quad (4.5)$$

Where p^i is the probability vector for grid cells and t is a vector of taxes in each cell.

As showed by Skaperdas (1994), the only contest function which fulfill the Assumption 1 take the following form:

$$p_i = \frac{\exp(m * t_i)}{\sum_{j=1}^{num.of\ cells} \exp(m * t_j)} \quad (4.6)$$

Where $m \in R, m > 0$ is an effectiveness parameter. Higher m means that peasants choose cell to move on more strictly with respect to its tax rate; on the other hand, lower m implies lower effectiveness of tax competition and the peasants migrate in more random manner.

We will make also the bandits mobile. For the reason to avoid artifactual results, they will use the same algorithm for movement as peasants. This will cause their movement to area with lower taxation, where are more peasants whose output can be looted.

Tax competition between lords

We will now make the determination of the tax rates endogenous. Firstly, we will use the same algorithm which lords use to optimize their others decisions. That is, each lord will try to change his tax rate randomly, and observe the immediate (i.e. after 1 round) effect on his utility. If this effect will be positive, he will be inclined to do the same change next round, if the effect will be negative, lord will prefer the change in opposite direction.

We choose $m=3$ and run the experiment (the results can be found in table 1). As we can see, this setting practically didn't lead to emergence of tax competition – the average tax rate selected by lords was $t=0.947$, and tax rate for average peasant was $t=0.940$, as peasants migrated to territories with lower taxation. Due to the stochastic nature of lords optimization algorithm and due to the fact that that tax rates are bounded by $t=1$, we practically cannot distinguish this result from the setting without mobility.

As we know from Chapter 3, it may be the case that low mobility of peasants lead to corner solution with taxes set to maximum (or close to maximum, in case of our stochastic agent-based model).

Thus, we will run another experiment with $m=15$. In this setting, the peasants select the grid cell where they will move very strictly with respect to its tax rate. (For example, if peasant compare two grid cells for his movement, one with tax rate $t=0.8$ and second one with tax rate $t=0.7$, the latter one has approximately 4 times higher probability to become the final destination of peasant's movement). This selection of parameter m should bring enough incentives for lords to significantly decrease their tax rates, in order to attract more peasants on their territory.

However, if we run the experiment (results can be found in table 2), we see that the results are almost identical to previous experiment with much lower parameter m . Thus, we may ask why lords didn't decrease their tax rates in this setting, where the lower taxes may be definitely very strong competitive advantage. To answer this problem, we will now examine the factor of time and time preference.

Factor of time and time preference

The reason why lords didn't lower their tax rates in order to obtain more peasants is the time dimension of tax competition. The migration of peasants is a gradual process – if lord decrease his tax rate, it will firstly attract only few more peasants from the neighborhood of his territory. But peasants who are too far from the borders of given lord will not immediately change their routine, and will keep moving across the map in unchanged manner. However, if this routine of random walk across the map bring them to the borders of the lord who lower his taxes, the effect of lower taxes will finally affect their behavior. But it will take some time before this moment occurs. How much, depends on the speed of peasant's migration and the parameters of contest function which peasants use to decide where they will move, and also on the actual state of the model (for example, the population of peasants may be concentrated on some territories, or they may be distributed uniformly across the map.) This relatively well correspond to real world, where the movement of mobile production factors is also relatively slow and gradual process.

The algorithm for lord's optimization of tax rate which take into accounts only immediate impact on lord's income may be realistic for population of lords with very strong time preference. If lords strongly discount their future profit and are interested only by immediate gain, the algorithm which we used may be a realistic model of their decision-making. In such environment, the price competition between lords will not take place. Despite the mobility of peasants, peasants' utility will not increase (compared to model without mobility), since tax rate in each territory will be set close to maximum.

However, we may more realistically assume that lords are aware of the long-term impact of their choice of tax rate on their income and they take it into account in their decision-making. Thus, we will now modify their optimization algorithm of tax rate selection to capture this effect. After the lord change his tax rate, he will wait for some time until he evaluate the impact on his profit. That is, he will not change his taxes each round, but in a longer interval, and his decision will be based on changes in average profit which he made in this interval.

We will now select delay between lords evaluation and selection of their tax rates to be 25 rounds, and we will also return mobility parameter to be $m=3$. We then run the experiment (results can be found in table 3), and we may finally observe the emergence of tax competition between lords, as the tax rates decrease to significantly lower level than in the case where lords evaluate only immediate effect of their decisions. The snapshot of this simulation can be found on Picture 2 – as we can see, peasants are accumulated on territories of lords who selected lower tax rates than their competitors.

Finally, we will run the experiment with $m=15$ and delay between lords decision set to 25 round. In this setting, the average tax rate imposed on peasants was much lower than in previous experiments. Therefore, we may conclude that peasant's mobility and lord's time preference are complementary factors, which together create the incentive for lords to compete against each other by lower tax rates.

Impact of tax competition

We may also observe that the emergence of tax competition significantly decreases the average number of warriors hired by lords. The explanation is that the tax competition works as substitute for military competition. If lord whose tax rate is high will hire more warriors and conquer larger territory, the peasants who he capture will move away to territories with lower tax rates. Also lords with low tax rates don't have much incentives to try conquer larger territory, as the peasants in their neighborhood will migrate to their territory on their own.

Another aspect of tax competition is its impact on level of collective protection provided by lords. Lords tax income is linked to the number of guards who provide collective protection against bandits. If lord decrease his tax rate and therefore his tax income, his incentive to hire guards also

decrease, and lord will provide lower level of collective protection to peasants living on his territory.

Factor of military technology

We will now fix other parameters and run experiments with different parameters of military technology. The results of these experiments can be found in table 5. The more offensive technology leads to higher number of warriors hired by lords, as the expansion of lord's territory become easier and lords have incentives to compete more intensely by their warriors. The more offensive technology also lead to lower profit for lords, as they need to pay their larger armies. We may also saw that more defensive technology leads to slightly lower taxes, as lords use tax competition as substitute to military competition, but this effect is relatively small. The impact on other variables showed to be practically very small and insignificant.

Factor of collective protection technology

We will now fix other parameters and run the experiments with different technology of collective protection. We tried 3 different collective protection technology (the results can be found in table 6).

Firstly, we can see that better technology of collective protection lead to lower number of bandits, as it become more difficult for them to loot peasants output. Also the peasants' utility increases with better technology of collective protection. But the most interesting effect is the impact on intensity of competition between lords. As we can see, the military and also tax competition between lords is much more intense with better collective protection technology. That is, lords hire higher numbers of warriors and also set lower tax rates. This is caused by the fact that better collective protection technology allow lords to obtain higher payoff from their peasants. The peasants are therefore more valuable for lords, and thus they compete for them more intensely.

Conclusion of agent-based model

The agent-based computational model examined in this chapter allowed us to describe some aspects of competition on market with protection which we were unable to describe by conventional analytical methods. The artificial environment which we created is still much simpler than real world, but unlike the mathematical approach it allow us to model the spatial dimension of

lords' competition. The optimization algorithm of bounded-rational agents are also far from the way how real world people make their decisions, but this simplicity allow us to relatively easily examine the factor of time preference on lords' behavior. It may be interesting to model decision making of agents in our environment by some more complex kind of artificial intelligence, which task can be subject of further research.

Chapter 5: Conclusion

In this thesis, we examined the provision of protection in setting where exogenous enforcement of property rights does not exist.

We firstly provided a survey of literature and described how different forms of technologies and possibility of various action available to agents can influence their behavior.

Then, in the analytical part, we examined the model from paper *The market for protection and the origin of the state*. We modified this model by adding the mobility of consumers of protection (peasants), and we saw that high mobility may induce the tax competition between protection providers (lords). On the other hand, if the mobility of peasants is low enough, the behavior of lords will remains unchanged from the original model.

Finally, we examined an agent-based computer simulation created in order to model the market with protection. This approach allowed us to explicitly model the spatial dimension of security provision, and also to examine the factor of time and time preference. We found out that the ability of protection providers to consider long-term impact of their actions is a complementary factor to mobility of consumers of protection, and these factors together create an incentive for lords to lower their tax rates. Furthermore, the tax competition may work as a substitute for military competition. Therefore, in environment with high mobility of consumers, we may expect that the industrial organization of security provision may be relatively similar to market with ordinary goods and services.

In our model, the incentive for lords to compete for peasants comes from their ability to collect taxes by their guards. If this ability to collect taxes (which is linked to provision of collective protection) increases, lords will compete more intensely, both by military forces and by tax competition.

Appendix A: analytical background

Proof of proposition 2: We will show that our function fulfills Assumptions 4-6. Let

$$q(N_{wl}, N_{w-l}, t_l, t_{-l}) = \frac{(N_{wl})^\alpha * \exp(-\beta * t_l)}{\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * t_j))} \quad (\text{A.1})$$

This function is in the form required by Assumption 4. In the following text, we will denote $q(\cdot) = q(N_{wl}, N_{w-l}, t_l, t_{-l})$ for clarity.

Now, we will show that $q(\cdot)$ is homogenous of degree zero in vector of warriors. Let $s \in R, s > 0$. Then:

$$\begin{aligned} & q(s * N_{wl}, s * N_{w-l}, t_l, t_{-l}) \\ &= (s * N_{wl})^\alpha * \exp(-\beta * t_l) / [\sum_{j=1}^{N_l} ((s * N_{wj})^\alpha * \exp(-\beta * t_j))] \\ &= s^\alpha * (N_{wl})^\alpha * \exp(-\beta * t_l) / [s^\alpha * \sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * t_j))] \\ &= (N_{wl})^\alpha * \exp(-\beta * t_l) / [\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * t_j))] \\ &= q(N_{wl}, N_{w-l}, t_l, t_{-l}) \end{aligned} \quad (\text{A.2})$$

Now, we will show that also Assumption 6 is fulfilled. Let $c \in R$ and $c_{-l} \in R^{N_l-1}$ be a vector with each component equal to c such that $t_i + c \in [0,1] \forall i \in \{1, 2 \dots N_l\}$. Then:

$$\begin{aligned} & q(N_{wl}, N_{w-l}, t_l + c, t_{-l} + c_{-l}) \\ &= (N_{wl})^\alpha * \exp(-\beta * (c + t_l)) / [\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * (c + t_j)))] \\ &= (N_{wl})^\alpha * \exp(-\beta * c) * \exp(-\beta * t_l) / [\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * c) * \exp(-\beta * t_j))] \\ &= (N_{wl})^\alpha * \exp(-\beta * t_l) / [\sum_{j=1}^{N_l} ((N_{wj})^\alpha * \exp(-\beta * t_j))] \\ &= q(N_{wl}, N_{w-l}, t_l, t_{-l}) \quad \blacksquare \quad (\text{A.3}) \end{aligned}$$

Proposition 3 - properties of Nash equilibrium: Applying F.O.C., the following conditions must hold in equilibrium:

$$\frac{\partial V_l}{\partial N_{wl}} = \frac{\partial q(\cdot)}{\partial N_{wl}} * N_p * TI_l * t_l - U_{pl} = 0 \quad (\text{A.4})$$

$$\frac{\partial V_l}{\partial N_{gl}} = \frac{\partial TI}{\partial N_{gl}} * q(\cdot) * N_p * t_l - U_{pl} - \frac{\partial U_{pl}}{\partial N_{gl}} * (N_{gl} + N_{wl} + N_{pr}) = 0 \quad (\text{A.5})$$

and

$$\begin{aligned} \frac{\partial V_l}{\partial t_l} = \frac{\partial q(\cdot)}{\partial t_l} * N_p * TI_l * t_l + q(\cdot) * N_p * TI_l + \frac{\partial TI_l}{\partial t_l} q(\cdot) * N_p * t_l \\ - \frac{\partial U_{pl}}{\partial t_l} * (N_{gl} + N_{wl} + N_{pr}) = 0 \end{aligned} \quad (\text{A.6})$$

Where U_{pl} is a utility of peasants living in territory owned by lord l defined as:

$$\begin{aligned} U_{pl}(t_l, N_{gl}) = p(x^*) * (1 - x^*) + (1 - t_l) \\ * \left[p \left(x^* + f \left(\frac{N_{gl}}{q(\cdot) * N_p} \right) \right) - p(x^*) \right] * (1 - x^*) \end{aligned} \quad (\text{A.7})$$

TI_l is a taxable income of peasants living in territory of lord l defined as:

$$TI_l(t_l, N_{gl}) = \left[p \left(x^* + f \left(\frac{N_{gl}}{q(\cdot) * N_p} \right) \right) - p(x^*) \right] * (1 - x^*) \quad (\text{A.8})$$

And x^* is a level of private defense strategically selected by peasants, defined as follows:

$$x^* = \arg \max U_p \quad (\text{A.9})$$

We can use the fact that due to symmetry of game, equilibrium $q(\cdot)$ is equal to $1/N_L$ (where N_L is exogenously given number of lords), that is, each lord

capture the same number of peasants. We will now compute the marginal effect of N_{wl} and t_l on $q(\cdot)$. By differentiation, we obtain:

$$\frac{\partial q(\cdot)}{\partial N_{wl}} = \frac{\alpha * (N_L - 1)}{N_{wl} * N_L^2} \quad (\text{A.10})$$

and

$$\frac{\partial q(\cdot)}{\partial t_l} = \frac{-\beta * (N_L - 1)}{N_L^2} \quad (\text{A.11})$$

Part (I): For case of equilibrium tax rate t_l , we will now check for the appearance of corner solution $t_l = 1$ where no tax competition between lords exist. Let

$$t_l^* = \arg \max V_l \quad (\text{A.12})$$

where $t_l^* \in (0,1]$. Then, the tax competition between lords will emerge if:

$$\begin{aligned} \frac{\partial V_l}{\partial t_l} = & \frac{\partial q(\cdot)}{\partial t_l} * N_p * TI * t_l^* + q(\cdot) * N_p * TI + \frac{\partial TI}{\partial t_l} q(\cdot) * N_p * t_l^* \\ & - \frac{\partial U_{pl}}{\partial t_l} * (N_{gl} + N_{wl} + N_{pr}) > 0 \end{aligned} \quad (\text{A.13})$$

The first term represent the effect of peasants mobility, that is, the effect that lord will lose some peasants if he increase his tax rate. The second term represent the drop of lord's income from peasants who he already own. This effect may be practically very large and will be the primary cause of emergence of corner solution with $t_l^* = 1$. The third term represent the effect on peasant's output – if lord decrease his tax rate, peasants will spend slightly less resources on private protection and thus produce more. We may expect this effect to be practically very small. Finally, the fourth term represent the fact that lower taxation lead to higher utility of peasants, and since lord's guards, warriors and praetorians demand the same payoff as peasant, lords costs will increase. We expect also this effect to be practically very small.

The condition for corner solution can be rewritten as:

$$\beta < \frac{N_L}{(N_L - 1) * t_l^*} + \frac{\partial TI(t_l^*, N_{gl})}{\partial t_l} * \frac{N_L}{(N_L - 1) * TI} - \frac{\partial U_{pl}}{\partial t_l} * \frac{(N_{wl} + N_{gl} + N_{pr}) * (N_L)^2}{(N_L - 1) * N_p * TI} \quad (\text{A.14})$$

Since the $\frac{N_L}{(N_L - 1)} \in (1, 2]$, $t_l^* \in (0, 1]$, and since the effect of tax rate TI_l may be reasonably expected to be practically very small as described above, there may exist some threshold $\tilde{\beta}$ such that if $\beta < \tilde{\beta}$, the corner solution $t_l = 1$ emerges in equilibrium, and the whole model works in the same manner as the original model from Konrad and Skaperdas (2010)

Part (II): We know that following condition must holds in equilibrium:

$$\frac{\partial V_l}{\partial N_{wl}} = \frac{\partial q(.)}{\partial N_{wl}} * N_p * TI * t - U_{pl} = 0 \quad (\text{A.15})$$

That is,

$$\frac{\alpha * (N_L - 1)}{N_{wl} * N_L^2} N_p * TI * t - U_{pl} = 0 \quad (\text{A.16})$$

Assumption on positivity of number of guards hired by lord ensure the positivity of TI , that is, lord can obtain positive profit from new peasants. Also, we assumed that equilibrium tax rate $t_l > 0$.

For contradiction, let's assume that $N_{wl}^* = 0$. Then, we have

$$\frac{\partial q(.)}{\partial N_{wl}} (N_{wl} = 0^+) = +\infty \quad (\text{A.17})$$

Which together with the positivity of TI and t implies

$$U_{pl} = +\infty \quad (\text{A.18})$$

Which is a contradiction, since U_{pl} is a real number bounded by $U_{pl} = 1$.
Therefore, it must be that equilibrium number of $N_{wl} > 0$. ■

Part (III): For non-corner solution, the following equation holds in equilibrium:

$$\begin{aligned} \frac{\partial V_l}{\partial t_l} = & \frac{-\beta * (N_L - 1)}{N_L^2} * N_p * TI * t + \frac{N_p}{N_L} * TI + \frac{\partial TI}{\partial t_l} q(.) * N_p * t_l \\ & - \frac{\partial U_{pl}}{\partial t_l} * (N_{gl} + N_{wl} + N_{pr}) = 0 \end{aligned} \quad (\text{A.19})$$

Which can be rewritten as:

$$\begin{aligned} 1 = & \frac{(N_L - 1) * t_l}{N_L} * \left[\beta + \frac{\partial TI(t_l, N_{gl})}{\partial t_l} * \frac{N_L}{(N_L - 1) * TI} - \frac{\partial U_{pl}}{\partial t_l} \right. \\ & \left. * \frac{(N_{wl} + N_{gl} + N_{pr}) * (N_L)^2}{(N_L - 1) * N_p * TI} \right] \end{aligned} \quad (\text{A.20})$$

We will denote:

$$\begin{aligned} c = & \frac{\partial TI(t_l, N_{gl})}{\partial t_l} * \frac{N_L}{(N_L - 1) * TI} - \frac{\partial U_{pl}}{\partial t_l} \\ & * \frac{(N_{wl} + N_{gl} + N_{pr}) * (N_L)^2}{(N_L - 1) * N_p * TI} \end{aligned} \quad (\text{A.21})$$

Which is positive and bounded, as follows from the assumption of lower bound of N_{gl} , which implies the lower bound for TI_l .

Then, we can rewrite the whole expression as:

$$t_l = \frac{(N_L)}{(N_L - 1) * (\beta - c)} \quad (\text{A.22})$$

Therefore, we have

$$\lim_{\beta \rightarrow \infty} \frac{(N_L)}{(N_L - 1) * (\beta - c)} = 0 \quad (\text{A.23})$$

Which implies

$$\lim_{\beta \rightarrow \infty} t_l = 0 \quad \blacksquare (\text{A.24})$$

Appendix B: specification of agent-based model

This section provides description of agent-based model from Chapter 4.

Technically, the model is written in Java programming language using open-source modeling environment Repast Simphony. While the code is relatively long and will not be much useful for readers not familiar with Java, we will at least briefly describe the main algorithm.

The program consist of peasants, bandits and lords, placed on grid. Each agent type is represented by its class, from which the agent object are created. The program also contains class GridCell which represent the terrain.

The program started by creating 1000 peasants, 200 bandits and 10 lords.

We chose to use protection technology $p(x)=x$ and collective protection technology $f(x) = x^\gamma$, where $\gamma \in R$ is a parameter. For all experiments except experiment 6, we used $\gamma = 1/2$.

The initial number of warriors hired by each lord was set as a random number from uniform distribution between 20 and 40. Initial level of collective protection was set for each lord to a random number between 0.1 and 0.4. The initial level of peasant's private protection was set to random number from uniform distribution between 0 and 1. The mobility of peasants and bandits is a random number between 0.5 and 1.

Then, the main cycle is run, and agents start to perform their actions. Each round consist of the following steps:

1. Lords choose if they want to increase or decrease their numbers of warriors and guards, and also tax rates, and they remember their choice.
2. Each grid cell evaluate the contest of lords for this cell by equation (4.1), and lord with highest influence will become owner of this cell. The cell's tax rate is set to tax rate of its owner.
3. Movement of peasants and bandits. Each agent draw a random number from uniform distribution between 0 and 1, and if his parameter of mobility is higher that this number, he will move. If he move, he prospect the cells in Moore neighborhood of range 4, and these grid cells contest by their tax rates for agent's final destination by contest function described by equation (4.6).
4. Peasants produce output, which is distributed to them, their lords and bandits in their neighborhood.

5. Bandits and peasant look around and prospect the utility of other agents in their neighborhood (that is, in Moore neighborhood of range 4). If they found out that their utility is lower than minimum utility of agents in their neighborhood + 0.01, they will make a random decision: Peasant will increase his private protection with 20% probability, decrease his private protection with 20% probability, or become a bandit with 10% probability. The size of change in private protection is a random number between 0.00 and 0.05. Peasant may also decide to do nothing with 50% probability. The bandit may become a peasant with 10% probability.

6. Lords evaluate their actions, that is, the increment or decrement of their number of warriors and guards made in step 1. If they find out that this change provide them higher utility, they will do the same change also next round with 80% probability, and opposite change with 20% probability. And vice versa, if this change brings them lower utility, they will incline to do change in opposite direction. The sizes of changes are 1 warrior, collective protection of 0.02 and 0.05 change in tax rate, respectively.

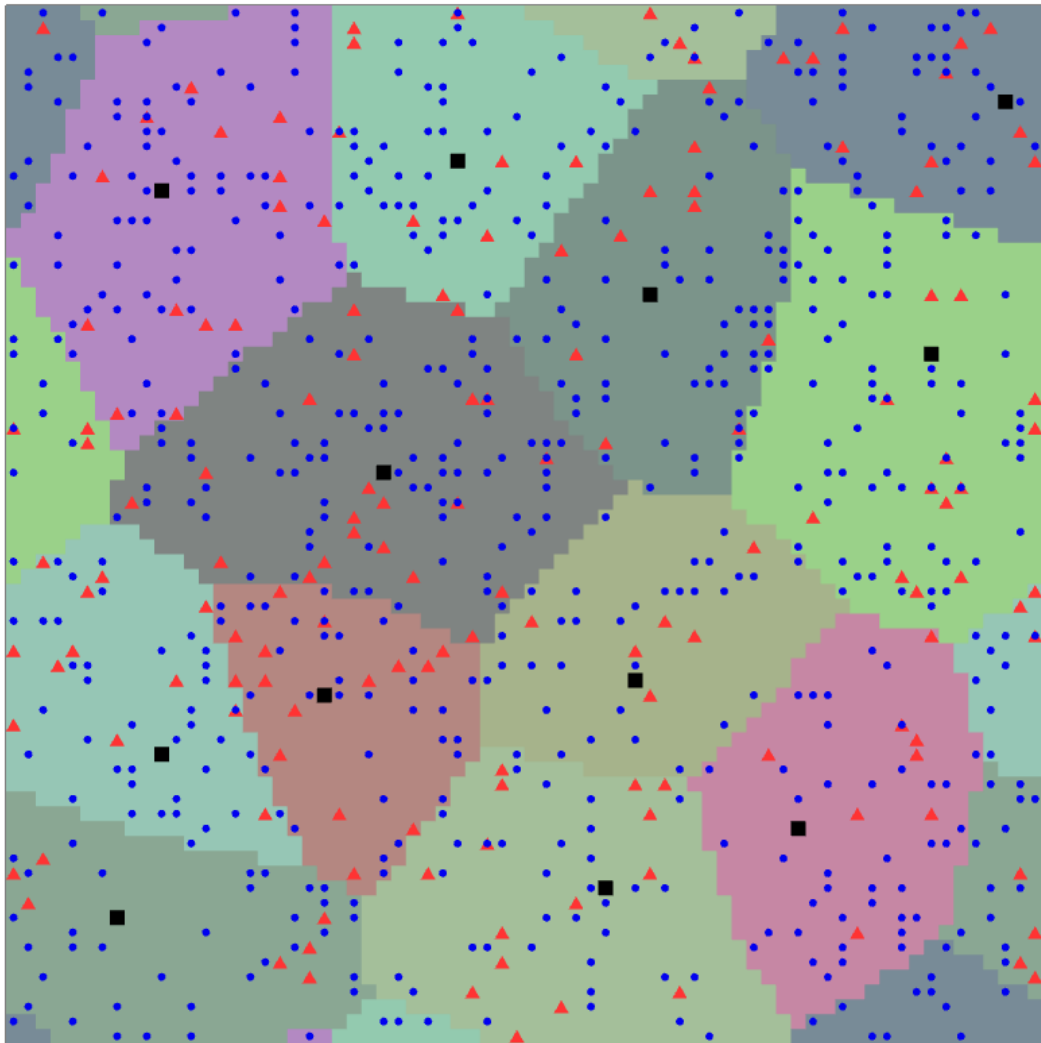
Then, new round begin and the process repeats. After 5000 rounds, the game is stopped and the data are gathered.

Appendix C: results from agent-based model

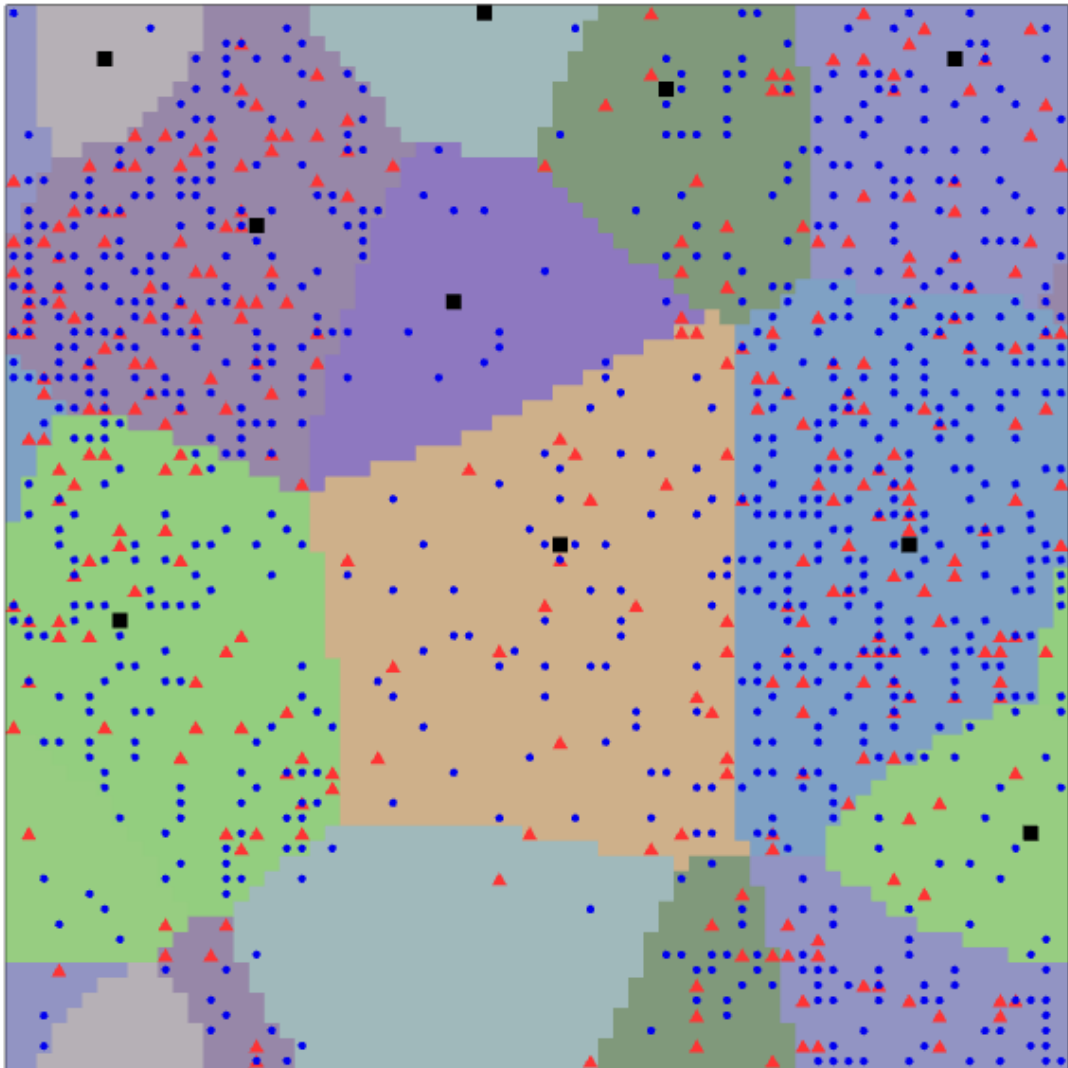
This section contains pictures, graphs and tables outputted by the agent-based model described in Chapter 4.

Pictures

Picture 1: A snapshot of the model at the start of simulation. Lords (or their capital cities) are represented by black squares, and their territories are distinguished by different colors. Peasants are represented by the blue circles and bandits by the red triangles.

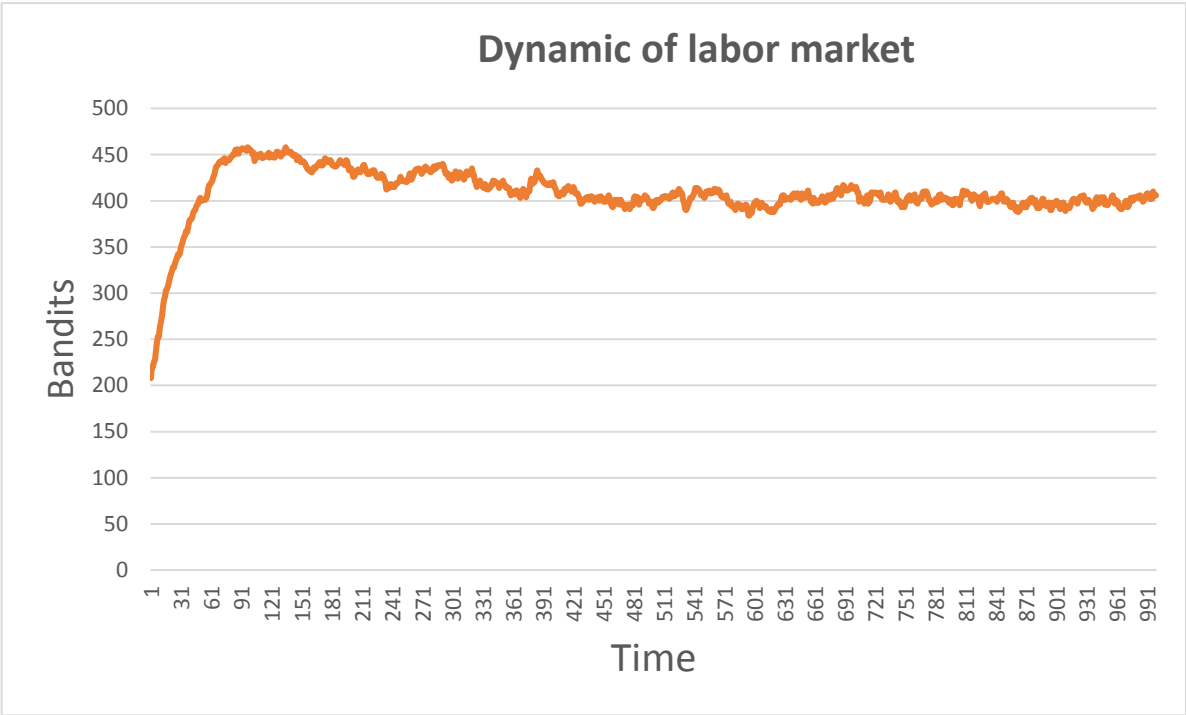


Picture 2: A snapshot of the simulation with tax competition. The peasants are accumulated on territories of lords who selected lower tax rates.

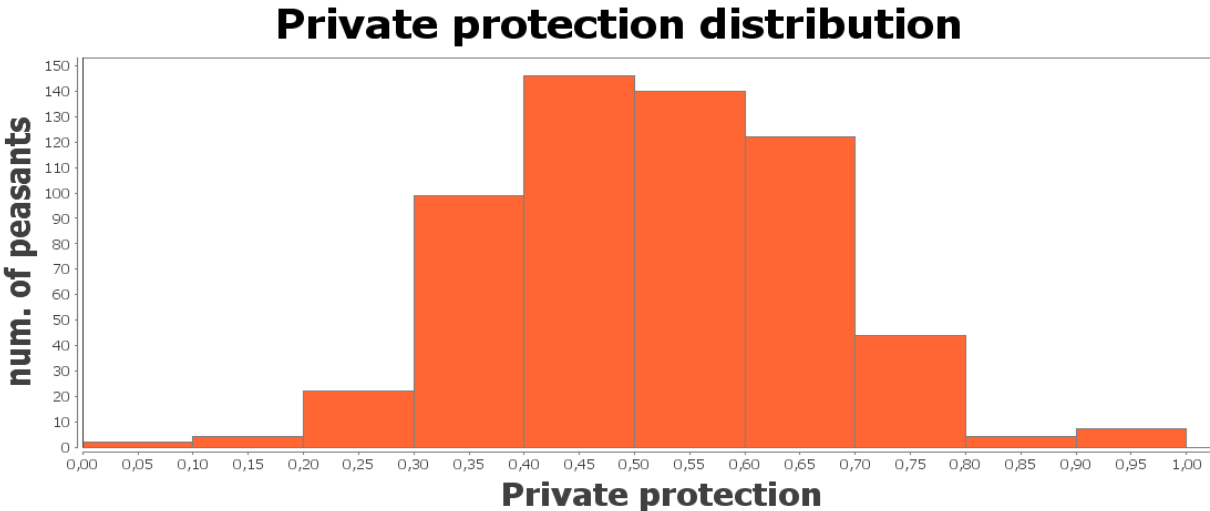


Graphs

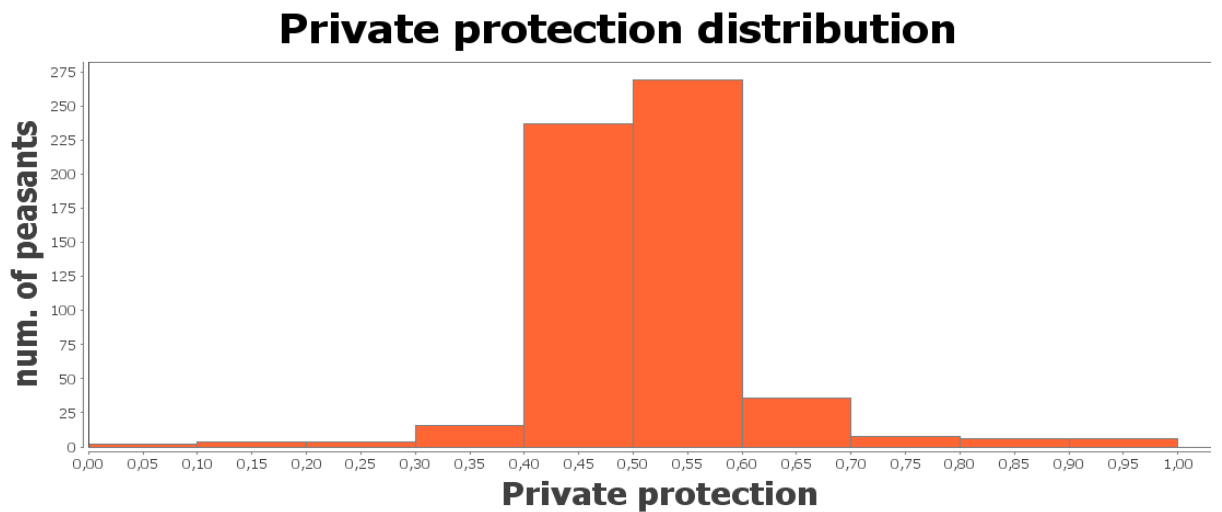
Graph 1: Dynamic of labor market.



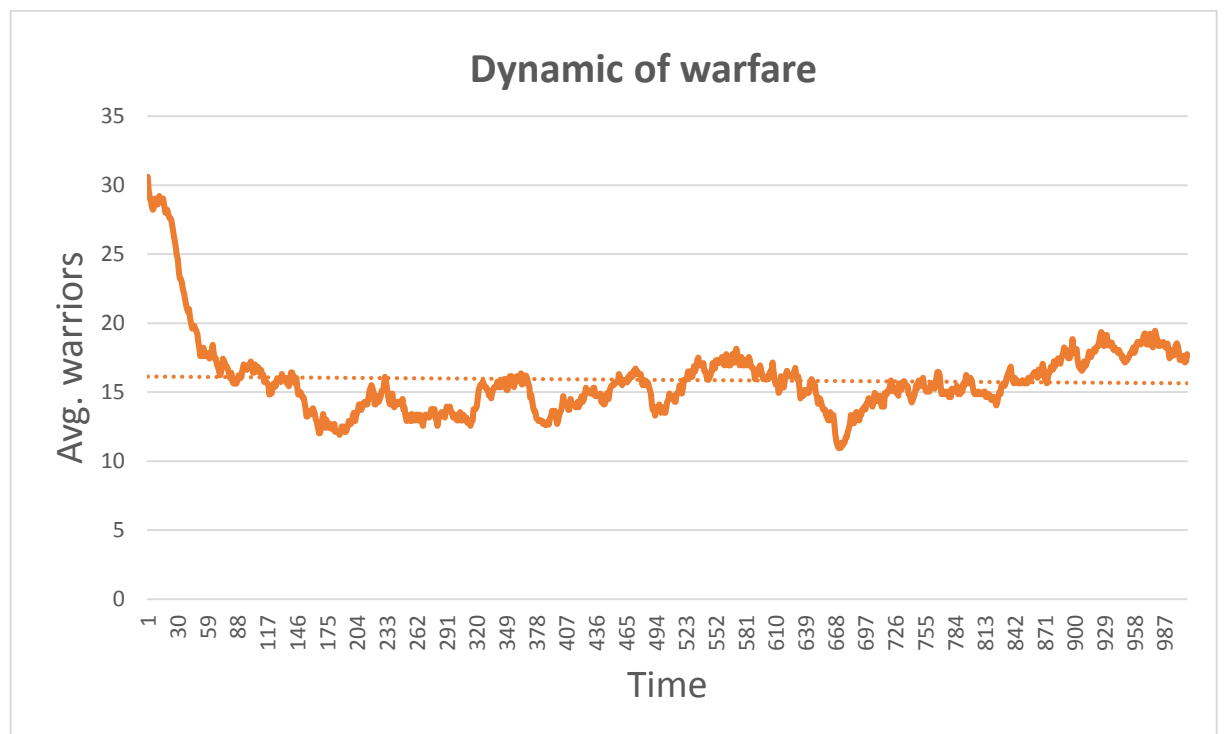
Graph 2: Histogram of private protection distribution among peasants after 200 rounds.



Graph 3: Histogram of private protection distribution among peasants after 1000 rounds.



Graph 4: Time series data of average number of hired warriors.



Tables

Following tables present results of experiments performed in our agent-based simulation. All experiment are run with initial numbers of 10 lords, 1000 peasants and 200 bandits. The length of each experiment is 5000 rounds.

The top number is the average value of variable in experiment. The standard deviations are reported in parentheses.

Table 1: Mobility parameter $m=3$, lords change taxes each round.

	40,93 (4,28)
Warriors	
Lords' tax rates	0,947 (0,022)
Level of collective protection	0,255 (0,024)
Peasant's taxes	0,940 (0,028)
Peasants' utility	0,242 (0,006)
Lords' profit per round	1,207 (0,549)
Level of private protection	0,509 (0,0039)
Bandits	401,8 (20,25)

Table 2: Mobility parameter $m=15$, lords change taxes each round.

	39,464 (6,344)
Warriors	
Lords' tax rates	0,934 (0,0297)
Level of collective protection	0,2469 (0,0351)
Peasant's taxes	0,895 (0,0635)
Peasant's utility	0,246 (0,008)
Lords' profit per round	0,85 (1,01)
Level of private protection	0,506 (0,005)
Bandits	406,41 (25,22)

Table 3: Mobility parameter $m=3$, lords change taxes after 25 rounds

Warriors	24,93 (3,85)
Lords' tax rates	0,738 (0,062)
Level of collective protection	0,173 (0,021)
Peasants' taxes	0,6786 (0,075)
Peasants' utility	0,2607 (0,0077)
Lords' profit per round	0,751 (0,364)
Level of private protection	0,494 (0,005)
Bandits	478,81 (20,24)

Table 4: Mobility parameter $m=15$, lords change taxes after 25 rounds.

Warriors	9,81 (6,179)
Lords' tax rates	0,608 (0,094)
Level of collective protection	0,0953 (0,0413)
Peasant's taxes	0,3654 (0,145)
Peasant's utility	0,2629 (0,01)
Lords' profit per round	0,2025 (0,499)
Level of private protection	0,4878 (0,005)
Bandits	504,6 (25,57)

Table 5: Effect of different military technologies. Higher parameter a implies less decisive (that is, more defensive) military technology. Mobility parameter is $m=5$ and lords change taxes after 25 rounds.

Military technology	a=1.3	a=2	a=3.5
Warriors	21,99 (4,73)	16,45 (5,36)	12,237 (5,144)
Lords' tax rates	0,660 (0,062)	0,604 (0,071)	0,607 (0,087)
Level of collective protection	0,148 (0,026)	0,128 (0,032)	0,126 (0,028)
Peasant's taxes	0,575 (0,084)	0,484 (0,104)	0,485 (0,098)
Peasant's utility	0,264 (0,0079)	0,264 (0,0084)	0,264 (0,0085)
Lords' profit per round	0,199 (0,598)	0,337 (0,3768)	0,567 (0,314)
Level of private protection	0,490 (0,0039)	0,488 (0,0042)	0,4865 (0,0049)
Bandits	493,2 (20,19)	507,67 (21,92)	496,63 (23,13)

Table 6: Different technologies of collective protection. The collective protection technology takes the form $f(N_{gl}) = (N_{pl}/N_{gl})^\gamma$, where γ is a parameter. Lower γ implies more effective protection technology. Mobility parameter is $m=5$ and lords change taxes after 25 rounds.

Collective protection	$\gamma = 5/6$	$\gamma = 1/2$	$\gamma = 1/3$
Warriors	2,420 (2,43)	16,07 (5,02)	31,885 (10,61)
Lords' tax rates	0,744 (0,131)	0,572 (0,090)	0,478 (0,131)
Level of collective protection	0,010 (0,013)	0,1253 (0,032)	0,244 (0,0748)
Peasant's taxes	0,385 (0,2078)	0,484 (0,1158)	0,378 (0,133)
Peasant's utility	0,237 (0,0061)	0,263 (0,0084)	0,313 (0,017)
Lords' profit per round	-0,248 (1,099)	0,354 (0,425)	8,55 (44,89)
Level of private protection	0,508 (0,0052)	0,486 (0,0043)	0,447 (0,011)
Bandits	558,56 (28,30)	493,2 (25,83)	412,9 (37,39)

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