Charles University in Prague

Faculty of Social Sciences
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MASTER'S THESIS

Modeling and Forecasting Volatility: Evidence from Bosnia and Herzegovina

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, June 14, 2016

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Signature

Acknowledgments

Abstract

The purpose of this thesis is to research stock market volatility in Bosnia and Herzegovina and provide comparison with regional and European stock markets. We employ symmetric and asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) models in order to estimate the conditional volatility of benchmark stock market indices in Bosnia and Herzegovina (SASX-10, BIRS), former Yugoslavia region (CROBEX, BELEX15, SBI TOP) and Europe (EURO STOXX50). Additionally, we analyze the evolution of conditional standard deviations for selected markets and develop dynamic GARCH volatility forecasts for SASX-10 and BIRS.

Our results suggest that Bosnia and Herzegovina markets are characterized with relatively high persistence and long memory in volatility. However, compared with regional and European markets, SASX-10 and BIRS exhibit lower persistence. Although significant leverage effect was found both for regional and European markets, asymmetric modeling produced insignificant and negative leverage effect for SASX-10 and BIRS time series. Bosnia and Herzegovina stock markets display moderate to low levels of synchronization with regional and European stock markets. In general, SASX-10 was found to be more volatile than BIRS. The latter is, surprisingly, the least volatile among all analyzed time series for the observed period.

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Acronyms

ACF	Autocorrelation function
ADF	Augmented Dickey–Fuller test
AIC	Akaike information criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
ATX	Austrian Traded Index
BEKK-GARCH	Baba-Engle-Kraft-Kroner multivariate GARCH
BELEX15	Belgrade Stock Exchange benchmark index
BELEXline	Belgrade Stock Exchange index
BIC	Bayesian information criterion
BIFX	Bosnian Investment Funds Index
BIRS	Banja Luka Stock Exchange benchmark index
BLSE	Banja Luka Stock Exchange
CAC 40	Cotation Assistée en Continu (French stock market index)
CEE	Central and Eastern Europe
CROBEX	Zagreb Stock Exchange benchmark index
CSD	Conditional standard deviation
DAX	Deutscher Aktienindex (German stock market index)
DCC-GARCH	Dynamic Conditional Correlation multivariate GARCH
DJIA	Dow Jones Industrial Average
EGARCH	Exponential GARCH
ERS10	Index of the Power Utility Companies of Republic of Srpska
EU	European Union
EURO STOXX50	Eurozone stock market benchmark index
EWMA	Exponentially Weighted Moving Average

FIRS	Investment Funds Index of Republic of Srpska
FTSE MIB	Milano Italia Borsa (Italian stock market index)
FTSE	Financial Times Stock Exchange
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GJR-GARCH	Glosten-Jagannathan-Runkle GARCH
HAR	Heterogeneous autoregressive model
KPSS	Kwiatkowski–Phillips–Schmidt–Shin test
LM test	Lagrange Multiplier test
MA	Moving Average
MAE	Mean absolute error
MBI 10	Macedonian Stock Exchange benchmark index
MONEX 20	Montenegro Stock Exchange benchmark index
MSCI	Morgan Stanley Capital International
NASDAQ	National Association of Securities Dealers Automated Quotations
PACF	Partial autocorrelation function
PGARCH	Periodic GARCH
QGARCH	Quadratic GARCH
RMSE	Root mean square error
SASE	Sarajevo Stock Exchange
SASX-10	Sarajevo Stock Exchange benchmark index
SBI TOP	Ljubljana Stock Exchange benchmark index
SEE	South Eastern Europe
SSR	Sum of squared residuals
SWARCH	Switching ARCH
TGARCH	Threshold GARCH
TIC	Theil inequality coefficient
TO	Turnover
UK	United Kingdom
US	United States
VaR	Value at risk

Master Thesis Proposal

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Motivation:

After the breakup of Yugoslavia and war that followed, Bosnia and Herzegovina kept independence but got politically and economically separated into two administrative "entities": The Federation of Bosnia and Herzegovina and the Republic of Srpska. Over time, distinct capital markets were established in each entity. In 2001, Sarajevo Stock Exchange (SASE) was founded by brokerage firms in the Federation. The same year, Banja Luka Stock Exchange (BLSE) was born in the Republic of Srpska following the agreement of major banks.

Since companies operating in different entities to some extent have different legislation (including general conditions for starting and operating business, requirements for stock market listings, access to state resources, etc.) public discourse is periodically flooded with comparisons and effectiveness of economic policies implemented by each entity.

However, there is still remarkably little academic research on these issues. This thesis aims to provide some answers. Specifically, we plan to do GARCH volatility modeling and forecasting of returns for main stock market indices both on SASE (SASX-10) and BLSE (BIRS). Modeling would help us understand volatility (and thus risk) of each market and relative position to other markets in the region (i.e. former Yugoslavia markets),

Why is this important?

First, as Bosnia and Herzegovina moves closer to EU accession and thus inspires confidence to the investors, market volatility will become focus of interest for the investors.

Second, if two capital markets are ever to merge into single, it would be useful to identify legislation and economic ecosystem that contributes to a more stable markets.

And finally, modeling will allow us to see the magnitude of global financial crisis impact on Bosnia and Herzegovina markets.

Hypotheses:

1. Stock market indices on SASE and BLSE can be properly modeled using GARCH process.
2. Stock markets in former Yugoslavia countries show similar GARCH processes, i.e. have similar volatility.
3. Stock markets in Bosnia and Herzegovina demonstrate higher volatility than the benchmark index for Europe (EURO STOXX50).

Methodology:

Historical data (price quotes) for selected indices (SASX-10, BIRS, BELEX15, CROBEX, SBI TOP, EURO STOXX50) will be obtained via Reuters Wealth Manager platform. If Reuters Wealth Manager proves to be insufficient, data will be supplemented with historical prices listed on offical stock market websites.

Hypothesis #1 will be tested by fitting data to follow GARCH process.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$
$$a_t = \sigma_t \epsilon_t$$

where $\alpha_0 > 0, \alpha_i > 0, \beta_j > 0$ and $\alpha_i + \beta_j < 1$.

Forecast of the returns will be obtained as:

$$\sigma_{t+1}^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_t^2 + \sum_{j=1}^{s} \beta_j \sigma_t^2$$

Hypothesis #2 and #3 will be tested by comparing estimates for respective GARCH models and plotting conditional standard deviations. More specifically, we will focus our attention on comparison of conditional variance, i.e. GARCH term and ARCH term.

Expected Contribution:

While neighbouring markets volatility, particularly the Croatia and Serbia markets, have been extensively researched and modeled, this is the first attempt (to the best of our knowledge) to model Bosnia's stock markets volatility. Lack of previous research may be due to the fact that stock markets in Bosnia and Herzegovina have emerged relatively late and remain fairly unfamiliar to broader public. In addition, since longer times series are needed for proper modeling, it could be argued that the lack of data previously stalled any serious academic interest. With ten years of data at our hand, this thesis will hopefully contribute to awakening of enthusiasm.

In practical terms, this thesis will add to the understanding of emerging Bosnia markets and provide crucial volatility information for the interested investors. As Bosnia moves to Euro-Atlantic integration, this will become increasingly important as a part of risk assesment. In addition, our research will allow us to compare former Yugoslavia markets in order to idenftify how diverse volatility across region is and provide policy recommendations.

Outline:

1. Introduction and Motivation: explaining the aim of the paper, scope of the research and contribution.
2. Literature Overview: summarising history of GARCH models application to stock markets in former Yugoslavia countries
3. Data: describing datasets that were used in research
4. Methods: conducting econometric analysis to test hypotheses.
5. Empirical results: Interpreting the results of econometric analysis and answering hypotheses.
6. Conclusion: summarising the paper and practical implications of findings. Providing pointers for future research.

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1   Introduction

Volatility is the central concept of modern financial markets and it is usually defined as a statistical measure of the dispersion of returns for a given asset. Modeling and forecasting of volatility is fundamental for risk management, portfolio management and option pricing. Thus, volatility is closely related to risk: the higher the volatility, the riskier is the asset.

The most influential innovation in modeling volatility was the introduction of autoregressive conditional heteroskedasticity (ARCH) models by Engle (1982) and subsequent generalization of the ARCH(p) model by Bollerslev (1986).

The intention of this thesis is to estimate the volatility in Bosnia and Herzegovina stock markets and to compare the results with volatility of several regional markets (Croatian, Serbian and Slovenian) as well as European market. As a proxy for these markets we use following indices: SASX-10 (Sarajevo Stock Exchange), BIRS (Banja Luka Stock Exchange), CROBEX (Zagreb Stock Exchange), BELEX15 (Belgrade Stock Exchange), SBI TOP (Ljubljana Stock Exchange) and EURO STOXX50 as a benchmark index for the European market. Besides modeling volatility, our goal is to describe the evolution of conditional standard deviation for selected markets and dynamically forecast volatility both for SASX-10 (Sarajevo Stock Exchange) and BIRS (Banja Luka Stock Exchange).

The decision to compare Bosnia and Herzegovina with above mentioned countries is motivated by the fact that Bosnia and Herzegovina, Serbia, Croatia, Slovenia, Montenegro and Macedonia were once part of the same country – Yugoslavia. Belgrade Stock Exchange and Zagreb Stock Exchange are the most capitalized among all former Yugoslavia markets while Ljubljana Stock Exchange was the only market integrated with European markets throughout observed period (2006 -2015). Thus, we gave these markets precedence over comparatively less capitalized, less integrated and less developed Montenegrin and Macedonian capital markets.

Following the violent dissolution of Yugoslavia, Bosnia and Herzegovina was politically and economically separated into two administrative "entities": The

Federation of Bosnia and Herzegovina and the Republic of Srpska. Over time, distinct capital markets were established in each entity. In 2001, the Sarajevo Stock Exchange (SASE) was founded by brokerage firms in the Federation while the Banja Luka Stock Exchange (BLSE) was born in the Republic of Srpska following the agreement of major banks.

Our study is foremost motivated by the lack of academic research about volatility in Bosnia and Herzegovina two capital markets. Lack of previous academic effort is probably due to the fact that stock markets in Bosnia and Herzegovina have emerged relatively late and remain fairly unfamiliar to broader public. In addition, since longer times series are needed for proper modeling, it could be argued that the lack of data previously stalled any serious academic interest.

As the country accelerates towards European Union accession, it is reasonable to expect increased interaction of institutional and individual investors with Bosnia and Herzegovina markets. Since volatility translates into risk, both investors and policy makers could benefit from proper modeling of volatility. For the investors, understanding volatility is prerequisite of risk assessment. Policy makers, on the other hand, through modeling of volatility might gain additional insights on building an economic ecosystem that mitigate risks, attracts investors and contribute to a more stable investing environment altogether. Moreover, studying the evolution of volatility over time will allow us to better understand the impact of global financial crisis on Bosnia and Herzegovina markets.

Our results suggest that Banja Luka Stock Exchange market (BIRS) is not only less volatile than Sarajevo Stock Exchange (SASX-10) but surprisingly the least volatile among all analyzed time series. Furthermore, Bosnia and Herzegovina markets are characterized with high persistence and long memory in volatility - comparable to regional markets. No significant leverage effect was found for Bosnia and Herzegovina markets as opposed to regional and European markets.

The structure of this thesis is following. Chapter 2 summarizes previous studies related to analysis and modeling of volatility in former Yugoslavia region. Chapter 3 presents methodology behind modeling and forecasting of volatility including models specification. Chapter 4 describes the data used in this study and justifies potential

transformations of the data. Chapter 5 discusses empirical analysis and presents results. Chapter 6 provides concluding remarks and suggestions for further research.

2 Literature Review

Autoregressive conditionally heteroscedastic models (ARCH) were first introduced by Engle in 1982. Although ARCH models are today best known for their application in finance, Engle initially used ARCH models on macroeconomic data in a seminal paper[1] published in Econometrica. This lay down theoretical foundations for analysis of time-varying volatility. Four years later, Generalized ARCH (GARCH) models were introduced by Engle's student Tim Bollerslev. This generalization allowed for broader application of the models and thus world of econometrics has been changed forever. In 2003, Engle was awarded with Nobel Prize for Economics "for methods of analyzing economic time series with time-varying volatility (ARCH)".

There have been various extensions and upgrades to these models over time. In fact, in 2008 Bollerslev compiled "Glossary to ARCH (GARCH)" where he listed 145 variations and extensions to the ARCH/GARCH models. On average, there have been almost 5 extensions to ARCH every year since Engle's breakthrough paper in Econometrica. These models have been widely used on innumerable economic and financial data and have spurred substantial academic interest.

Still, most frequent application of these models has been in the domain of time-varying modeling and forecasting of volatility. We are interested in this utilization too: volatility modeling and forecasting of stock market returns in Bosnia and Herzegovina, to be more precise. However, it would be tedious job to summarize abundance of literature related to volatility modeling on wordwide stock markets. Thus, we narrow our focus to former Yugoslavia region whose stock markets will be modeled in this thesis and compared to Bosnia and Herzegovina stock markets.

In order to avoid citing articles arbitrarily and to make this section more readable, we will categorize and compare most important articles by country.

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[1] Engle, R. (1982): "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation."

2.1. Croatia

Murine and Poshakwale (2001) were among the first to model the volatility in Croatia. In the paper named "*Volatility in the emerging stock markets in Central and Eastern Europe: Evidence on Croatia, Czech Republic, Hungary, Poland, Russia and Slovakia*" authors applied symmetric and asymmetric GARCH to model daily returns volatility. Their results suggest that returns volatility shows conditional heteroscedasticity and non-linearity that is properly captured by GARCH. No asymmetric effect was found in the case of Croatia. Additionally, volatility is found to be persistent and did not explain expected returns. In other words, explanation provided by the model is not significant in predicting future volatility.

Erjavec and Cota's (2007) seminal paper "*Modeling stock market volatility in Croatia*" used GARCH models to test the hypothesis that volatility depends on the volume of trade. They also wanted to test whether international stock markets (DJIA and NASDAQ in US; DAX and FTSE in EU) have any influence on Croatia stock market's volatility. GARCH (1,1) was used for modeling CROBEX index. Although volume of trade variable proved to be significant in the model, its value was negligible. With regards to the other hypothesis, authors found that European stock markets have significant influence on Croatian stock market index over the same day of trading. For US markets, results indicated that movements from previous days provide valuable signal for the direction of Croatian market movement on the next day.

Sajter and Ćorić (2009) expanded Erjavec and Cota's (2007) research in a paper "*(I)rationality of Investors on Croatian Stock Market – Explaining the Impact of American Indices on Croatian Stock Market*". They were interested to detect and explain comovements and spillover effects between United States and Croatia stock markets. Authors found substantial correlation and comovements between the two markets although intra-sectoral relationships between US and Croatian business sectors were not strong at all. Thus, detected comovements and dependence of Croatian companies cannot be explained by business results. It is rather the case, authors suggest, psychological factors come into play.

Petrovski (2011) compared stock market volatilities in Central and South Eastern Europe, including Croatia. His results prove that Croatian market was the most integrated SEE market with the Central European markets among all analyzed markets. Petrovski did not found any evidence for volatility spillovers from European markets to the Croatia market.

Minović (2012) published an empirical analysis of liquidity for the Croatian stock market (Zagreb Stock Exchange). To measure liquidity, she employed three different measures: Zero Rates return by Lesmond et al. (1999), Price Pressure of non-trading by Bekaert et al. (2007), as well as Turnover (TO). Her results suggested that liquidity of the market was very low. Minović also went to compare Zagreb Stock Exchange liquidity with Serbian stock market (Belgrade Stock Exchange) by using Zero Rates return methodology and came to conclusion that Serbian market was more liquid.

Horvath and Petrovski (2012) examined stock market comovements between Western Europe and Central/South Eastern Europe using multivariate GARCH. The degree of Croatia's correlation gradually increased over time: there was no correlation at the very beginning of the sample. However, at the dawn of the global financial crisis correlation was as high as in Central European countries. Authors have attributed increasing correlation to the process of growing economic integration of Croatia towards the EU.

Baumohl and Lyocsa (2013) observed relationship between time-varying correlations and conditional volatility of several European emerging stock markets (including Croatian) and MSCI World stock market index used as a proxy for developed markets. None of the emerging stock markets exhibited asymmetric volatility. In addition, authors found evidence that the relationship between correlations and volatility might be considered positive.

Dajčman (2013) analyzed dependence between Croatian and European stock markets (Austrian, French, German, Italian and United Kingdom) using copula GARCH approach. Dajčman found out that dependence between stock markets is dynamic and non-linear. Thus, it cannot be captured by "ordinary" measures of dependence that Erjavec and Cota (2007) used. It can be properly captured by dynamic normal (for

indices CROBEX-CAC40, CROBEX-DAX, CROBEX-FTSE-MIB) or Joe-Clayton copula GARCH approach (CROBEX-ATX, CROBEX-FTSE100). Dependence is shown to increase during turbulent times when returns are very volatile.

Miletić and Miletić (2015) investigated performance of Value at Risk (VaR) models during global financial crisis in Central and Eastern European (CEE) emerging capital markets, including Croatia (CROBEX stock market index). They used GARCH-type models with time varying volatility and heavy tails of the empirical distribution of returns and compared them to the estimation given by GARCH-type models with normal errors. Results indicate that GARCH-type models with t error distribution provide better VaR estimation.

2.2. Macedonia

Kovačić (2007) was probably the first researcher to model daily returns volatility on Macedonian stock exchange in a paper "*Forecasting volatility: Evidence from the Macedonian stock exchange*". He found that Macedonian stock exchange shows volatility clustering (large changes in returns are followed by large changes and small changes are followed by further small changes), has high kurtosis and low starting as well as slow-decaying autocorrelation function of squared returns. Kovačić employed five GARCH-in-mean models to find out more about impact of conditional variance on stock returns: symmetric GARCH model and four asymmetric models (EGARCH, TGARCH, PGARCH and GJR). Considering that higher risk normally bears higher rewards, his results were somewhat strange: increase in volatility led to decrease in stock returns. Author attributed this anomaly to the early stage of emerging stock market such as Macedonian stock market.

Horvath and Petrovski (2012) examined stock market comovements between Western Europe and Central/South Eastern Europe including Macedonia. As opposed to the results observed by authors in the case of Croatia, Macedonian stock market did not display any correlation with Western European stock markets. Thus, authors concluded that Macedonian market was not integrated with Western European markets. This is in line with Petrovski's (2011) earlier conclusion in his paper "*Comparison of stock market volatilities in Central Eastern Europe and South Eastern Europe*"

Bucevska (2013) used daily returns of the Macedonian stock exchange index MBI 10 to test the performance of selected GARCH models in delivering volatility estimates. Author was also interested in VaR estimation. She found out that volatility was best captured by asymmetric EGARCH with Student's t-distribution, the EGARCH model with normal distribution and the GARCH-GJR model.

2.3. Serbia

Zemčik and Seskar (2008) assessed liquidity of the Serbian stock market using daily stock market data. Analysis suggests that market is liquid and efficient. Authors emphasize that number of transactions on Serbian stock market does affect volatility while arrival of new public information does not affect trading frequency.

Stamenović (2011) examined the relationship between stock market volatility and political instabilities in Serbia. His findings suggest that unexpected political events have immediate effect on Belgrade Stock Exchange main index BELEX15. Stamenović used asymmetric GARCH models that lead him to conclude that asymmetric volatility is present: political events that are assigned "negative" connotation influence market more severely than those with "positive" connotation.

Horvath and Petrovski (2012) examined stock market comovements between Western Europe and Central/South Eastern Europe using multivariate GARCH. Serbia was one of the countries in focus. According to their results, there is no correlation between Serbian and Western European stock markets.

Chocholata (2013) investigated stock market integration of Slovakia and Serbia into Western European stock markets. She used bivariate BEKK-GARCH(1,1) to model daily data between 2008 and 2012. In case of Serbia, conditional variation was around 0.2 during the whole observed period. This indicates low integration of Serbian stock market with Western European stock markets.

Miletić and Miletić (2013) evaluated various symmetric and asymmetric GARCH models in order to forecast the market risk of Serbian stock market. Most adequate model for this purpose proved to be EGARCH model under the assumption that residuals follow normal distribution as well as GARCH (1,1) model once residuals were assumed to follow Student's t-distribution.

Njegić et al. (2013) modeled Serbian stock market volatility for the period 2010 - 2012, focusing on the post–crisis period. They found that time series are mean-reverting with volatility clustering and heavy tails. No "leverage effect" was found for Serbian stock market. Volatility was best captured with AR(1) - GARCH(1,1) model with Student's t-distribution. Interestingly, their results suggest that the direction of Serbian stock market (BELEXline index) can be successfully predicted by using Stoxx Balkan and Stoxx E. Europe indices.

Milojević and Terzić (2014) modeled market risk on Serbian stock market using five different VaR models: historical simulation with rolling window of 500 days, Risk Metrics, exponentially-weighted moving average (EWMA) with optimized decay factor, VaR based on models from GARCH family under three distributional assumptions (normal, generalized error, and Student-t), and Filtered historical simulation. Since standard VaR models are developed for liquid and efficient markets, authors found that these models underestimate forecast of market risk on Serbian stock market.

2.4. Montenegro

Janjušević (2008) performed statistical analysis on Montenegrin stock market daily returns time series. She found lack of normality in time series and correlation among returns.

Vulić and Karadžić (2014) conducted econometric analysis of MONEX20 index in order to understand the main characteristics of Montenegrin capital market. In particular, authors tested market efficiency and estimated volatility. Using ADF and Run tests they came to the conclusion that there was a weak form of efficiency on the capital market in Montenegro. By using the measure of historical volatility they have found volatility to be low.

Cerović et al (2015) wanted to know whether extreme value theory outperform econometric and quantile evaluation of VaR based on GARCH models in emerging stock markets such as Montenegrin. They analyzed daily returns of MONEX20 index for the period 2004-2014. According to their results, extreme value theory outperforms all alternatives.

2.5. Slovenia

Egert and Koubaa (2004) found that Slovenian stock market index returns can be properly modeled by using asymmetric GARCH models (in particular, GJR and QGARCH models).

Syllignakis and Kouretas (2012) conducted research on the volatility of new EU members in a paper titled *"Switching volatility in emerging stock markets: Evidence from the new EU member countries"*. By using weekly stock market returns they were interested to know whether volatility of stock returns changed as the results of accession to the EU. Authors used Markov-Switching ARCH (SWARCH) and found gradual decrease in volatility as new members came closer to the accession.

Dajčman and Festić (2012) examined comovements and spillovers between Slovenian and various Europęan stock markets return series. Authors used dynamic conditional correlation GARCH (DCC-GARCH) and concluded that comovements between Slovenian and European stock markets were time-varying for the period 1997-2010 and that there were return spillovers among these markets. In addition, results imply that global financial crisis increased comovements.

2.6. Bosnia and Herzegovina

There has been remarkably little research done on the volatility of stock market returns in Bosnia and Herzegovina. This may be due to the fact that stock markets in Bosnia and Herzegovina have emerged relatively late and remain fairly unfamiliar to broader public. In addition, since longer times series are needed for proper modeling, it could be argued that the lack of data previously stalled any serious academic interest. For example, Petrovski (2011) excluded Sarajevo stock exchange index SASX-10 from his comparison of stock market volatilities in Central and South Eastern Europe *"as a result of the high number of missing observations and the low level of liquidity and correlation with the European markets."*

Furthermore, Horvath and Petrovski (2013) opted not to include Bosnia and Herzegovina as a part of their research on South Eastern stock markets integration to Western Europe *"since the European Bank for Reconstruction and Development show that the degree of financial reforms is still rather low"*.

Having that said, there are still several papers that are worth mentioning.

Rovčanin et al. (2015) forecasted SASX-10 index by using multiple regression based on Principal Component Analysis. They have initially used 17 macroeconomic indicators as independent variables in a regression model only to find out that model suffers from multicollinearity. Principal Component Analysis helped solve multicollinearity problem and the model was subsequently reduced to 9 independent variables that were able to explain 81,10% of variation in SASX-10 index.

Possibly the most relevant paper to us was written by Okičić (2014): "*An empirical analysis of stock returns and volatility: The case of stock markets from Central and Eastern Europe*". Okičić focused on the relationship between returns and conditional volatility. She concludes that ARIMA and GARCH processes provide good approximation of mean and volatility in case of CEE region, including Bosnia and Herzegovina. Author also observed "leverage effect": negative shocks increased the volatility more than positive shocks. Additionally, Okičić found market information inefficiency reflected as a possibility to earn abnormal gains by using historical data.

3   Methodology

3.1. Stylized facts about volatility

Poon (2005) defines volatility as a spread of all likely outcomes of an uncertain variable. More specifically and in the context of financial markets, volatility is a statistical measure of the dispersion of returns for a given asset.

As such, volatility is closely related (but should not be equated) with risk. This is because risk is usually associated with potential losses, i.e. negative outcome while volatility encompasses positive outcomes too. Furthermore, volatility only measures dispersion of a distribution but bears no information about distribution itself which makes volatility less than perfect measure for risk.

From the abundance of empirical research some well documented facts about volatility of returns have emerged:

a) **Volatility clustering.** Large movements of returns, of either sign, tend to be followed by further large movements and small movements are followed by further small movements. As a consequence, correlogram and Box-Ljung statistics usually show correlations among squared returns.

b) **Fat tails.** Distribution of returns exhibit fatter tails than those of a normal distribution do. As a result, extreme values are more frequently observed compared to normal distribution. For example, Reider (2009) mentions that S&P 500 returns between 1987 and 2009 have kurtosis od 33 while normal distribution has kurtosis of only 3.

c) **Mean reversion**. Unconditional variance tends to revert to a mean, i.e. there is a "normal" level of volatiltiy to which series gravitate in the long run.

d) **Leverage effect**. Black (1976) noted that volatility increases more if previous day returns were negative compared to positive returns of the same magnitude. In other words, there is an asymmetric response of volatility to "positive" and "negative" innovations. This is referred to as "leverage effect".

e) **Volatility comovements**. Volatiltiy of different assets returns and different markets tend to move together, e.g. large movements in one markets are usually matched by large movements in other markets.

3.2. Returns and prices

Time series can be defined as a sequence of continuous data points captured over discrete time interval where *t; t=1…T*. Time intervals between individual data points are usually equal in length and can range from milliseconds to years depending on the data.

When analyzing time series of a stock markets researchers normally use stock market returns instead of prices. The main reason for this lies in the fact that time series analysis of prices would be extremely difficult. This is because prices are correlated. In addition, variance of the prices tends to increase over time.

Campbell, Lo and MacKinlay (1997) gave two fundamental reasons why assets returns should be used instead of prices. First, returns represent complete summary of investment opportunity for the average investor. Second, returns have more attractive statistical properties than prices. Thus, it is more convenient to use returns, i.e. price changes. Hence, we adopt this approach in our research.

Furthermore, distinction should be made between arithmetic and geometric returns. Aas and Dimakos (2004) define daily arithmetic returns as:

$$r_t = (P_t - P_{t-1})/P_{t-1}$$

where P_t is observed price of the asset P in time t. Annualized arithmetic returns are thus defined as:

$$R = (P_T - P_0)/P_0$$

where P_0 is the first observed price of the asset (e.g. price of the asset on the first day of trading) and P_T is the last observed price (e.g. price of the asset on the last day of trading).

On the other hand, daily geometric returns are defined as:

$$d_t = \log(P_t) - \log(P_{t-1})$$

Hence, annualized geometric return can be written as:

$$D = \log(P_T) - \log(P_0)$$

There is at least one significant advantage of using logarithm-scaled returns: if normal distribution can be assumed for geometric returns, this means that prices can never be negative, which is in line with economic intuition. On the other hand, normal distribution of arithmetic returns still may lead to negative prices which would be in contrast with observed economic reality. Therefore, using geometric returns may be more practical.

Another important remark mentioned by Aas and Dimakos (2004) must be noted When volatility of the prices is small and time interval is high, geometric and arithmetic are similar. However, once volatility increases and time interval decreases, difference between the two types of returns grows larger.

3.3. Stationarity

Stationarity is the core concept of time series analysis. In simple words, (weak) stationary implies that the mean, variance and autocorrelation do not change over time.

More formally, strict stationary is achieved when joint distribution of $(r_{t_1,\dots,t_k})$ is identical to $(r_{t_{1+t},\dots,t_{k+t}})$ for all t, while k is positive integer and $(t_1, \dots, t_k)$ is a collection of positive k integers. In other words, joint distribution of time series does not variate depending on time. Thus, variance and any other moment are exactly the same. Strict stationarity does not assume any specific distributions but stipulates that distribution is the same throughout the time series development. However, this is almost never the case in practice.

Therefore, weak stationarity must be assumed. Time series are weakly stationary if the mean of r_t and covariance between r_t and r_{t-l} are time invariant. Thus, weak stationarity is essential assumption for making future predictions, i.e. forecasts. If time series r_t follow normal distribution then weak stationarity is the same as strict

stationarity. In real world application, we usually work with weak stationarity of time series.

Non-stationary time series are not mean reverting and autocorrelations do not decay with time. By contrast, in stationary time series shocks are only temporary and disappear over time. In other words, they do not impact new time series values infinitely. So the persistence of shocks lasts longer for non-stationary time series than for stationary time series. Hence, any estimation on non-stationary time series can lead to spurious regression.

Non-stationary time series can be transformed to stationary time series by means of differencing. Given the non-stationary time series r_i we use differencing to obtain stationary time series y_i:

$$y_i = r_i - r_{i-1}$$

It is usually enough to difference data just once (integration of order one). Differencing will always lead to certain data loss, i.e. differenced data would have one data point less.

There are several formal tests for stationarity, most famous being the (Augmented) Dickey-Fuller (unit root) test and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) stationarity test.

Considering AR (1) process

$$y_t = \theta y_{t-1} + \epsilon_t$$

The Dickey-Fuller test sets the null hypothesis of unit root against the alternative of stationarity.

$$H_0: \theta = 1$$

$$H_A: \theta < 1$$

The Augmented Dickey-Fuller (ADF) test is just the Dickey-Fuller test extended to AR (p) model.

On the other hand, KPSS takes stationarity as the null hypothesis.

$$H_0: \sigma_v^2 = 0$$

$$H_A: \sigma_v^2 > 0$$

Thus, if $\sigma_v^2 = 0$, then $\vartheta_t = \vartheta_0$ for all t in

$$y_t = \vartheta_t + e_t$$

and y_t would be stationary.

3.4. Autocorrelation and Autocorrelation Function

Autocorrelation represents linear dependence of a variable with itself at different points in time. If time series are stationary, autocorrelation between two observations depends on time lag k between them.

$$Cov(y_t, y_{t-k}) = Y_k$$

$$\rho_k = Corr\ (y_t, y_{t-k}) = \frac{Y_k}{Y_0}$$

where Y_0 is unconditional variance of the process.

Autocorrelation function was first proposed by Box and Jenkins (1976). It is normally used to identify non-randomness in the data or to approximate appropriate model when data is not random. If autocorrelation is employed to detect non-randomness, then usually only first lag is observed. However, to approximate time series model, more lags must be plotted.

$$\widehat{\rho_k} = \frac{\sum_{T=k+1}^{T}(y_t - \overline{y})(y_{t-k} - \overline{y})}{\sum_{t=1}^{T}(y_t - \overline{y})^2}$$

where $y_1, ..., y_T$ is time series with mean $\overline{y}$.

Slow decay of autocorrelation function (once plotted as correlogram) implies long-run effect of random shocks and thus non-stationarity of data.

3.5. Autoregressive models (AR)

Autoregressive models are the simplest type of models used in time series analysis. Autoregressive model of order one, also known as AR (1) process, can be considered as basis for derivation of more complex models. In essence, AR (1) is just a linear dependence of variable on previous data points. Thus, we can write it as:

$$r_t = \phi_o + \phi_1 r_{t-1} + \varepsilon_t$$

where r_t=1,…,t is a time series we observe and ε_t is residual (white noise) with zero mean and constant variance σ_ε^2. To satisfy stationarity condition $|\phi| < 1$ must hold. For $\phi = 1$, the model is transformed to a random walk.

It is evident that simple AR model has the same form as simple linear regression model where r_t is dependent variable while r_{t-1} is explanatory variable.

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

However, for AR (1) model ϕ must be less than 1 in order to have meaning, as we already mentioned. For simple linear regression model this would not be the case. The coefficient β_1 can be any real number and the model will still be meaningful.

Let us now take a look at some basic properties of AR (1) process.

If we compute expected value of AR (1) process, we get:

$$E\left[r_t | r_{t-1}\right] = \phi_o + \phi_1 r_{t-1}$$

while the variance is:

$$Var\left(r_t | r_{t-1}\right) = Var\left(\varepsilon_t\right) = \sigma_\varepsilon^2$$

In simple words, given past return r_{t-1}, return r in time t will be $\phi_o + \phi_1 r_{t-1}$. Generalization of AR (1) model to AR (p) would bring us to the following form:

$$r_t = \phi_o + \phi_1 r_{t-1} + \cdots + \phi_p r_{t-p} + \varepsilon_t$$

The form of the AR (p) model is identical to the multiple regression model. According to Tsay (2005), model should be interpreted in such way that p values

r_{t-i} $(i = 1, \ldots, p)$ together determine conditional expectation of r_t based on the past data.

3.6. Simple Autoregressive Moving-Average Models (ARMA)

Sometimes, we need a higher-order model than AR or MA can provide. To achieve this, we turn to ARMA models. Autoregressive Moving-Average models of orders p,q combine AR(p) and MA(q) models. They were introduced by Box, Jenkins, and Reinsel (1994).

Simple MA (1) process is represented with the following form:

$$r_t = \theta_o + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

where ε_t is white noise while θ_o is the constant.

Generalized form of MA process can be written as:

$$r_t = \theta_o + \varepsilon_t - \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

Thus, by combining AR and MA process we get ARMA (1,1):

$$r_t - \phi_1 r_{t-1} = \phi_o + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

where left side of the equation is represented by the AR model and the right side of the equation by MA model. Once more, ε_t is white noise and ϕ_o is constant. Process will be meaningful only if $\phi_1 \neq \theta_1$. Otherwise, it would be reduced to white noise.

Generalized ARMA (p, q) takes the form of:

$$r_t = \phi_o + \sum_{i=1}^{p} \phi_i r_{t-i} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

where ε_t is white noise with zero mean and time invariant variance while p and q are non-negative integers.

ARMA methodology can be applied only to stationary time series. Thus, both AR(p) and MA(q) processes must be stationary. In fact, MA(q) satisfies this condition by

definition. However, this is not necessarily the case with AR(p) process. Hence, ARMA(p,q) will satisfy stationarity condition if $\left|\sum_{i=1}^{p} \phi_i\right| < 1$.

It is possible to extend ARMA(p,q) model to get autoregressive integrated moving average model by allowing AR process to have unit-root. Hence, ARIMA (p, d, q) is unit-root nonstationary. Common approach for turning ARIMA into stationary series is to use differencing.

ARIMA models are known to have "strong memory" meaning that the shocks to the model would have permanent effect to time series. The reason for this can be attributed to MA model. In particular, θ_i coefficient does not decay to zero with time.

Usually, it is sufficient to have ARIMA (p, 1, q) model, meaning that differencing has been done just once. Additional differencing may be justified if series contains multiple unit roots but this would lead to a loss of information.

3.7. Conditional Heteroscedastic Models

Conditional Heteroscedastic Models represent essential tool for modeling volatility of an asset return. Volatility is defined in this context as a conditional standard deviation.

Why model volatility at all? Among many reasons, we will emphasize two. First, volatility modeling allows estimation of risk trough Value at Risk metrics. Second, volatility modeling improves accuracy of forecast.

For the purpose of this thesis, autoregressive conditional heteroscedastic model (ARCH), generalized autoregressive conditional heteroscedastic model (GARCH), threshold autoregressive conditional heteroscedastic model (TGARCH) and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) will be described in more details.

3.7.1. Autoregressive Conditional Heteroscedastic Model (ARCH)

Autoregressive Conditional Heteroscedastic Model (ARCH) was first proposed by Engle (1982). As Tsay (2004) mentiones, this was the first systematic framework to model volatility. The concepts of historical volatility and exponential volatility

preceded ARCH models but they were not efficient enough. In fact, Campbell, Lo, and MacKinlay (1997) insist that it is logically inconsistent to use volatility measures that are based on the assumption of constant volatility while time series move over time.

Simple linear regression model assumes homoscedasticity: expected value of squared error terms is same at any point. In contrast, heteroscedasticity occurs if we have data in which error terms differ in size at some points or ranges. With the presence of heteroscedasticity, regression coefficients remain unbiased but confidence intervals and standard errors are too narrow and can lead to incorrect inferences, as explained by Engle (2001). ARCH (and GARCH) model attempt to fix least squares flaws by modeling variance.

Main idea behind ARCH is that returns are serially uncorrelated but dependent. This dependence can be adequately captured as a function of lagged values:

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2$$

where $\{\epsilon_t\}$ represent independent and indentically distributed random variables (iid) with zero mean and variance of one. For the model to hold, $\alpha_0 > 0$, $\alpha_i \geq 0$ and i must be non-negative integer. Setting $\alpha_i \geq 0$ is a necessary condition for stationarity. Model works under assumption that ϵ_t follow normal distribution, Student's t-distribution or generalized error distribution. Variance of the error term is dependent on a squared error term in the previous period.

ARCH model specification allows volatility clustering. Large past squared shocks lead to large conditional variance. In other words, as defined by Mandelbrot (1963), "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." This is so-called ARCH effect.

To test for ARCH effect, we use Lagrange Multiplier Test (LM test) with null hypothesis of no ARCH effect:

$$\epsilon_t^2 = \alpha_0 + \left(\sum_{s=1}^{q} \alpha_s \, \epsilon_{t-s}^2 \right) + u_t$$

$$H_0 = \alpha_1 = \cdots = \alpha_s = 0$$

Another option for testing ARCH effect is Ljung-Box statistics with null hypothesis stating that the first s lags of ACF for the σ_t^2 are zero.

ARCH itself has some notable weaknesses. First, ARCH applies same weight to positive and negative shocks to volatility when it has been empirically proven that negative shocks tend to have bigger impact on volatility than positive shocks.

Second, as Luthkepohl and Kratzig (2007) mention, ARCH needs rather large number of squared lagged residuals in order to obtain correct specification of the model.

Third, ARCH has no explanatory power over causes for variations in time series. It merely gives a description of conditional variance behavior without going into the underlying reasons for this behavior.

Fourth, it has been shown that ARCH tends to over predict volatility.

Tse (1990) notes that both ARCH and GARCH disadvantage is dependance on long series of past data. He also observes that ARCH/GARCH results may be dissapointing in periods when volatility changes very abruptly.

Generalization of ARCH (p) model was proposed by Bollerslev (1986). This brings us to famous generalized ARCH model (GARCH).

3.7.2. Generalized Autoregressive Conditional Heteroscedastic Model (GARCH)

GARCH models the variance as the weighted average of long-run variance (α_0), new information that is represented by current period's variance (α_i) and the variance predicted for this period (β_j).

In a paper that introduced GARCH, Bollerslev (1986) argued that even very simple GARCH models represent better fit than ARCH models with large number of lags.

Tsay (2004) illustrate this point on a real word data. He worked with excess returns of S&P 500 index and was able to model volatility with ARCH (9) process. Alternatively, better fit was achieved by using GARCH (1, 1).

GARCH (m, s) can be written in a following form:

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_s \sigma_{t-s}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

Once more, $\{\epsilon_t\}$ represent independent and indentically distributed random variables (iid) with zero mean and variance of 1. For the GARCH model to hold, $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$. Stationarity condition dictates that $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$. We refer to α_i as ARCH paramter while β_i is GARCH paramter. Thus, if s = 0 GARCH equation will reduce to ARCH equation presented before. As with ARCH models, ϵ_t follows normal, Student's t-distribution or generalized error distribution.

In essence, GARCH model builds upon ARCH by allowing lagged conditional variances of GARCH to enter the equation. Thus, ARCH can be considered as special case of GARCH specification, i.e. GARCH (1,0).

3.7.3. Threshold GARCH

As we have seen before, in standard GARCH model conditional variance is just a linear function of lagged squared past returns and conditional variances. While this allows understanding of dependence structure in conditional variance, standard model still fails to account for characteristic that has been well documented in the literature: conditional variance tends to be higher after a decrease in return than after an increase of equal size.

Thus, asymmetric GARCH models were introduced in order to allow asymmetric response in the volatility to the sign of shock. One of the most notable asymmetric GARCH models is the threshold GARCH (TGARCH) proposed by Zakoian (1994).

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i |a_{t-i}| + \sum_{i=1}^{m} \gamma_i S_{t-i} |a_{t-i}| + \sum_{j=1}^{s} \beta_j \sigma_{t-j}$$

where S_{t-i} is an indicator function:

$$S_{t-i} = \begin{cases} 1 \ if \ a_{t-i} < 0 \\ 0 \ if \ a_{t-i} \geq 0 \end{cases}$$

In equations above, a_t represent series of return while σ_t^2 is a conditional variance of returns at time t and ϵ_t are independent and identically distributed random variables (*iid*) with zero mean and variance equal to 1. If $\gamma_i = 0$ then TGARCH is equal to GARCH.

Hence, a_{t-i}^2 will have different impact on σ_t^2 depending on whether a_{t-i} is above or below zero threshold. When $a_{t-i} \geq 0$, then the impact of a_{t-i}^2 on σ_t^2 reduces to $\alpha_i a_{t-i}^2$. Thus, we get:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i |a_{t-i}| + \sum_{j=1}^{s} \beta_j \sigma_{t-j}$$

However, when $a_{t-i} < 0$ then the impact of of a_{t-i}^2 on σ_t^2 is equal to $(\alpha_i + \gamma_i) a_{t-i}^2$.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \gamma_i) |a_{t-i}| + \sum_{j=1}^{s} \beta_j \sigma_{t-j}$$

3.7.4. The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)

GJR-GARCH was proposed by Glosten, Jagannathan and Runkle (1993) and belongs to the asymmetric GARCH models. Thus, it responds differently to the positive and negative innovations. Hence, it is very similar to the TGARCH model in this regard. GJR-GARCH is formalized as follows:

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{i=1}^{m} \gamma_i S_{t-i} a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

The only difference in comparison to TGARCH models is the use of standard deviation instead of variance.

According to Bollerslev et al. (2007), γ_i is usually found to be positive when modeling equity returns. Thus, volatility increases more after negative shocks than it does following a positive shock. This is known as a "leverage effect".

3.8. Information Criteria

Model selection is based on information criteria that measure the quality of statistical model given the data at hands. In particular, information criteria measure balance between goodness of fit (log-likelihood estimation) and parsimony, i.e. complexity (number of parameters) of the model. In simple words, criterion helps us find model specification that has good fit but not many parameters. The model with smallest information criterion is usually preferred.

Two most often used criteria are Akaike information criterion (AIC) and Bayesian information criterion (BIC) proposed by Schwarz (1978).

$$AIC = 2k - 2L$$

$$BIC = 2k \log(n) - 2L$$

where L is maximum log likelihood function formulated as

$$L = -\frac{n}{2}(1 + \log(2\pi)) - \frac{n}{2}\log(\frac{SSR}{n})$$

where SSR is sum of squared residuals, n is number of observations in the data and k is number of estimated parameters.

In general, it is considered that AIC punishes size of the model less strongly than BIC does. In other words, if parsimony is very important then BIC should be used. Yang (2005) showed that AIC is asymptotically optimal in selecting the model with the least mean squared error while BIC is not asymptotically optimal.

3.9. Forecasting abilities

Why forecast volatility at all? Reider (2009) mentions that there are at least three reasons to forecast volatility:

1. risk management,

2. asset allocation, and

3. trading on future volatility.

In fact, without volatility forecast it would not be possible to calculate potential losses for a portfolio. Thus, it is understandable that estimation of future volatility is at the very heart of risk management and asset allocation. In addition, many investors forecast volatility to make informed investing choices, especially among option traders. For example, the Black-Scholes option pricing formula requires volatility to be estimated and any errors made in such estimation may lead to option mispricing.

There are several standard methods for forecasting volatility:

  1. The Historical Variance

  2. The Exponential Weighted Moving Average (EWMA)

  3. The Generalized Autoregressive Conditional Heteroscedastic (GARCH).

All three methods belong to the Linear Squared Deviation (LSD) class of estimators since forecasted variance is always just a linear combination of the squared deviation of recent returns.

The historical variance assigns equal weights to the squared deviations and can be formally expressed as:

$$\overline{\sigma_t^2}(k) = \frac{1}{k-1} \sum_{i=0}^{k-1} (r_{t-i} - \overline{r})^2$$

where $\overline{r}$ is the estimate of mean μ.

Exponential Weighted Moving Average model (EWMA), on the other hand, assings exponentially declining weights to the squared deviations. Simply said, larger weights are assigned to the newer observations and smaller weights to the older observations. EWMA estimates volatility as follows:

$$\sigma = \sqrt{(1-\lambda)\sum_{i=0}^{k-1}\lambda^i (r_{t-i} - \overline{r})^2}$$

where λ respresent the decay factor which determines weights assigned to observations.

ARCH/GARCH models are the most widely used class of models for volatility forecasting. As such, these models are method of choice in our research. Similar to EWMA, weightning scheme is exponentially declining. ARCH/GARCH models allow us to obtain both one-step and multi-step forecasts of volatility as formulated below.

One-step forecast can be obtained recursively as

$$\sigma_{h+1}^2 = \alpha_0 + \alpha_1 a_h^2 + \beta_1 \sigma_h^2$$

For multi-step forecast $a_t^2 = \sigma_t^2 \epsilon_t^2$ is used:

$$\sigma_{h+1}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_h^2$$

$$\sigma_{h+2}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_{h+1}^2$$

$$\sigma_{h+l}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_{h+l}^2$$

4   Data Description

Data used in our analysis were obtained mainly via Reuters Wealth Manager platform. For certain indices, when Reuters Wealth Manager did not contain sufficiently long historical data, datasets were supplemented with historical quotes available via official stock market websites or through direct communication with stock market officials.

The sample period for analysis is between 4th April 2006 and 22nd October 2015. Thus, observed time period spans over 9 years and includes pre-crisis, crisis and post-crisis periods. We analyze daily closing levels of following indices: SASX-10 (Sarajevo Stock Exchange), BIRS (Banja Luka Stock Exchange) BELEX15 (Belgrade Stock Exchange), CROBEX (Zagreb Stock Exchange), SBI TOP (Slovenia Stock Market), EURO STOXX50 (benchmark index for European stock markets).

All indices are in respective national currencies. Throughout our analysis we used several software solutions for statistical analysis. Most notably, we used GRETL, JMulti, Mathematica and EViews.

Necessary precondition for a meaningful analysis is transformation of level quotes to returns.

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) * 100$$

where P_t is daily closing value of index at time t, and P_{t-1} is the closing value of the same index day before.

All data are formally tested for stationarity by using Augmented Dickey-Fuller (ADF) test and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. Augmented Dickey-Fuller (ADF) test is a unit root test. Hence, rejection of null hypothesis leads to a conclusion that series are stationary. Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test is, on the other hand, stationarity test. Hence, null hypothesis assumes stationarity so rejection of null hypothesis would mean that time series are not stationary. Outputs of these tests are presented in the Table **1** and Table **2**. In case of

non-stationarity, we perform differencing of the data and repeat formal testing to make sure transformation is adequate. Tests are initially performed with GRETL and results are additionally supported with JMulti and EViews calculations.

Below we briefly introduce each stock market index and provide additional insight into the data at our hands.

4.1. Sarajevo Stock Exchange (SASX-10)

In 2001, The Sarajevo Stock Exchange (SASE) was founded by eight brokerage firms operating in the Federation of Bosnia and Herzegovina. It took one year for the actual trading to begin. In 2003, first stock market index was founded - Bosnian Investment Funds Index (BIFX). The Bosnian Investment Fund Index (BIFX) consists of eleven investment funds registered on the territory of the Federation.

The Sarajevo Stock Exchange Index 10 (SASX-10) index was officially presented in February 2006. This is the main index on Sarajevo Stock Exchange and the focus of our interest in this thesis. SASX-10 consists of top 10 companies registered on the Sarajevo Stock Exchange (not counting investment funds) measured by market capitalization and frequency of trading. Base value of the index was 1.000 points and individual influence of any company included in the index cannot exceed 20%.

Figure **1** and Figure **2** present both levels and logarithm differenced returns plots for SASX-10 index. After initial enthusiasm and rapid growth, peak value was achieved in 2007 just before global financial crisis. In following two years, index has plunged below its starting value. It has never really recovered since. Index is still fluctuating around initial base value indicating almost non-existent recovery.

Returns plot indicate that the most volatile period for the index was between 2007 and 2010. This is in line with our expectations due to financial crisis meltdown and something we observe on other markets too. Subsequent years are marked with lower volatility and periodical jumps in 2012, 2014 and 2015. Volatility memory seems to be relatively long.

Looking at the SASX-10 levels series plot it is evident that series are not stationary since mean is not being constant over time. This is visually confirmed by looking at

the autocorrelation function (ACF) plot. Plot does not decay over time indicating absence of stationarity.

Formal testing using ADF test confirms this conclusion. Large p-value indicates that we cannot reject null hypothesis of unit root. The risk to reject the null hypothesis while it is true is 70.21%. Thus, data are not stationary.

Result is additionally supported by KPSS test. Small p-value proves that we can reject null hypothesis of stationarity. Hence, time series are not stationary. Same conclusions can be derived by inspecting the test statistics of both tests and comparing them to critical values.

Upon differencing the data, stationarity is achieved. We visually observe this by looking at the fast decay of the ACF plot. This was further formally endorsed both by ADF test and KPSS (see Table **1** and Table **2**). Thus, SASX-10 is no longer non-stationary, it is now described as a percentage growth of the index.

Our dataset for SASX-10 contains total of 2389 observations with minimum historical daily closing value being 663,550 and maximum capped at 6040,630 points. Index mean for the observed period was 1439,308 points.
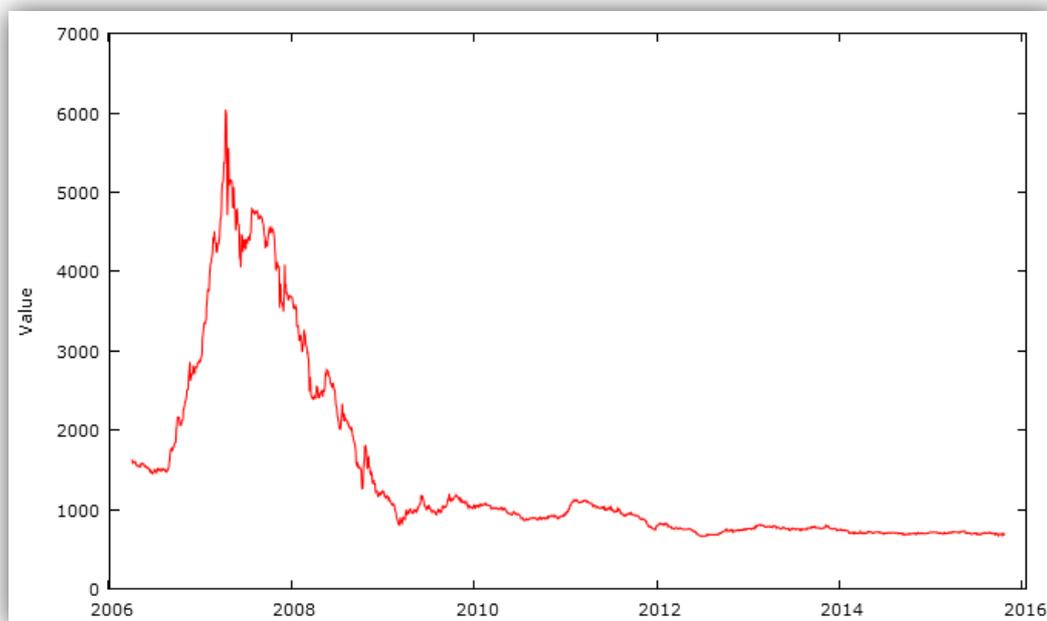

Figure 1: SASX-10 levels from April 2006 to October 2015
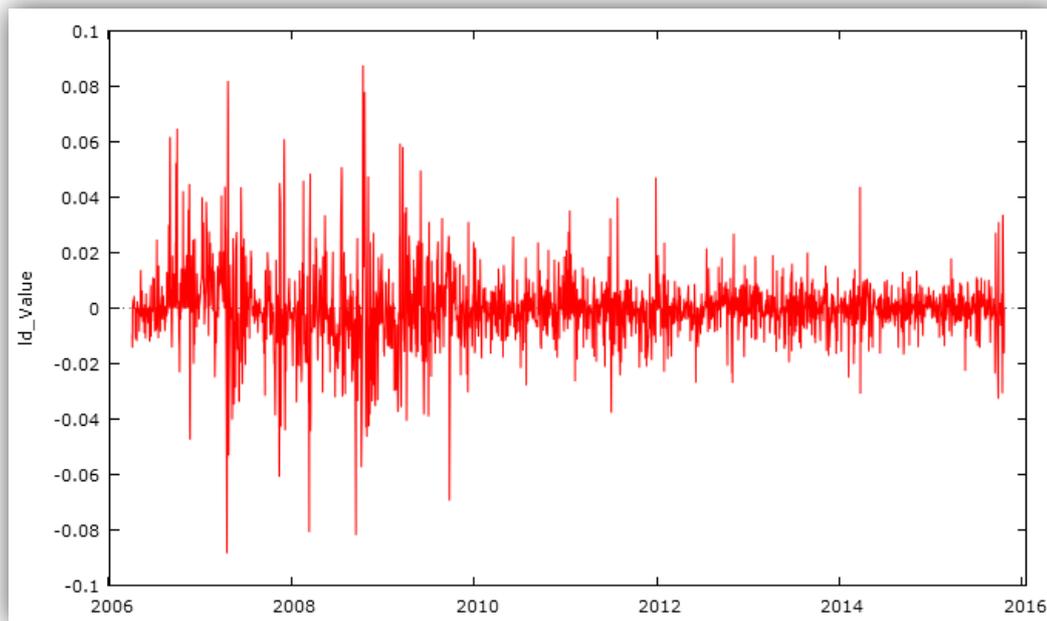
Figure 2: SASX-10 returns from April 2006 to October 2015

4.2. Banja Luka Stock Exchange (BIRS)

Banja Luka Stock Exchange (BLSE) was born in the Republic of Srpska following the agreement of major banks in 2001. Similar to Sarajevo Stock Exchange, the Banja Luka Stock Exchange also has investment fund index (FIRS) and index for largest power utility companies (ERS10).

In 2004, main index BIRS was founded with the initial value of 1.000,00 base points. BIRS stands for „*Berzanski indeks Republike Srpske*", which can be translated as „The stock market index of Republic of Srpska". BIRS is constituted of 12 largest companies listed on the Banja Luka Stock Exchange where individual influence of any company included in the index cannot exceed 25%.

Figure **3** shows the levels plot for BIRS and we see similar development as with SASX-10 index. After initial phase of rapid growth and reaching peak value at the dawn of financial crisis, BIRS experienced sharp decline in years to follow. During more than two years of constant decline, BIRS has lost over 5 times of its peak value. In 2011, there was a slight rise in index value and a glimmer of hope towards

recovery. However, index has further declined since and recovery seems non-existent to present day.

Returns plot can be seen as Figure **4.** Once more, volatility was highest between 2007 and 2010 following bankruptcy of Lehman Brothers. Jumps in volatility reappear during 2012 and 2015 probably as a reaction to European debt crisis. Volatility memory seems to be shorter than for SASX-10.

To test for stationarity, ADF and KPSS have been employed on BIRS time series. ADF produced large p-values indicating null hypothesis of unit root cannot be rejected. The risk to reject the null hypothesis while it is true is 41.68%. KPSS test results further confirmed our suspicions of non-stationarity.

These results are visually confirmed by looking at the ACF plot. ACF decays extremely slowly suggesting that random shocks can have a long run effect. To achieve stationarity, we transformed data by using logarithm differencing.

Tests were performed again on transformed data. Resulting p-values and test statistics suggest that series are now stationary. BIRS is integrated of order 1. ACF plot confirms stationarity too.

BIRS dataset contains total of 2365 observations with minimum daily historical value of the index being 673 points and maximum value peaking at 5218,180. BIRS mean closing value for our focus period was 1302,005 points.

Both SASX-10 and BIRS data exhibit strange characteristics not seen with other analyzed time series: large number of identical consequent index values. During the observed period, SASX-10 has had the same value for two consequent days exactly 51 times while same value for three consequent days was recorded 2 times. At the same time, BIRS has had the identical index value for two consequent days 78 times while same value for 3 subsequent trading days was observed 8 times. Why was this happening?

Most probably because of low to non-existing volume of trading on certain days. In fact, most of these occurrences for SASX-10 are recorded in the early days of the trading on Sarajevo Stock Exchange. We could argue that, at the time, public was still sceptic about capital markets and thus sustained from investing. In case of BIRS,

most episodes of identical consequent values were observed right after the global financial crisis which shook investors' confidence and thus may have halted investing activities to a certain extent.
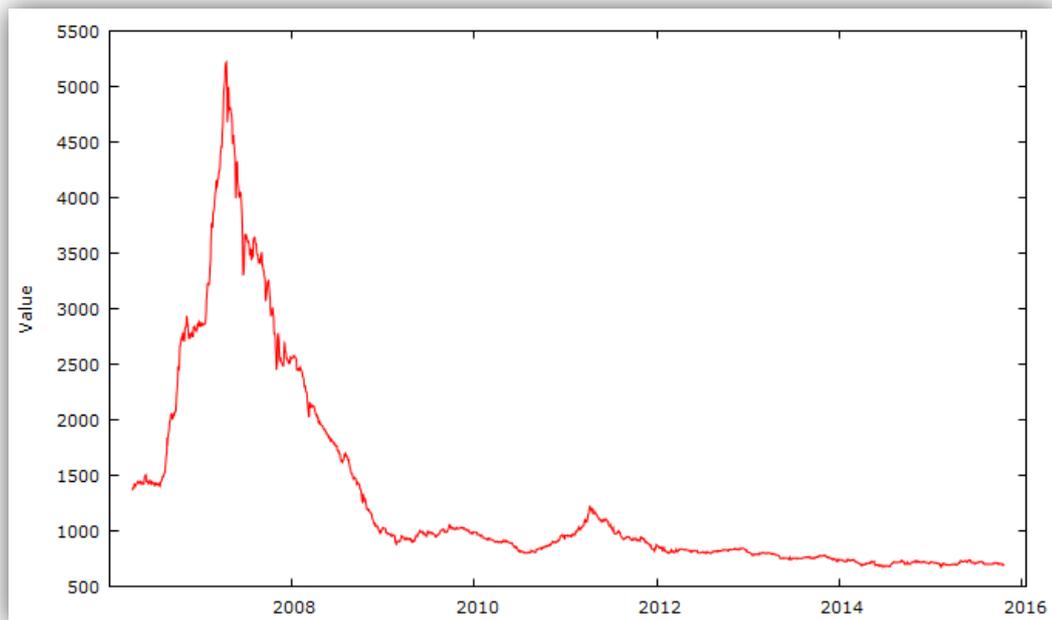

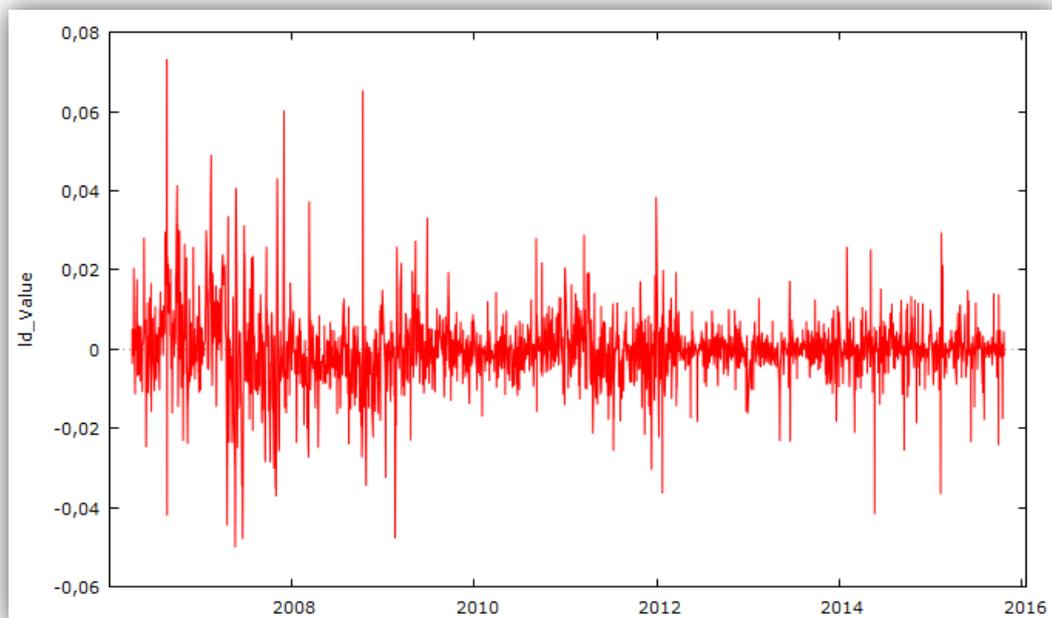Figure 3: BIRS levels from April 2006 to October 2015

Figure 4: BIRS returns from April 2006 to October 2015

4.3. Belgrade Stock Exchange (BELEX15)

Belgrade Stock Exchange was initially founded in the late 19th century but was abolished during 1950s following anti-capitalist sentiment in Yugoslavia. In 1992, Belgrade Stock Exchange was reestablished as a part of widespread economic reforms. However, it took more than ten years for Belgrade Stock Exchange to establish its first stock market index.

BELEX15 is the blue chip index of Belgrade Stock Exchange founded in 2005. Fifteen largest and most liquid companies listed on Belgrade Stock Exchange are index constituents.

BELEX15 levels plot (seen as Figure **5**) shows similar movements to those of SASX-10 and BIRS indices. Once more, peak value was reached just before the crisis followed by a sharp decline. Recovery is slow but somewhat more agile compared to Bosnia and Herzegovina markets. However, no serious signs of recovery towards pre-crisis levels can be observed since index is fluctuating around the same value for the past 3 years.

Returns plot (Figure **6**) confirms that the period of highest volatility was 2007- 2009 with three major shocks hitting the market during this timeframe. Volatility is strong again in 2012 probably as a reaction to European debt crisis. Volatility memory seems to be, however, somewhat shorter than on BIRS and SASX-10.

ADF test resulted in large p-values leading to conclusion that BELEX15 time series are non-stationary since null hypothesis cannot be rejected. The risk to reject the null hypothesis while it is true is 81.89%. Conclusion is enforced with KPSS test results. Small p-values allow us to reject KPSS null hypothesis that series is stationary. Test results are additionally confirmed by plotting autocorrelation function. Slow decay leaves no doubt: time series are not stationary.

Thus, we proceed with transformation of BELEX15 time series applying the same methodology as before. Upon transformation, both ADF and KPSS tests are repeated. This time, small p-value for ADF test supports stationarity of time series. In addition, stationarity is confirmed with KPSS test and visual inspection of the autocorrelation function plot.

BELEX15 dataset has 2409 observations in total. Minimum daily closing value was 354,390 while highest value was 3304,640. During observed period mean value of the index was 952,835 points.
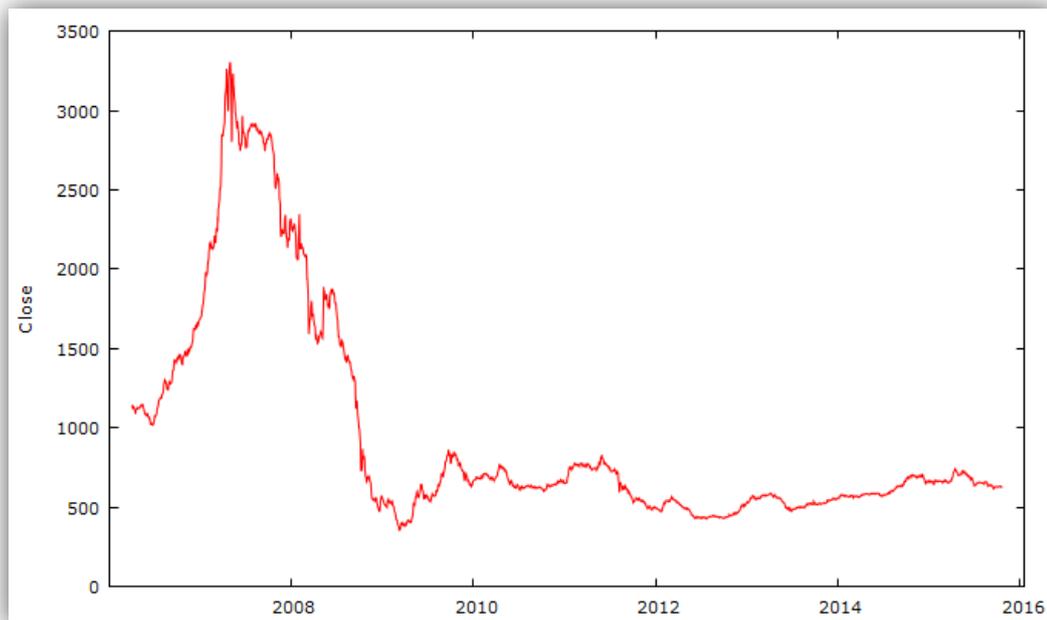

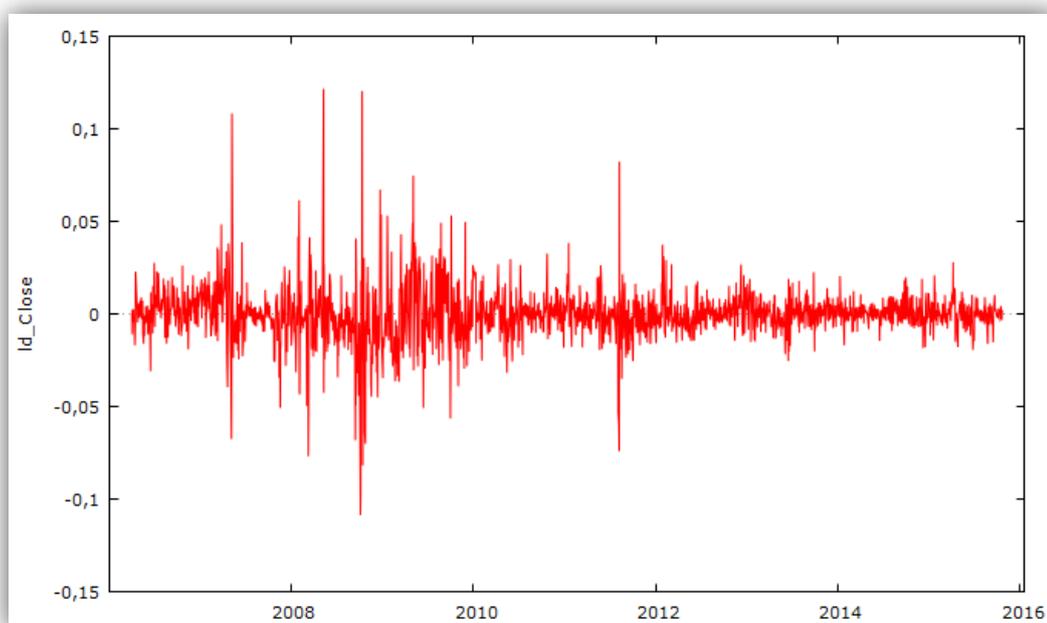Figure 5: BELEX15 levels from April 2006 to October 2015

Figure 6: BELEX15 returns from April 2006 to October 2015

4.4. Zagreb Stock Exchange (CROBEX)

Zagreb Stock Exchange was founded in 1991 following the agreement of 25 banks and 2 insurance companies. Among all former Yugoslavia countries, Zagreb Stock Exchange is often considered as the most integrated market (together with Ljubljana Stock Exchange) with European stock markets[2]. Zagreb Stock Exchange is a founder of the Federation of Euro-Asian Stock Exchanges and a member of the Federation of European Stock Exchanges.

Over 10 different indices are active on Zagreb Stock Exchange with CROBEX being benchmark index. CROBEX was founded in 1997 with base value of 1.000,00 points. Today CROBEX has 25 constituents with maximal weight of any individual constituent capped at 10%.

CROBEX levels plot (see Figure **7**) reveals familiar crisis pattern already observed on Serbian and Bosnia and Herzegovina markets with several differences worth mentioning. Although index has severely plunged as a result of global financial crisis, "free fall" did not take place until mid-2008. In 2009, CROBEX reached lowest level and recovery has been slow ever since with several ups and downs. Levels plot is more comparable to BELEX15 than with Bosnia and Herzegovina indices.

Returns plot (Figure **8**) shows substantial volatility clustering during 2009 followed by period of gradual stabilization. Smaller volatility jumps reappear again in 2011 and 2012, most probably as a reaction to European debt crisis. In general, post-crisis period was marked with relatively low volatility process.

Since both ADF and KPSS tests have confirmed that series are not stationary (in addition to slow decay of ACF plot), we transformed data with logarithm differencing. The risk to reject the null hypothesis while it is true is 74.79%. Upon differencing, tests are computed once again. This time, both ADF and KPSS results indicate that series are stationary. CROBEX is now integrated of order 1.

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[2] See Horvath, R., & D. Petrovski (2013).

Our dataset for CROBEX index has total of 2378 observations with minimum value being 1262,580 and maximum 5392,940 points. Mean closing value for CROBEX was 2393,518 points.
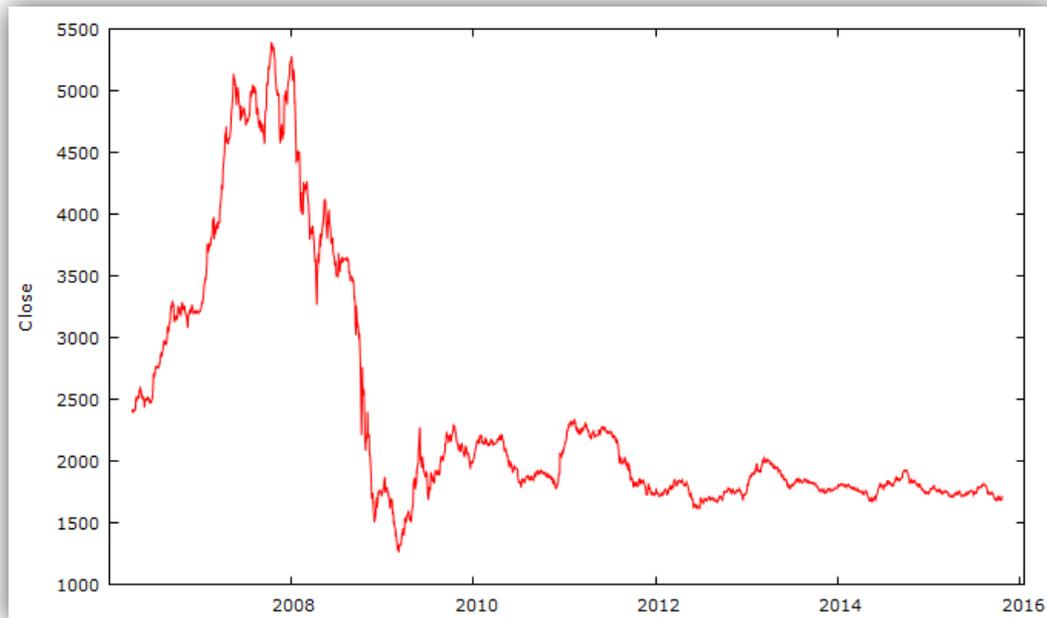

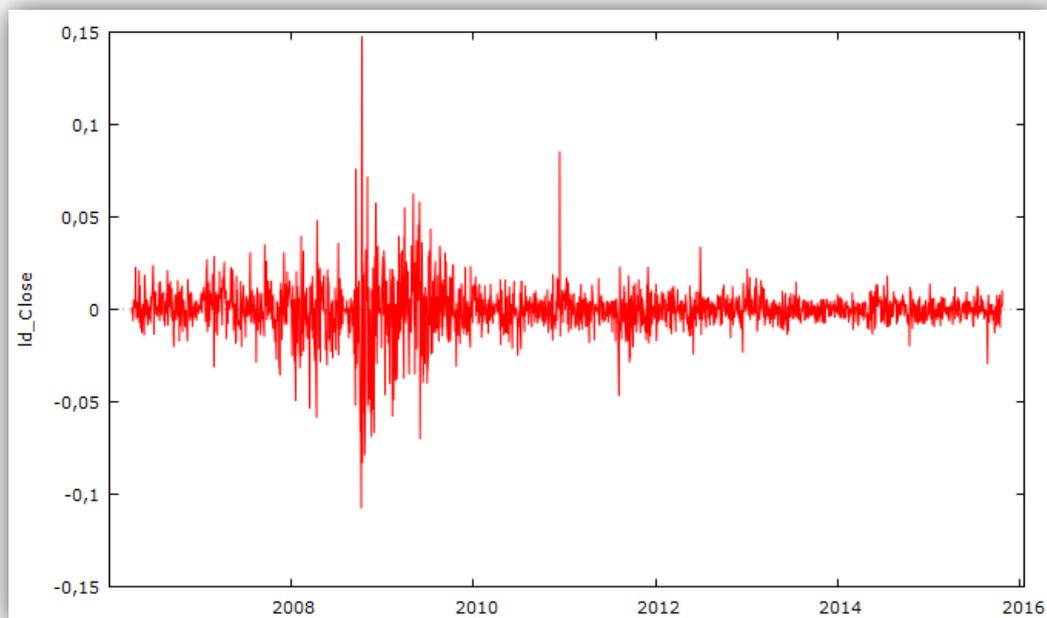Figure 7: CROBEX levels from April 2006 to October 2015

Figure 8: CROBEX returns from April 2006 to October 2015

4.5. Ljubljana Stock Exchange (SBI TOP)

Ljubljana Stock Exchange was founded in 1989 as a part of general economic reforms. However, similar to Belgrade Stock Exchange, this was not the very first iteration of stock market in Slovenia. Back in 1924, Slovenia had its first stock exchange that got suspended after Second World War. Ljubljana Stock Exchange has four official indices with SBI TOP being a leading index.

SBI TOP is an official blue chip index of Ljubljana Stock Exchange measuring the performance of 7 highest capitalized and most liquid companies[3] listed on Ljubljana Stock Exchange. Highest possible participation of any individual constituent is capped at 30% and base value of index was 1.000,00 points.

Figure **9** shows SBI TOP levels plot between April 2006 and October 2015. It can be seen that series follow pattern comparable to other indicies we analyzed. Growth was reversed with global financial crisis offset. Index reached lowest point in 2012 when already slow recovery was additionally threatened by looming European debt crisis.

Returns plot can be seen as Figure **10**. Volatility clustering is noticable throughout series with volatility jumps peaking during 2008-2009 period. Post-crisis period is characterised by periodic jumps in volatility: most notably in 2011, 2013 and 2015. Downward jumps had longer spikes implying that "negative" news impacted market more severely.

ADF test (producing high p-values) indicates non-stationarity of time series. The risk to reject the null hypothesis while it is true is 86.14%. This observation is confirmed with KPSS test (producing low p-values) and by plotting slow decaying autocorrelation function. After differencing, series have achieved stationarity and SBI TOP is integrated of order 1. This has been confirmed both by ADF and KPSS test as seen in **Table 1** and **Table 2**. We are using SBI TOP dataset with 2379 observations. Index reached historical low of 501,270 points while peak was at 2674,690 points. Mean for SBI TOP dataset in our observed period is 1040,349 points.

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[3] Number of companies included in SBI TOP index varies over time. Referenced data is conclusive with 21 September 2015.
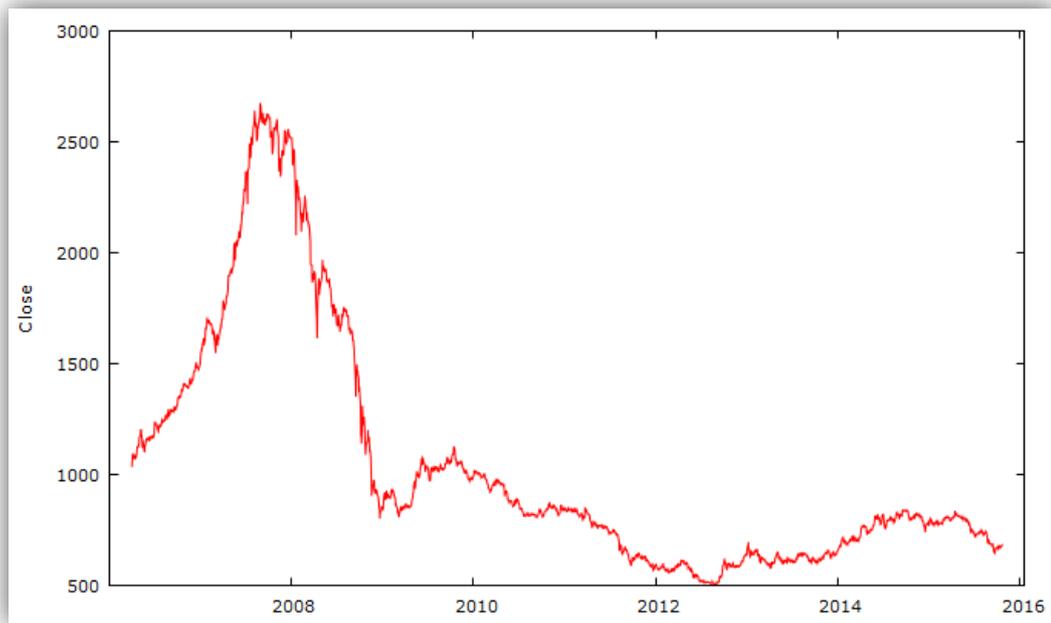
Figure 9: SBI TOP levels from April 2006 to October 2015
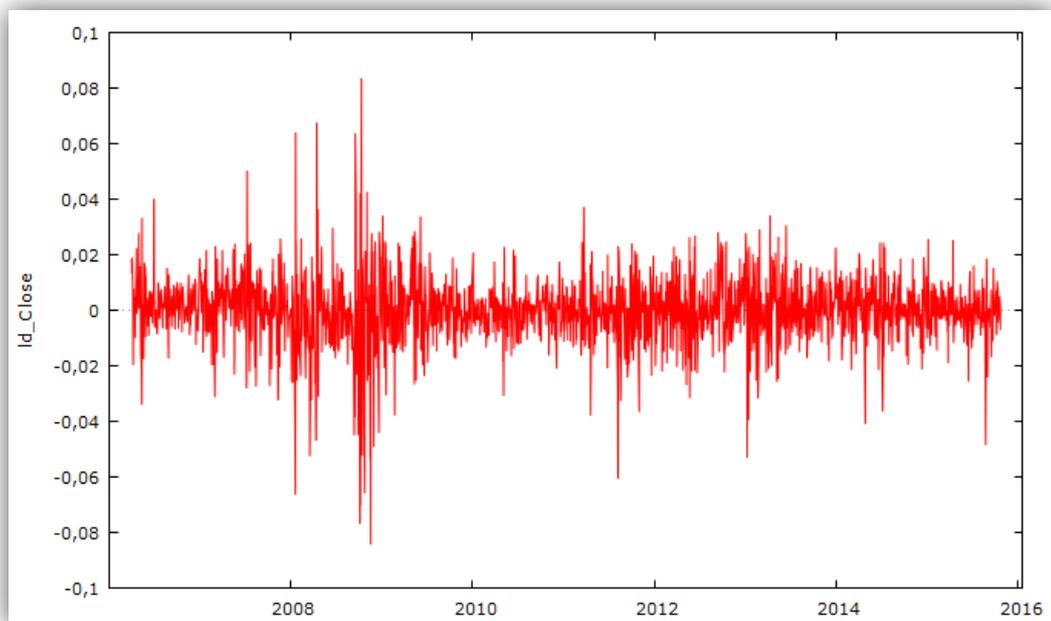
Figure 10: SBI TOP returns from April 2006 to October 2015

4.6. Eurozone (EURO STOXX50)

We decided to use EURO STOXX50 stock market index (designed by index provider STOXX and owned by Deutsche Börse Group) as a proxy for European markets. EURO STOXX50 was established in 1998 and consists of 50 largest and most liquid European stocks. We use it as a benchmark since it is the most liquid index for the Eurozone and thus suitable for benchmarking purposes.

Looking at the levels plot on Figure **11**, we see significantly different development compared to previously observed time series. As expected, index initially plunged with the financial crisis offset. After hitting lowest point in 2009, recovery began. However, recovery was endangered in 2011 after European debt crisis shattered investors' confidence.

Index rebounded again in following years and has recovered significant portion of its pre-crisis value by the end of our sample period. Nontheless, recovery has been slow and still in progress.

Returns plot (see Figure **12**) suggests that volatility was highest during the period that overlaps with financial crisis outset (2008-2010). Volatility memory is quite long both in crisis and post-crisis period. Major shock to the volatility occurred in late 2008, corresponding to Lehman brothers' failure.

ADF test and KPSS test prove that EURO STOXX50 time series were indeed non-stationary. The risk to reject the null hypothesis while it is true is 29.66%. Thus, we used logarithm differencing approach in order to transform series to stationary. Following transformation, stationarity was confirmed by both ADF test and KPSS as can be seen from **Table 1** and **Table 2**.

EURO STOXX50 dataset has 2448 observations with minimum value of 1809,980 and maximum value reaching 4557,570 points. Dataset mean is 3090,642 points.

Figure 11: EURO STOXX50 levels from April 2006 to October 2015
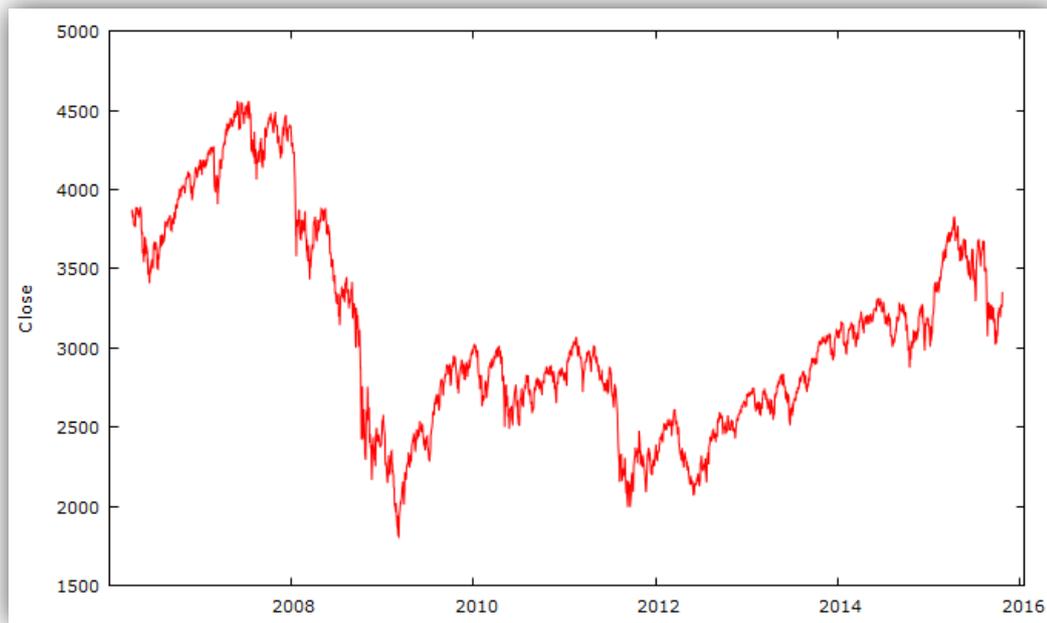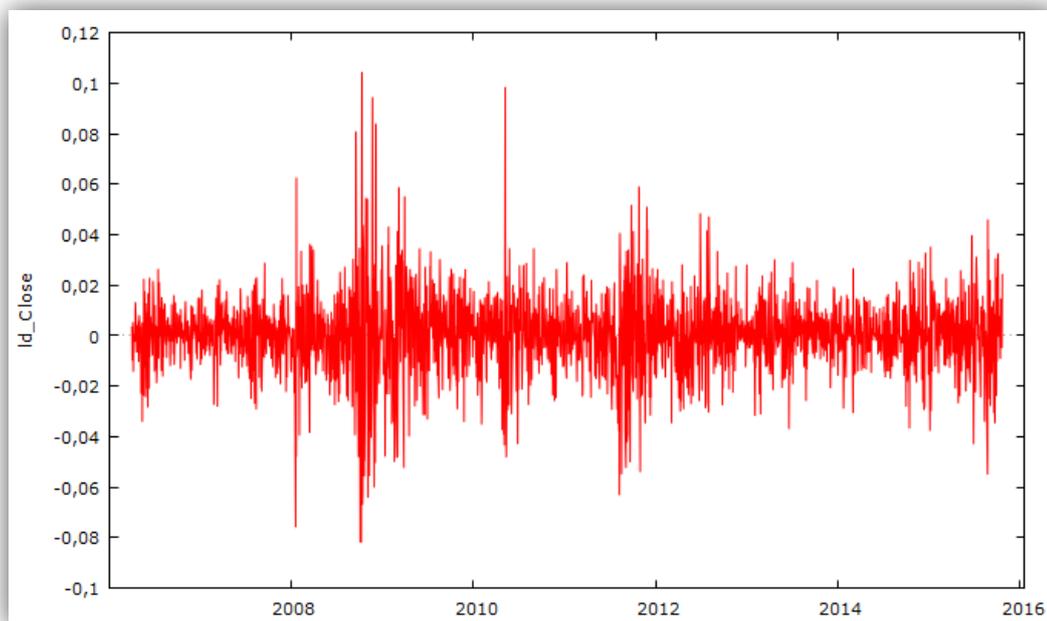

Figure 12: EURO STOXX50 returns from April 2006 to October 2015

To conclude, we have seen that all markets reacted with sharp decline to global financial crisis. The degree of recovery differs between markets. While European markets are characterized with slow but continuing recovery, it seems that former Yugoslavia markets are mostly stagnating.

As expected, highest volatility for most markets was observed in late 2008 (due to Lehman Brothers failure and consequent fear and panic that flooded worldwide markets). Judging by visual analysis of returns plots, it seems that volatility jumps were more severe on former Yugoslavia market compared to European markets. However, European markets may have had longer volatility memory than most of former Yugoslavia markets.

It is noticeable that European markets are characterized with more frequent volatility jumps although less severe. Former Yugoslavia markets, on the other hand, have less frequent volatility jumps, but their effect is seemingly more austere. Thus, returns plot reveal destructive effect that global financial crisis (and even European debt crisis) has had on developing and underdeveloped markets.

Table **1** and Table **2** present summary of the statistics behind tests (before and after transformation) computed in this chapter.

Table 1: Augmented Dickey-Fuller Test

Variable	Before transformation		After transformation	
	t-Statistics	p-value	t-Statistics	p-value
SASX10	-1.139535	0.7021	-10.60462	0.000
BIRS	-1.728382	0.4168	-6.318884	0.000
BELEX15	-0.798630	0.8189	-28.37782	0.000
CROBEX	-1.016356	0.7495	-18.57817	0.000
SBI TOP	-0.630326	0.8614	-34.68366	0.000
EURO STOXX50	-1.043940	0.2966	-6.284784	0.000

Source: Author's calculations

Table 2: Kwiatkowski-Phillips-Schmidt-Shin Test

Variable	Before transformation		After transformation	
	LM-Statistics	p-value	LM-Statistics	p-value
SASX10	3.212183	0.000	0.192808	0.5842
BIRS	3.284484	0.000	0.170143	0.5226
BELEX15	2.959491	0.000	0.201575	0.5837
CROBEX	3.216954	0.000	0.157692	0.6751
SBI TOP	3.549917	0.000	0.350966	0.6358
EURO STOXX50	1.907047	0.000	0.170376	0.8175

Source: Author's calculations

4.7. Descriptive Statistics

We used EViews to compute descriptive statistics for each of our datasets in order to get insights into key moments and features of the data. Data has been described by mean, median, maximum value, minimum value, standard deviation, kurtosis, skewness, and Jarque-Bera test. It should be noted that computations were done on transformed series (i.e. returns) and not levels.

The mean (or average) is a sum of all values in the dataset divided by number of observations (i.e. dataset values). The median is the middle value of the dataset once dataset is arranged in the ascending order. Maximum value is the largest value in the dataset while minimum is the smallest value.

Standard deviation is a measure of dispersion from the average value. Kurtosis is a measure of probability distribution shape relative to normal distribution. Skewness shows potential asymmetry of the distribution. Jarque-Bera test examines whether data have kurtosis and skewness matching a normal distribution.

Table **3** presents computation results.

Table 3: Descriptive statistics of time series returns

Index	Mean	Median	Max	Min	Std. Dev	Skewness	Kurtosis	Jarque-Berra
SASX-10	-3.6e-4	-6.8e-5	8.757e-2	-8.84e-2	1.309e-2	1.62632e-1	1.10348e1	6.43404e3
BIRS	-2.9e-4	0.0000	7.319e-2	-5.009e-2	9.1e-3	4.04941e-1	1.10212e1	6.40211e3
BELEX 15	-2.4e-4	-2.93e-4	1.2158e-1	-0.10861	-1.0861e-1	1.54021e-1	1.67605e1	1.90077e4
CROBEX	-1.4e-4	0.0000	1.4779e-1	-1.0764e-1	1.271e-2	-0.00485	-4.85e-3	3.0296e4
SBI TOP	-1.8e-4	1.09e-5	8.358e-2	-8.431e-2	1.197e-2	-4.489e-1	9.72831	4.56538e3
EURO STOXX 50	5.6e-5	1.13e-4	1.0438e-1	-8.208e-2	1.518e-2	1.5415e-2	8.26285	2.8241e3

Source: Author's calculations

Looking at the mean values it is noticeable that all former Yugoslavia markets had negative mean returns. This is not too surprising if we take into consideration global financial crisis and slow to non-existent recovery of these markets in the subsequent years. Mean return of EURO STOXX50 has a small albeit positive mean return. Since both European and former Yugoslavia markets have been hit severely by the crisis this disparity stems from the fact that European markets have had faster recovery. That being said, SASX-10 index has had worst performance with an average return of -0.0362% during observed period while EURO STOXX was the most successful with an average return of 0.00565%. Among former Yugoslavia countries most successful was CROBEX with a mean return of -0.0143%.

We see that CROBEX has had the largest maximum value indicating that index has recorded highest daily return (14,77%) within our observed timeframe. This is not very surprising since CROBEX also has the best mean return among former Yugoslavia countries, as said before. On the other hand, BELEX15 is the infamous winner of the minimum value category with the largest recorded negative return of -10.86%. Interestingly enough, CROBEX also had one of the largest negative returns: -10.76%.

EURO STOXX was seemingly the most volatile index having standard deviation of 1.5178%. On the other hand, BIRS had the lowest standard deviation of 0.9099% indicating lowest volatility.

Since normal distribution has skewness of zero and kurtosis of 3, we conclude that our time series do not follow normal distribution. These results were additionally confirmed by Jarque-Bera test.

4.8. Hypotheses

The focus of this thesis is the modeling and forecasting of stock markets volatility in Bosnia and Herzegovina, in particular Sarajevo Stock Exchange SASX-10 index and Banja Luka Stock Exchange BIRS index. In addition, we intend to compare volatility processes for these indices with volatility of other former Yugoslavia countries and European markets. Thus, we formulate our hypotheses as following:

1. SASX-10 index and BIRS index volatilities can be properly modeled with ARCH/GARCH. We expect this should be feasible.
2. Representative stock market indices in other former Yugoslavia countries (most notably Slovenia, Croatia and Serbia) can also be described with ARCH/GARCH and have volatility comparable to indicies in Bosnia and Herzegovina.
3. Stock markets in Bosnia and Herzegovina (SASX-10 and BIRS) demonstrate higher volatility compared to the European markets (EURO STOXX50 index).

5 Empirical Results

In this part of the thesis we present empirical results obtained by employing GARCH type models on our data.

5.1. ARMA Models Estimation

However, we start our empirical journey by researching whether ARMA analysis would be sufficient in describing time series at our hands. In doing so, we follow Box-Jenkins (1970) methodology. We have already established stationarity of the data and thus it is safe to proceed with model identification and model selection. Hence, we look at the autocorrelation function plot (ACF) and partial autocorrelation plot (PACF) and compare information criteria for various model specifications.

We estimated number of different models and compared the Akaike information criterion (AIC). Results are presented in Table 4.

Table 4: Akaike information criterion (AIC) for ARMA models

INDEX/ARMA	(1,0)	(0,1)	(1,1)	(2,0)	(0,2)	(2,1)	(1,2)	(2,2)
BELEX15	-14068,3	-14020,9	-14070,4	-14071,6	-14077,9	**-14078,8**	-14075,9	-14077,4
BIRS	-15721,2	-15690,8	-15723,1	-15723,3	-15721,1	-15721,5	**-15752,3**	15759,6
CROBEX	-14034	-14038,3	-14062,3	-14038,7	-14046,4	-14060,3	-14060,3	**-14077,5**
EURO STOXX50	-13549,7	-13550,1	**-13556,9**	-13553,6	-13554,3	-13556,2	-13556,2	-13554,3
SASX10	-14195.7	-14203,1	-14207,1	-14209,2	-14207,8	-14208,8	**-14216**	-14206,8
SBI TOP	-14355,5	-14360,8	-14359,7	**-14361,4**	14360	14360,1	-14359,5	-14358,1

Source: Author's calculations

As can be seen from the table above, the most appropriate model for BELEX15 is ARMA (2,1) since it has the lowest AIC criterion among alternatives. Looking at the ACF and PACF (see **Error! Reference source not found.**), we note that ARMA 2,2) could have been chosen too. We opted for ARMA (2,1) because of somewhat lower AIC and stronger significance of the coefficient parameters in the model. ACF and PACF offer yet another insight: model with higher number of AR terms (more

than 2) could have been chosen as well since PACF has significant third lag. However, due to parsimony we stick with ARMA (2,1).

ARMA (1,2) was chosen as the most fitting model for BIRS time series. Although ARMA (2,2) had somewhat lower AIC we opted for ARMA (1,2) due to insignificant additional MA parameter in ARMA(2,2). This observation is confirmed by looking at the ACF and PACF plots which suggest that either ARMA (1,2) or ARMA (2,2) may be appropriate although higher order models could be used too. Once more, due to parsimony reasons we do not consider ARMA (2,3).

Computed information criteria for CROBEX indicates that ARMA (2,2) should be used since this model has the lowest AIC. ACF and PACF provide us with visual proof that ARMA (2,2) is appropriate choice. However, it is noticeable that ARMA (3,3) could have been a good option too or even ARMA (5,5). Since we are trying to avoid over complex models, ARMA (2,2) seems to provide us with best tradeoff between complexity and accuracy.

According to the computed AIC, EURO STOXX50 is best modeled by using ARMA (1,1). ACF and PACF provide us with insight about significant second, third and fourth lag (for both plots) and thus there is an option to use models with more than 2 AR and/or MA terms.

SASX10 is best described by ARMA (1,2) and ARMA (2,0) if we are to judge by the information criteria. Both models have parameters (with the exception of constant) highly significant at 99% confidence level. However, since former has lower AIC, that makes ARMA (1,2) our model of choice. ACF and PACF plot suggest that ARMA (1,1) may be appropriate too.

Finally, ARMA (2,0) has the lowest AIC for SBI TOP. Thus it is model specification of our choice.

We now proceed to the estimation of these models as described by Box-Jenkins (1970). All AR (p) and MA (p) terms have highly significant parameters at 99% significance for all time series. The only exception is ARMA (2,2) model estimated on CROBEX time series with MA (1) term significant on 95% level. Constant is insignificant across all estimated models with p-value consistently over 0.5.

The third and final step in Box-Jenkins (1970) methodology is model checking. In particular, we proceed with residuals diagnostics in order to find out whether residuals are normally distributed, serially correlated and if there is an ARCH effect present. We also look for traces of clustering volatility by visually inspecting residuals and squared residuals plots.

Normal distribution of the residuals is tested by using Normality test and plotting histograms relative to normal distribution. Serial correlation is checked by using Ljung-Box test and plotting ACF and PACF of both residuals and squared residuals. Finally, presence of ARCH effect is investigated by employing ARCH-LM test.

If diagnostics prove that a) there is no clustering volatility, b) residuals are normally distributed, c) there is no serial correlation and d) no ARCH effect then ARMA model is an appropriate one. Otherwise, (G)ARCH models should be used to model respective residuals.

5.2. Residual Diagnostics

5.2.1. BELEX15

Figure **13** and Figure **14** present residuals and squared residuals plots for BELEX15 series. We can observe clustering volatility indicating that ARMA (2,1) may not be sufficient.
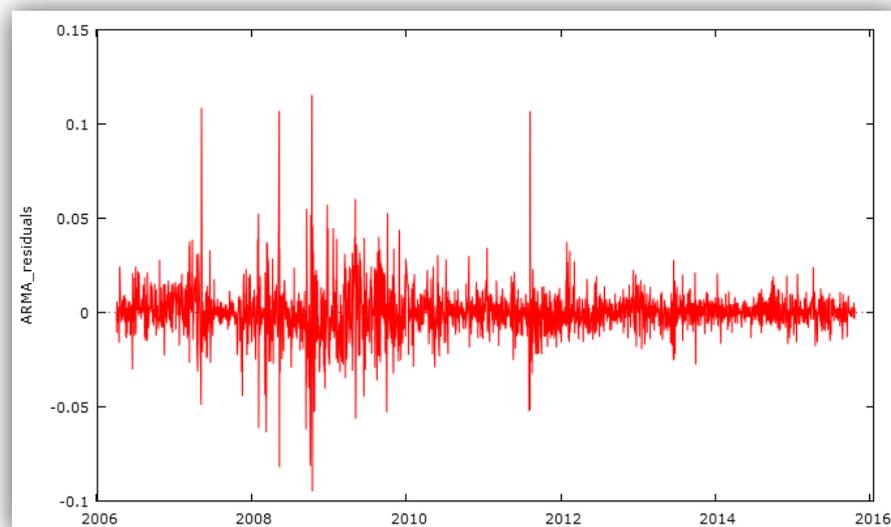
Figure 13: Time series plot of residuals for BELEX15
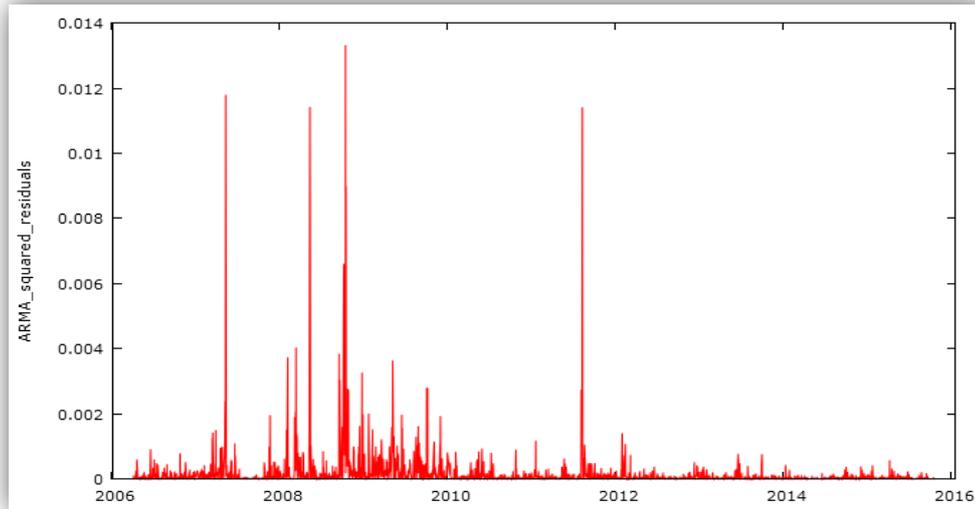
Figure 14: Time series plot of squared residuals for BELEX15

Upon conducting Normality test, resulting zero p-value suggests that we can reject null hypothesis of normal distribution and thus conclude that BELEX15 residuals do not follow normal distribution. This is obvious from the histogram as well (see Figure **15**).
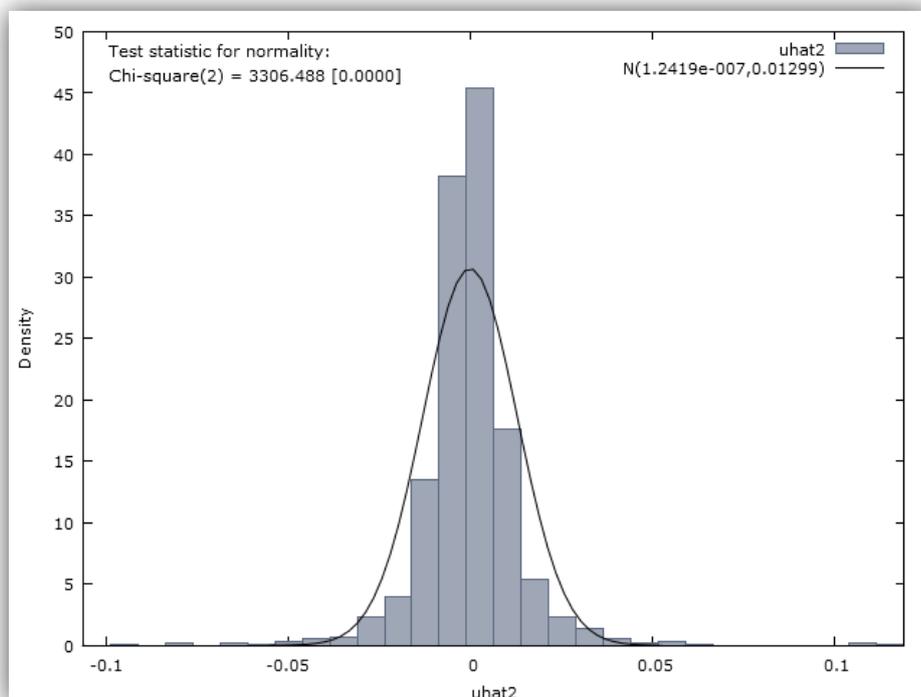

Figure 15: Distribution histogram of residuals for BELEX15

In order to check for serial correlation, we plotted ACF and PACF of residuals and squared residuals (see **Figure A.42** and **Figure A.43** and in Appendix) for BELEX15 residuals. The need for inspecting both residuals and squared residuals when testing for serial correlation arises from the fact that sometimes residuals are not correlated but squared residuals are. This observation dates back to Engle (1982).

Looking at the first 12 lags of autocorrelation function for BELEX15 residuals, we observe no serial correlation. In fact, if we conduct Ljung-Box test for first 5 lags p-value of 0.1479 indicates that null hypothesis of no correlation should not be rejected. However, higher order lags exhibit clear serial correlation confirmed both by Ljung-Box test and autocorrelation function.

In case of squared residuals, serial correlation is present throughout series and confirmed both by Ljung-Box test and autocorrelation function.

Finally, we proceed with ARCH-LM test in order to test for ARCH effect within residuals. With p-value of close to zero (see

Table **A.13** in Appendix) it is safe to reject null of no ARCH effect. In other words, ARCH effect is contained within residuals.

To conclude, all observed aspects of residuals diagnostics indicate that BELEX15 times series residuals should be modeled with (G)ARCH type models. Before doing so, we proceed with the residuals diagnostics for remaining time series.

5.2.2. BIRS

Looking at the residuals and squared residuals plots (see Figure **16** and Figure **17**, respectively) volatility clustering is clearly observed. In other words, small changes are followed by small changes and large changes tend to be followed by further large changes. This is our first indication that (G)ARCH model should be introduced to model BIRS residuals.
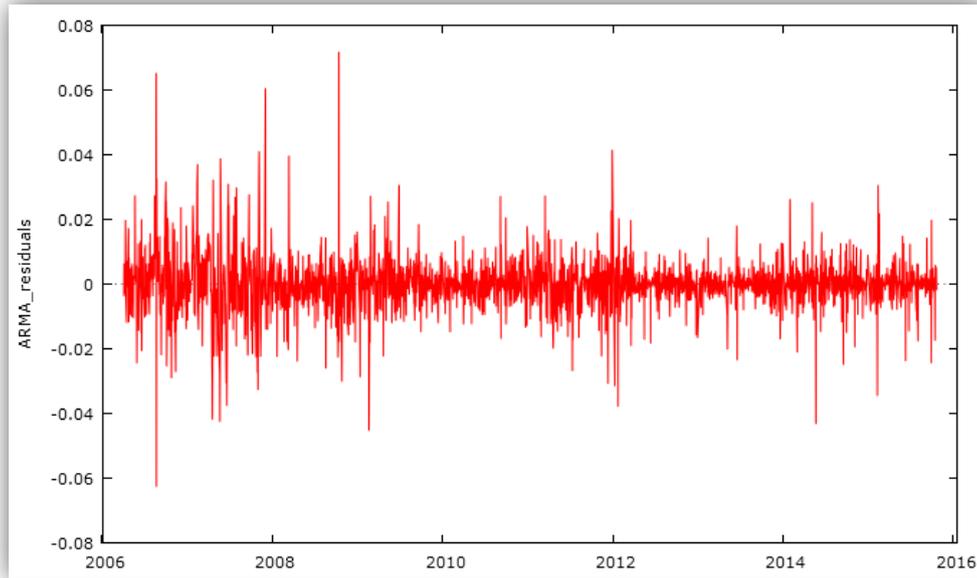
Figure 16: Time series plot of residuals for BIRS
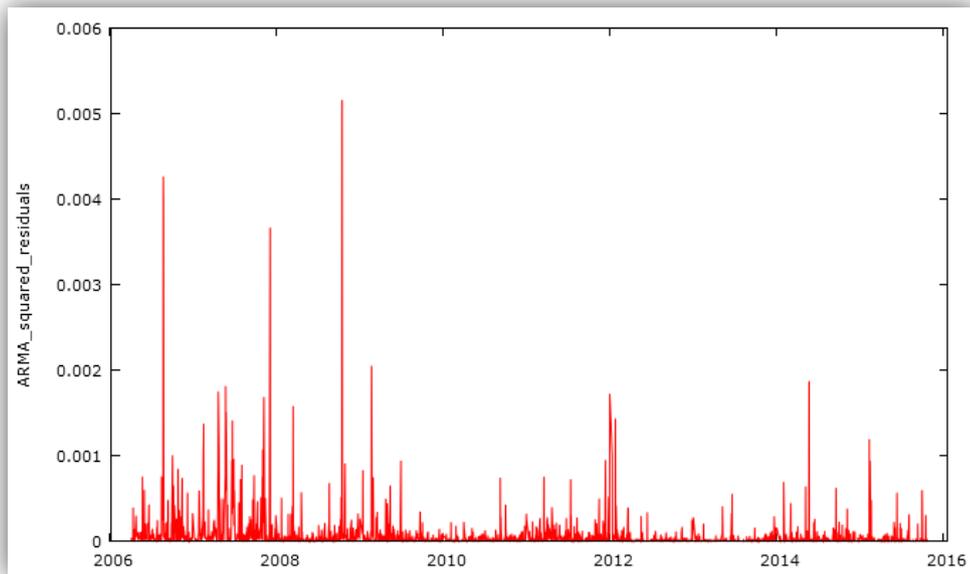
Figure 17: Time series plot of squared residuals for BIRS

Normality test produced zero p-value, suggesting that BIRS time series residuals do not follow normal distribution. This is further confirmed by visual inspection of histogram presented as Figure **18**.
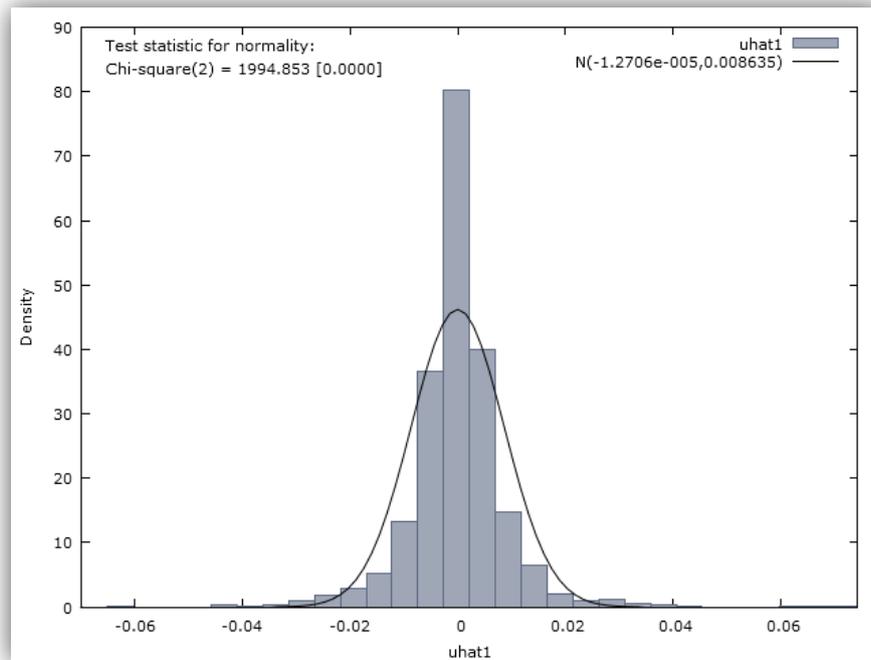
Figure 18: Distribution histogram of residuals for BIRS

According to the low p-values obtained from ACF and PACF (for both residuals and squared residuals) we confirm serial correlation. Ljung-Box test results support this inference.

Next we proceed with ARCH-LM test. With p-value close to zero, it is safe to say that ARCH effect is present within residuals and thus they will be modeled with (G)ARCH type models. With this conclusion we wrap up residual diagnostics for BIRS time series and proceed to CROBEX.

5.2.3. CROBEX

As before, we visually inspect residuals and square residuals plots (seen as Figure **19** and Figure **20,** respectively)**.** Once again, volatility clustering is evident and most notable during global financial crisis.

Normality test confirms our suspicions that CROBEX residuals do not follow normal distribution. In addition to formal testing results this can be seen from histogram presented as Figure **21**.
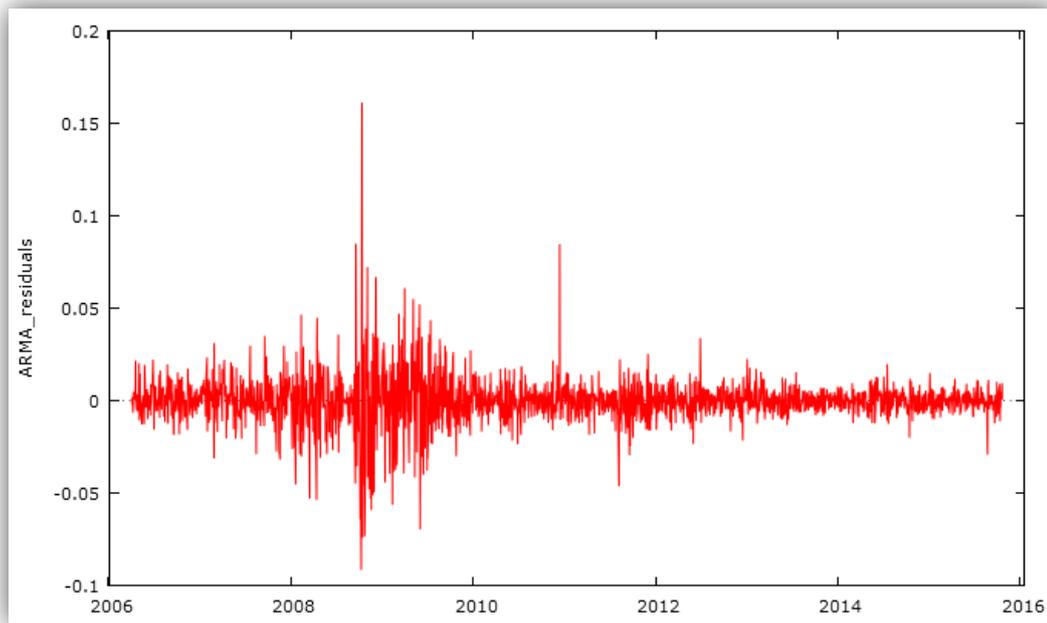
Figure 19: Time series plot of residuals for CROBEX
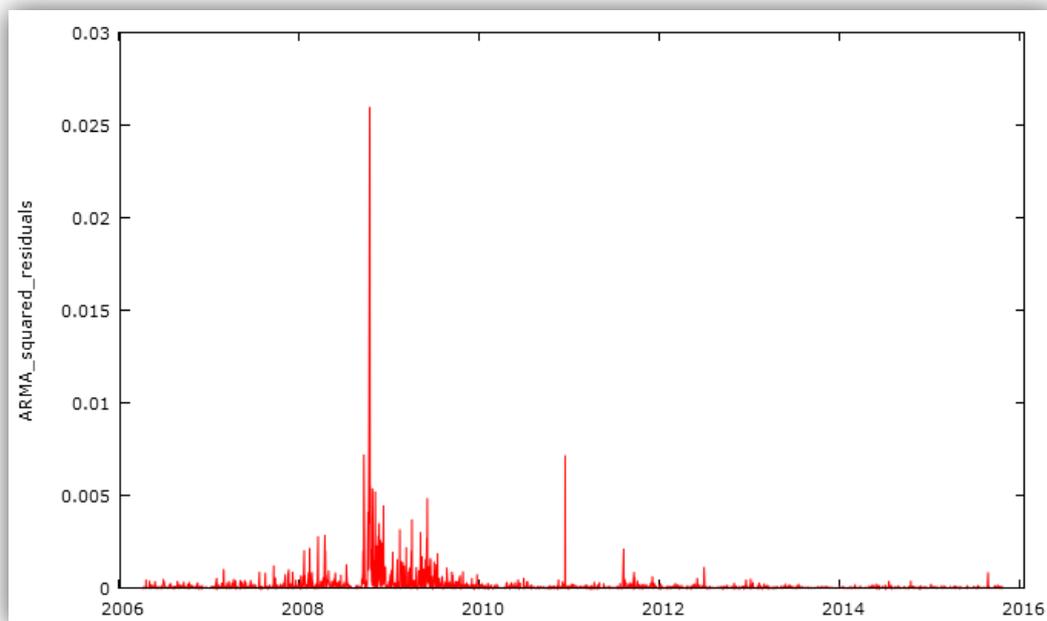
Figure 20: Time series plot of squared residuals for CROBEX
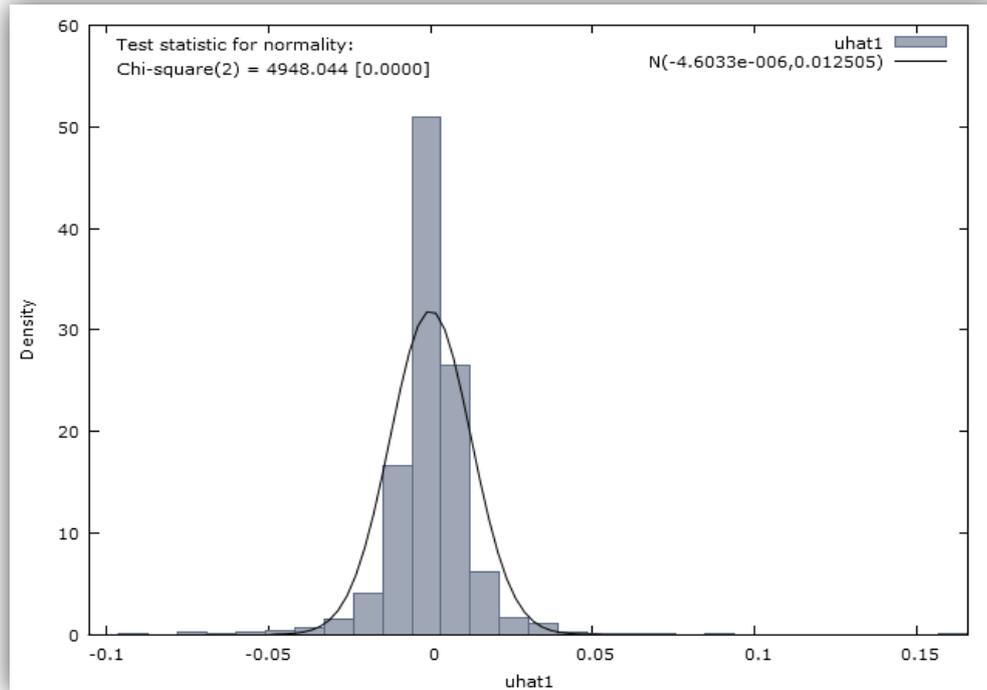
Figure 21: Distribution histogram of residuals for CROBEX

Similar to BELEX15 time series, ACF indicates that first 8 lags of CROBEX residuals exhibit no serial correlation. However, higher order lags show clear signs of serial correlation confirmed by Ljung-Box test. In case of squared residuals, serial correlation is evident for all observed lags.

Logical next step is to test CROBEX residuals for potential ARCH effect. To do so, we employed ARCH-LM test. Very low p-value gives us reason enough to conclude that ARCH effect is indeed present in the residuals.

Thus, diagnostics checking has provided us with sufficient evidence that residuals are supposed to be modeled with (G)ARCH. Hence, we continue with residuals diagnostics for EURO STOXX50.

5.2.4. EURO STOXX50

Once more, we begin diagnostics with visual inspection of residuals and squared residuals (Figure **22** and Figure **23**). No surprise here: once more we observe volatility clustering.

Figure 22: Time series plot of residuals for EURO STOXX50
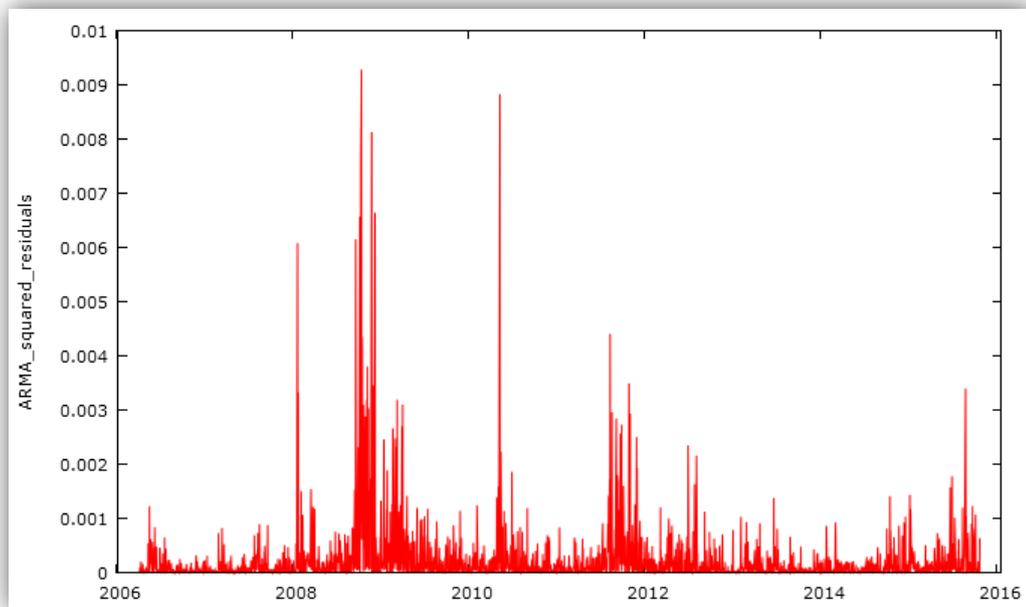
Figure 23: Time series plot of squared residuals for EURO STOXX50

Normality of residuals is rejected as p-value produced by Normality test is close to zero. Thus, EURO STOXX50 residuals do not follow normal distribution. Histogram can be seen below as Figure **24**.
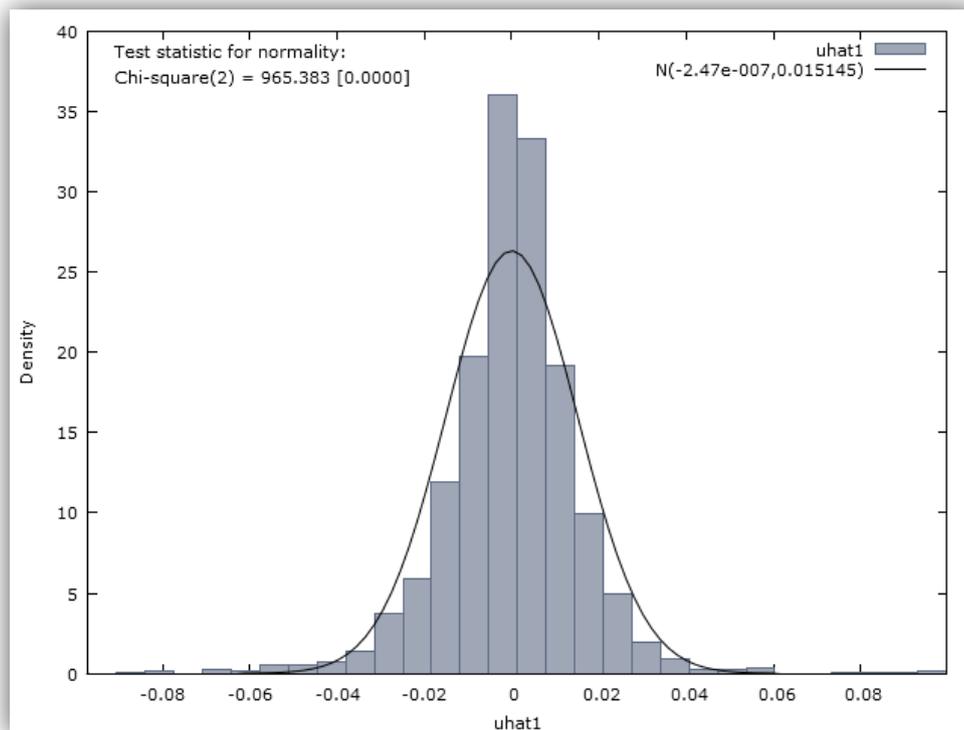
Figure 24: Distribution histogram of residuals for EURO STOXX50

Ljung-Box test with 33 lags show clear lack of independence both for residuals and squared residuals. If we take a closer look at each individual lag by plotting ACF and PACF, patterns seen with BELEX15 and CROBEX residuals emerge again: first several lags of residuals are not correlated while higher order lags are. Squared residuals are serially correlated throughout time series.

ARCH-LM test leaves no doubt regarding existence of an ARCH effect in EURO STOXX50 residuals. With p-values close to zero, null hypothesis of no ARCH effect can be rejected. SASX-10 series are next up for residual diagnostics.

5.2.5. SASX-10

Visual inspection of residuals and squared residuals confirms existence of clustering volatility. This can be seen from Figure **25** and Figure **26,** respectively.
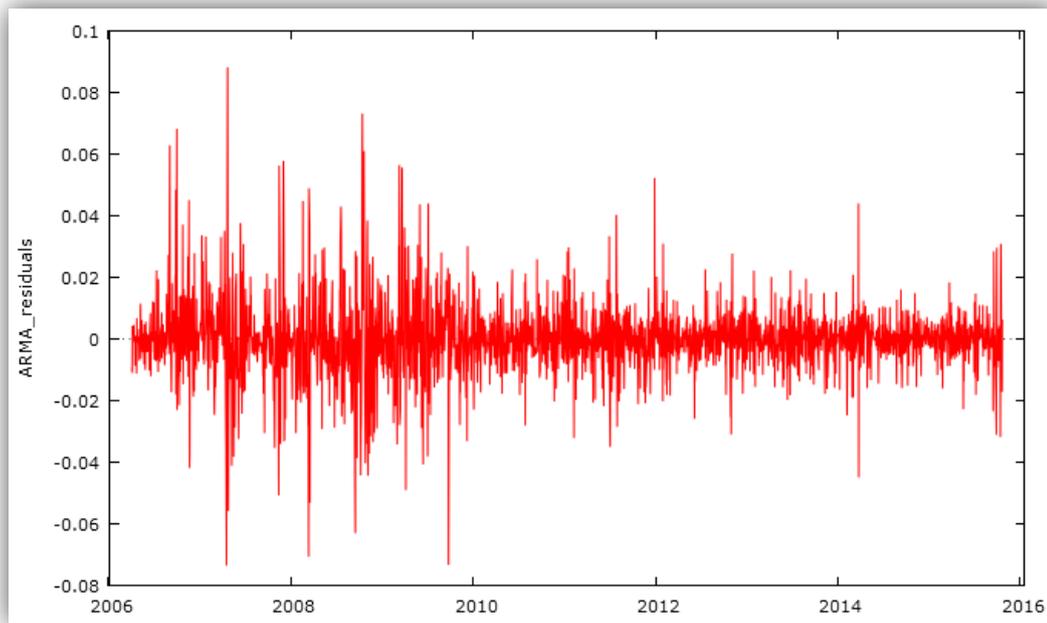
Figure 25: Time series plot of residuals for SASX-10
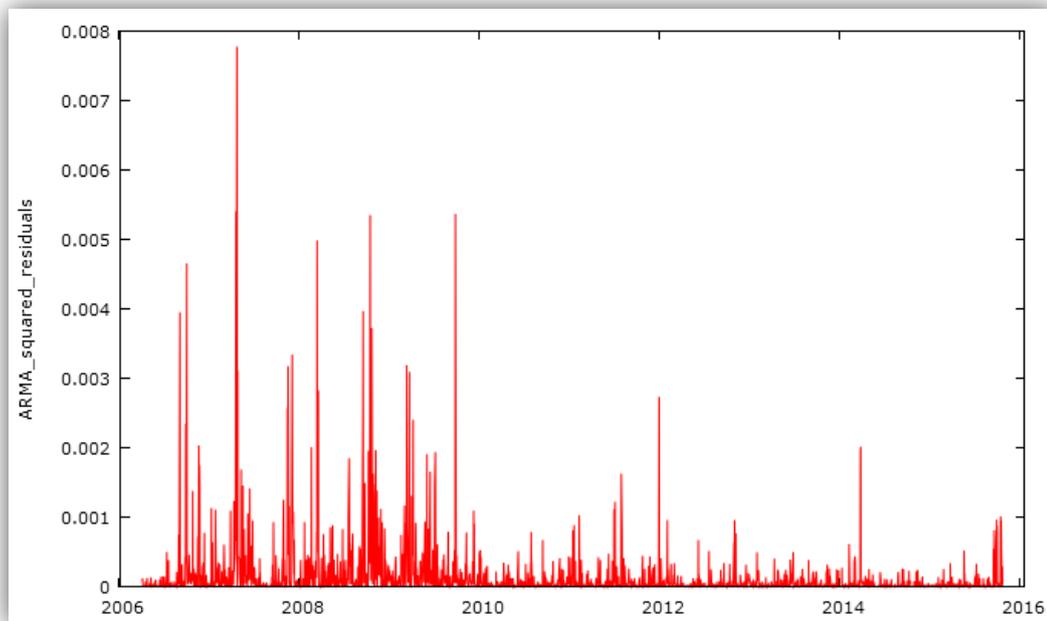
Figure 26: Time series plot of squared residuals for SASX-10

Normality test is conclusive: SASX-10 time series do not follow normal distribution since p-value is close to zero. Distribution histogram is presented below as Figure **27**.
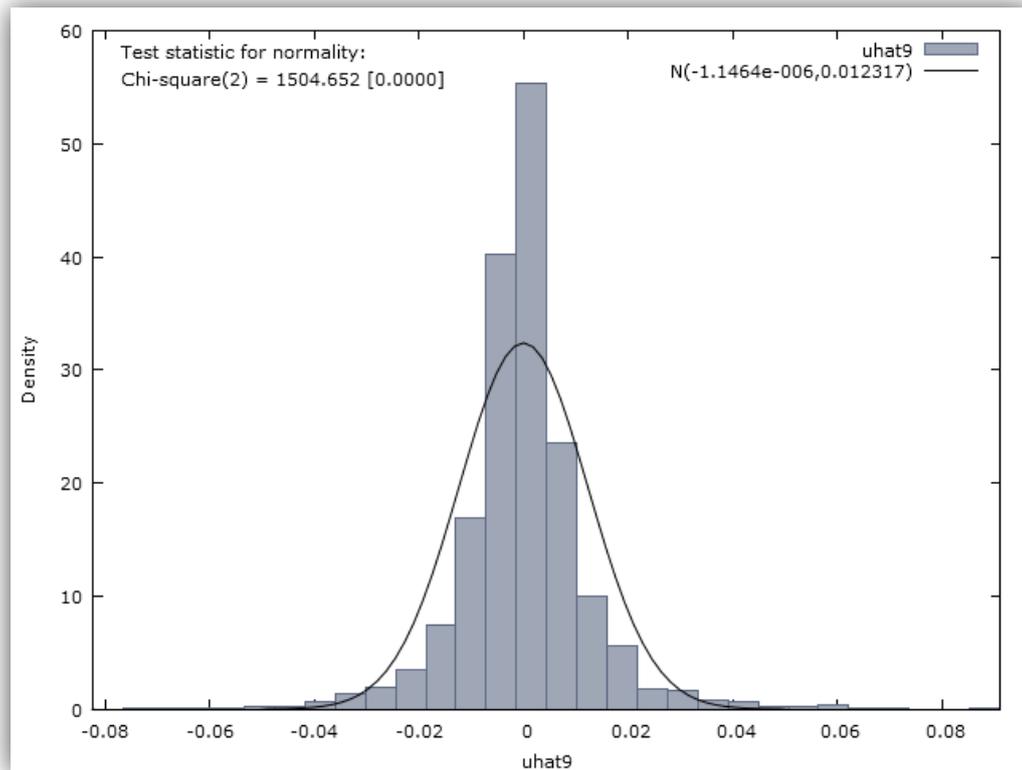
Figure 27: Distribution histogram of residuals for SASX-10

Ljung-Box test suggests that residuals and squared residuals are indeed correlated, as we suspected. This is further confirmed by ACF and PACF of residuals and squared residuals.

Finally, ARCH-LM test gives us enough reasons to conclude that ARCH effect is present within residuals at significance level of 99%. Thus, SASX-10 residuals will be modeled using (G)ARCH type models. Finally, we conclude this subsection with analysis of SBI TOP series residuals.

5.2.6. SBI TOP

SBI TOP residuals and squared residuals plots (Figure **28** and Figure **29**) follow the same pattern we already observed by looking at the residuals/squared residuals plots of other indices. In other words, clustering volatility is quite noticeable.
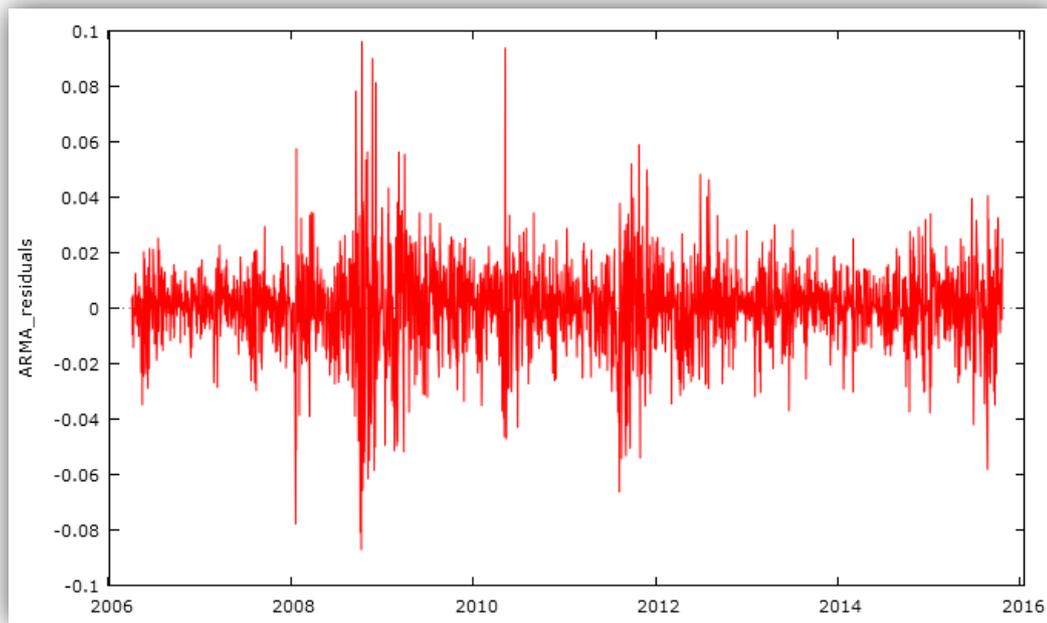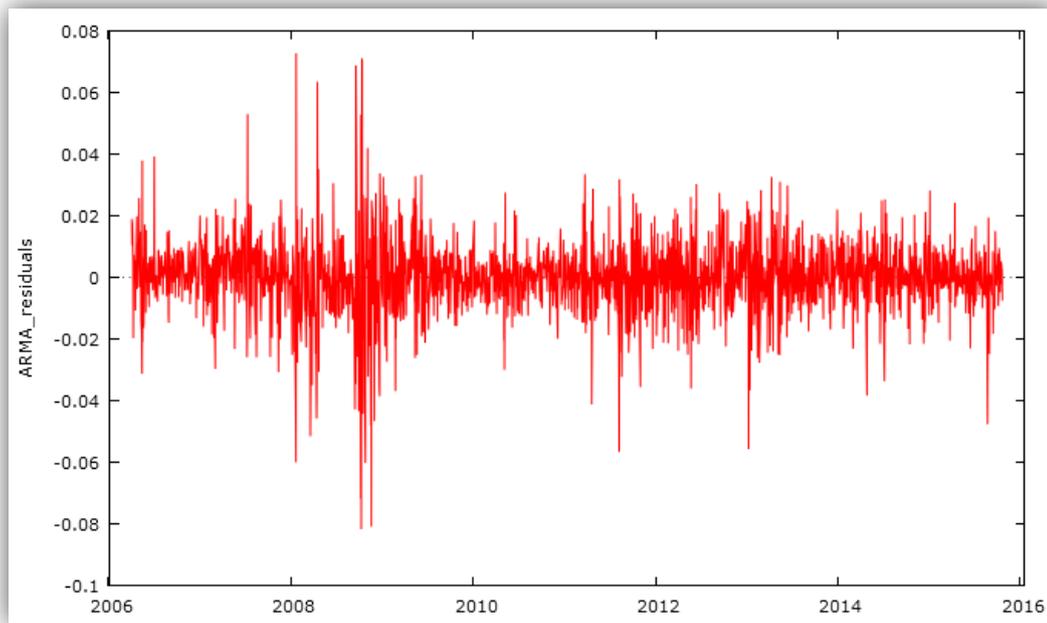
Figure 28: Time series plot of residuals for SBI TOP
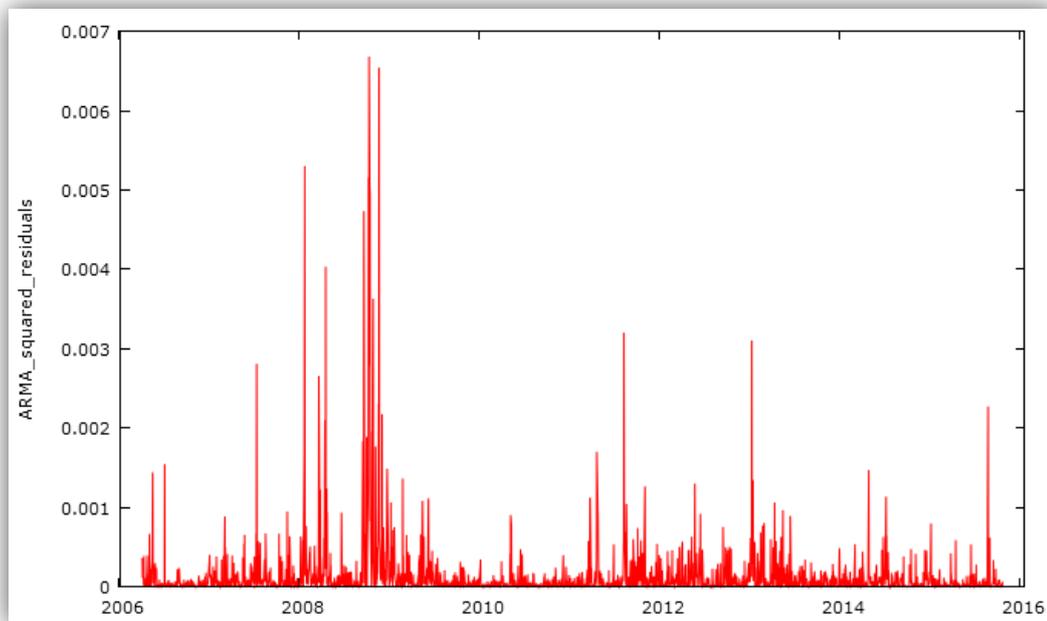
Figure 29: Time series plot of squared residuals for SBI TOP

Normality test results are able to assure us that SBI TOP residuals do not follow normal distribution. Disatribution histogram (presented as Figure **30**) further confirms this.
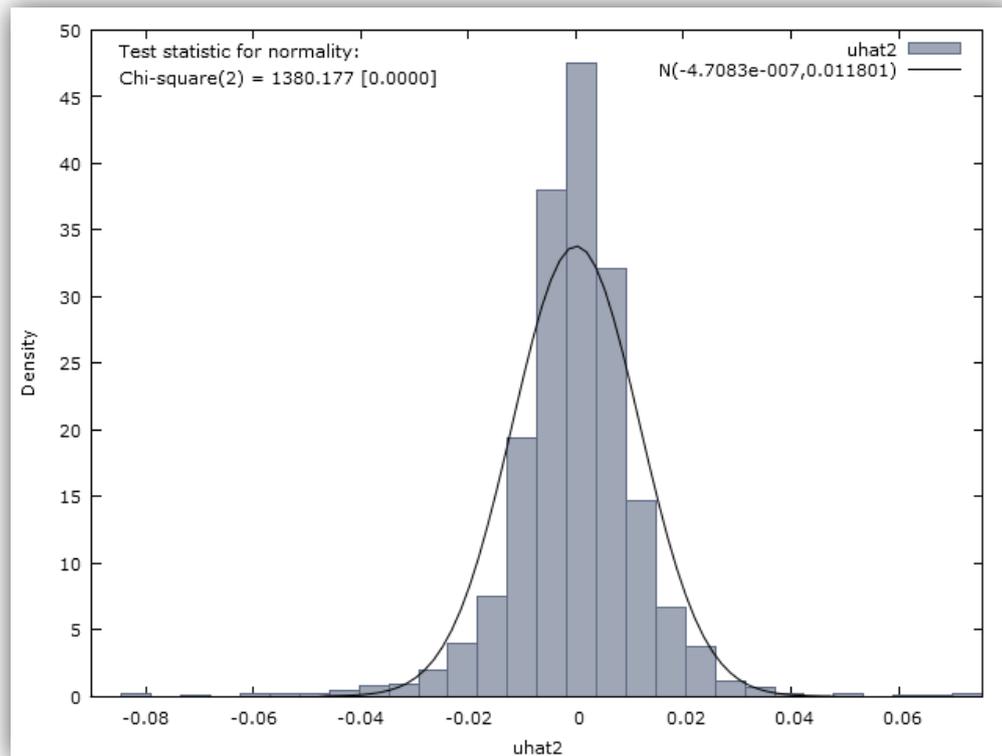
Figure 30: Distribution histogram of residuals for SBI TOP

Residuals seem to be independent during first 12 lags and then exhibit serial correlation on higher order lags. Squared residuals, on the other hand, are serially correlated throughout series.

ARCH-LM test proves that ARCH effect is present. Thus, residuals should be modeled with (G)ARCH.

5.3. GARCH Models Estimation

After completing residual diagnostics and justifying the application of (G)ARCH type models, we proceed with GARCH estimation.

After estimating a number of different GARCH models (with normal and t-Student's distributions) and comparing their information criteria, we came to conclusion that GARCH (1,1) is the most appropriate model for all observed time series. Furthermore, we draw upon similar conclusions made in seminal paper "*Does anything beat a GARCH (1,1)*?" by Hanse and Lunde.

In order to test for potential leverage effect, we additionally employed asymmetric GJR-GARCH (1,1) under the assumption of both normal and t-Student's distribution.

Hence, our models of choice are as following:

a) ARMA (2,1) – GARCH (1,1) and ARMA (2,1) – GJR-GARCH (1,1) for BELEX15 time series,

b) ARMA (1,2) – GARCH (1,1) and ARMA (1,2) – GJR-GARCH (1,1) for BIRS time series,

c) ARMA (2,2) – GARCH (1,1) and ARMA (2,2) - GJR-GARCH (1,1) for CROBEX time series,

d) ARMA (1,1) – GARCH (1,1) and ARMA (1,1) – GJR-GARCH (1,1) for EURO STOXX50 time series,

e) ARMA (1,2) – GARCH (1,1) AND ARMA (1,2) – GARCH (1,1) for SASX-10 time series, and finally

f) ARMA (2,0) – GARCH (1,1) and ARMA (2,0) – GJR-GARCH (1,1) for SBI TOP time series.

Upon estimating these models, we inspected residuals again. Residuals diagnostics convinced us that substantial improvement has been made compared to ARMA models. This time ARCH effect was eliminated for all models and serial correlation was not observable within residuals.

Although residuals were still not normally distributed they were (according to Jarque–Bera test statistics) much closer to the normal distribution than before.

Table **5** summarizes results of the GARCH and GJR-GARCH modeling for BELEX15 time series under the assumption of normal and t-Student's distribution.

Table 5: GARCH and GJR-GARCH estimation results for BELEX15

BELEX15			
Mean Equation			
	Coefficient	**z-statistics**	**p-value**	
Constant	-0.000246956	-0.6086	0.54281	
AR (1)	-0.395996	-3.3844	0.00071	***
AR (2)	0.283347	8.0443	<0.00001	***
MA (1)	0.707424	5.9427	<0.00001	***
Variance Equation			
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	4.05145e-06*** (4.5260)	3.70682e-06** (2.186)	3.97580e-06** (3.388)	3.68788e-06** (3.472)
ARCH term	0.247814*** (8.9524)	0.230407 *** (3.710)	0.246043*** (5.266)	0.229628*** (6.060)
Leverage effect	NA	NA	0.0475292 (250.3)	0.00726549 (282.4)
GARCH term	0.751661 *** (29.9193)	0.770042*** (12.98)	0.753338*** (43.99)	0.770789*** (51.78)

Source: Author's calculations

Both GARCH and GJR-GARCH coefficients are highly significant at 99% confidence level with the exception of GARCH/GJR-GARCH constant (under the assumption of t-Student's distribution for GARCH model and both distributions for GJR-GARCH model) being significant at 95% level. GARCH term is roughly around 0.75 (normal distribution) and 0.77 (t-Student's distribution) indicating long memory and persistence. In fact, the sum of α and β is ~ 1 showcasing tendency to stick to today's variance (i.e. high persistence).

GJR-GARCH showcases insignificant and rather small leverage effect. This happens to be the case under both normal and t-Student's distribution. Thus, we did not found evidence that negative innovations impact volatility more than positive do.

Table **6** summarizes results of the GARCH and GJR-GARCH modeling for CROBEX time series under the assumption of normal and t-Student's distribution.

Table 6: GARCH and GJR-GARCH estimation results for CROBEX

CROBEX			
Mean Equation			
	Coefficient	**z statistics**	**p-value**	
Constant	-0.00013328	-0.3009	0.76352	
AR (1)	0.318117	4.1544	0.00003	***
AR (2)	0.61128	9.3314	<0.00001	***
MA (1)	-0.177562	-2.5225	0.01165	**
MA (2)	-0.699919	-12.4525	<0.00001	***
Variance Equation			
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	9.33195e-07*** (3.8664)	7.11081e-07** (2.185)	8.48400e-07** (2.151)	6.02551e-07** (2.060)
ARCH term	0.0959032*** (8.3361)	0.0800894*** (3.730)	0.0884559*** (3.563)	0.0684581*** (3.395)
Leverage effect	NA	NA	0.128142 (1.149)	0.187470*** (2.658)
GARCH term	0.900918*** (82.3673)	0.915519*** (42.72)	0.907548*** (46.07)	0.925457*** (46.11)

Source: Author's calculations

Similar to BELEX15, most GARCH and GJR-GARCH coefficients are significant at 99% level. The GARCH term under GARCH (1,1) is substantial under both distributions (0.90 and 0.91, respectively) indicating longer memory and volatility persistence compared to BELEX15 time series. On the other hand, ARCH term is almost negligible with estimated coefficient of just 0.09. The sum of coefficients is close to 1 proving that decay towards long-run variance is very slow.

GJR-GARCH model provides us with some insights regarding leverage effect. While insignificant under normal distribution, leverage effect seems to be highly significant under t-Student's distribution. Although much lower (just below 0.19) compared to BELEX15 series, existing leverage effect still signals that negative innovations have more influence on volatility than the positive do. GARCH term is further boosted in GJR-GARCH model (to 0.92) while ARCH term remains low.

Table **7** summarizes results of the GARCH and GJR-GARCH modeling for SBI TOP time series under the assumption of normal and t-Student's distribution.

Table 7: GARCH and GJR-GARCH estimation results for SBI TOP

SBI TOP				
Mean Equation				
	Coefficient	**z statistics**	**p-value**	
Constant	-0.00017804	-0.6553	0.51227	
AR (1)	0.167352	8.1748	<0.00001	***
AR (2)	-0.0574472	-2.8059	0.00502	***
Variance Equation				
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	1.20177e-05*** (6.5332)	9.17946e-06*** (3.788)	1.22828e-05*** (4.180)	9.28476e-06*** (3.839)
ARCH term	0.256616*** (8.9393)	0.237133*** (5.826)	0.237742*** (6.047)	0.226680*** (5.689)
Leverage effect	NA	NA	0.154978*** (3.260)	0.108897*** (2.626)
GARCH term	0.660592*** (20.3834)	0.706468*** (14.92)	0.666615*** (14.06)	0.709452*** (14.98)

Source: Author's calculations

All estimated coefficients (for both models and under both distributions) are significant at 99%. The GARCH term seems to be somewhat smaller compared to CROBEX and BELEX15 time series at 0.66 (under normal distribution) and 0.70 (under t-Student's distribution). Thus, SBI TOP displays shorter memory and volatility persistence. To be more precise, sum of ARCH and GARCH terms coefficients for SBI TOP varies between 0.90 - 0.93 depending on distribution. In both cases, the sum is lower than the sum observed for BELEX15 and CROBEX signaling lower persistence of SBI TOP and comparably faster convergence to long-run variance.

GJR-GARCH detected leverage effect under both distributions: 0.15 (normal distribution) and 0.10 (t-Student's distribution). Smaller leverage effect signals that

although negative innovations indeed have stronger impact to volatility, the effect is less emphasized than for CROBEX and BELEX15 series. In contrast with BELEX15 and CROBEX series, leverage effect does not significantly alter either ARCH or GARCH term.

Table **8** summarizes results of the GARCH and GJR-GARCH modeling for EURO STOXX50 time series under the assumption of normal and t-Student's distribution.

Table 8: GARCH and GJR-GARCH estimation results for EURO STOXX50

EURO STOXX50				
Mean Equation				
	Coefficient	**z statistics**	**p-value**	
Constant	-5.83476e-05	-0.2281	0.81960	
AR (1)	0.681042	5.7179	<0.00001	***
MA (1)	-0.733374	-6.6468	<0.00001	***
Variance Equation				
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	4.42916e-06*** (4.3149)	3.21318e-06*** (3.092)	4.33148e-06*** (3.388)	3.64045e-06*** (3.472)
ARCH term	0.10674*** (7.7580)	0.0989255*** (6.099)	0.0452042*** (5.266)	0.0457095*** (6.060)
Leverage effect	NA	NA	1.00721*** (250.3)	1.00668*** (282.4)
GARCH term	0.874702*** (56.8191)	0.890201*** (54.41)	0.886730*** (43.99)	0.889134*** (51.78)

Source: Author's calculations

Similar to SBI TOP, all estimated coefficients for EURO STOXX50 are significant at 99%. The GARCH term of EURO STOXX50 varies from 0.87 to 0.89 across chosen models and distributional assumptions. In general, this implies long volatility memory of EURO STOXX50 - longer than SBI TOP or BELEX15 and shorter than CROBEX. With $\alpha + \beta \sim 0.98$ persistence is clearly higher than for SBI TOP albeit lesser compared to BELEX15 and CROBEX series.

GJR-GARCH showcases existing leverage effect: negative innovations increase volatility more than the positive do. Leverage effect seems to slightly increase the GARCH term and simultaneously decreases the ARCH term from 0.10 to mere 0.045. It must be noted that estimated coefficient for the leverage effect is extremely high (over 1) signifying disproportionate impact of negative "news" over "positive".

Table **9** summarizes results of the GARCH and GJR-GARCH modeling for BIRS time series under the assumption of normal and t-Student's distribution.

Table 9: GARCH and GJR-GARCH estimation results for BIRS

BIRS				
Mean Equation				
	Coefficient	**z statistics**	**p-value**	
Constant	-0.000261567	-0.4193	0.67499	
AR (1)	0.984333	130.7977	<0.00001	***
MA (1)	-0.735102	-35.2124	<0.00001	***
MA (2)	-0.208872	-10.6847	<0.00001	***
Variance Equation				
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	1.86416e-06*** (4.0896)	9.13298e-06* (1.950)	1.84601e-06* (1.399)	9.37956e-06** (2.003)
ARCH term	0.0994961*** (6.6427)	0.419841*** (3.001)	0.0987396** (2.140)	0.426982*** (3.075)
Leverage effect	NA	NA	0.0361523 (0.4636)	-0.0248441 (-0.4488)
GARCH term	0.879519*** (48.9431)	0.632855*** (5.428)	0.880279*** (15.29)	0.626793*** (5.478)

Source: Author's calculations

Under normal distribution, GARCH model coefficients for BIRS time series are significant at 99%. Not much has changed under t-Student's distribution except for constant which is now significant at just 90%. GJR-GARCH estimation has somewhat decreased coefficients significance: under normal distribution constant is significant at 90% while this increases to 95% under t-Student's distribution.

Both GARCH and ARCH term coefficients substantially change with the shift of distribution assumptions. Hence, the GARCH term in GARCH (1,1) model is ~ 0.88 under normal distribution but drops to 0.63 under t-Student's distribution. ARCH term, on the other hand, jumps from ~ 0.1 under normal distribution to ~ 0.42 under t-Student's distribution.

We observe similar scenario with GJR-GARCH model too. This shift cannot be attributed to the leverage effect since: a) we do not observe leverage effect under simple GARCH model, and b) its coefficients are very small and insignificant. It rather seems that different distributional assumptions favor this period actual variance (ARCH term) or the variance predicted for this period (GARCH term), respectively.

Hence, under normal distribution BIRS volatility is similar to EURO STOXX50 time series with relatively long memory. However, under t-Student's distribution volatility looks a lot more like SBI TOP with moderately long memory.

The sum of GARCH and ARCH term coefficients is close to 0.97 (under normal distribution) implying strong persistence. Compared to other analyzed series, persistence is stronger than for SBI TOP and weaker compared to BELEX15, CROBEX and EURO STOXX50.

Under t-Student's distribution, the sum of coefficients is over 1. This signals unit root in the variance and provides us with explanation on (previously mentioned) strange behavior under t-Student's distribution. Thus, GARCH (1,1) is not an appropriate model under t-Student's distribution for BIRS.

As already noted, leverage effect is insignificant under both distributional assumptions. Surprisingly, leverage effect coefficient under t-Student's distribution is negative implying that the positive innovations may have stronger influence on volatility than the negative innovations.

Table **10** summarizes results of the GARCH and GJR-GARCH modeling for SASX-10 time series under the assumption of normal and t-Student's distribution.

Table 10: GARCH and GJR-GARCH estimation results for SASX-10

SASX-10				
Mean Equation				
	Coefficient	**z statistics**	**p-value**	
Constant	-0.000348667	-0.4711	0.63754	
AR (1)	0.991515	195.6458	<0.00001	***
MA (1)	-0.658764	-33.8189	<0.00001	***
MA (2)	-0.31565	-16.7882	<0.00001	***
Variance Equation				
	GARCH (normal distribution)	**GARCH (t-Student's distribution)**	**GJR (normal distribution)**	**GJR (t-Student's distribution)**
Constant	6.07595e-06*** (4.3591)	9.15657e-06** (2.335)	6.13870e-06* (1.668)	9.21990e-06** (2.316)
ARCH term	0.178805*** (7.2493)	0.337952*** (3.790)	0.179428*** (2.701)	0.339766*** (3.762)
Leverage effect	NA	NA	-0.0214500 (-0.3756)	0.0169635 (0.3711)
GARCH term	0.788922*** (26.9326)	0.700192*** (9.580)	0.787682*** (9.386)	0.698873*** (9.447)

Source: Author's calculations

SASX-10 modeling produced highly significant coefficients for the most part. Only exception is the GJR-GARCH constant (under normal distribution) significant at 90%.

The GARCH term coefficient varies under different distributions (~ 0.69 under t-Student's to 0.78 under normal distribution) but this shift is not as large as with BIRS time series. ARCH term seems to vary a bit more: ~ 0.17 under normal distribution to ~ 0.33 under t-Student's distribution.

Compared to other analyzed time series, SASX-10 GARCH term is most similar to BELEX15 series. It showcases shorter memory than CROBEX or EUROSTOXX series but longer than SBI TOP. The sum of ARCH and GARCH term coefficients (~ 0.96 under normal distribution) signals that the persistence is similar to BIRS albeit

somewhat lower. Looking at α and β coefficients under t-Student's distribution we see once more that sum exceeds one. Thus, there is a unit root in the variance and thus GARCH (1,1) model (under t-Student's distribution) is not an appropriate one for modeling SASX-10 series.

Compared to the other Bosnia and Herzegovina stock market index - BIRS, SASX-10 displays shorter volatility memory and persistence as seen from lower GARCH term. ARCH term is higher under normal distribution (compared to ARCH term for BIRS series) but lower under t-Student's distribution.

As with BIRS series, leverage effect is insignificant under both distributions. While BIRS had a surprising negative coefficient for the leverage effect (under t-Student's distribution) SASX-10 has negative leverage coefficient under normal distribution. Again, this is not very common since it is well documented in the literature that leverage effect usually comes with positive coefficient and indicates prevalence of "negative" news impact to volatility over "positive" news.

Among all analyzed time series, CROBEX has the longest memory and persistence as signaled by highest GARCH term coefficient (~ 0.92). Highest ARCH term coefficient has been detected for BIRS time series under t-Student's distribution (~ 0.42). Lowest GARCH term coefficients was detected for BIRS time series (under t-Student's distribution), approximately 0.62, while lowest ARCH term coefficient is observed for EURO STOXX50 (~ 0.04).

5.4. Conditional Standard Deviations Comparison

In order to compare the volatility of selected time series with SASX-10 and BIRS we will be plotting conditional standard deviations over time. First, we compare regional indices (BELEX15, CROBEX, SBI TOP) and European benchmark index (EURO STOXX50) with both Sarajevo stock exchange index (SASX-10) and Banja Luka stock exchange index (BIRS).

Afterwards, we will compare SASX-10 and BIRS with each other. These graphical comparisons were derived from previously estimated GARCH (1,1) models under the assumption of normal distribution.
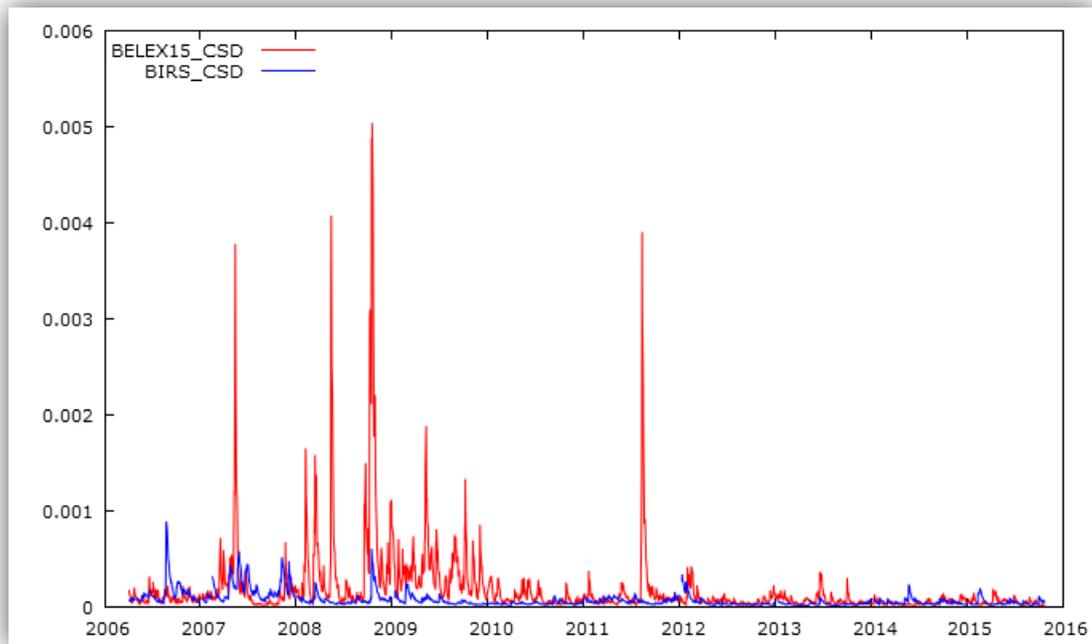
Figure 31: Comparison of CSD for BELEX15 and BIRS

Comparing the conditional standard deviation graphs for BELEX15 and BIRS time series (see Figure **31**) it is evident that BELEX15 exhibited higher volatility most of the observed period, the only exception being late 2006 and early 2014. Both indices displayed highest conditional standard deviations during crucial stages of global financial crisis and were somewhat synchronized at the very offset of crisis (late 2007) and during 2009. However, BIRS had substantially smaller shocks intensity compared to BELEX15. BELEX15 was characterized by short memory manifesting as sharp jumps and fast decaying peaks on the graph. BIRS had longer memory as well as persistence manifested in slow decaying shocks. Estimation results confirm these findings since GARCH term is higher for BIRS than for BELEX15.

In the post-crisis period conditional standard deviations for both indices have decreased and remained relatively low by the end of the observed period. The only exception was BELEX15's severe response to Greece debt crisis in late 2011. BIRS reaction to debt crisis was somewhat delayed (compared to BELEX15) and is seen during the early 2012 albeit much lesser in the magnitude of conditional standard deviation jump. BIRS suffered several smaller shocks during 2014 and 2015 most probably due to violent social unrest that took place in Bosnia and Herzegovina

during early 2014 and periodical risk that Greece could default on its debt and thus bring turmoil to the markets.
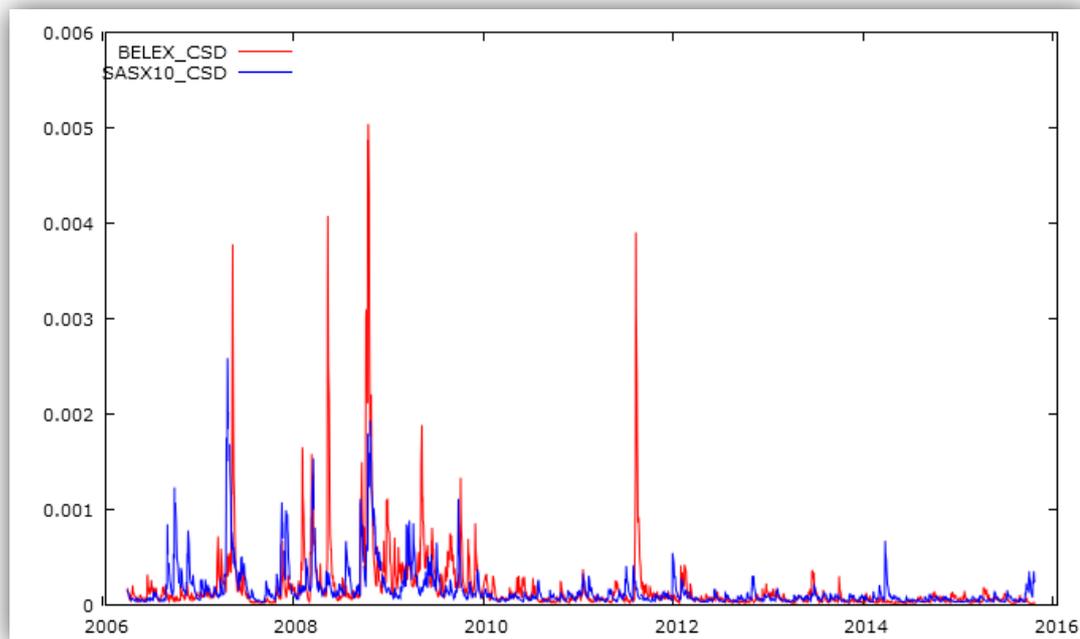
Figure 32: Comparison of CSD for BELEX15 and SASX-10

Judging by the conditional standard deviation graph seen on Figure **32**, SASX-10 followed BELEX15 volatility pattern more closely than BIRS did. As expected, both indices had highest conditional standard deviations during global financial crisis as well as similar pattern (although different in magnitude).

Conditional standard deviation is consistently higher for BELEX15 indicating much more volatile time series. They only exception is the very beginning of the crisis and early 2014 when social unrests shook Bosnia and Herzegovina market.

Post-crisis volatility pattern was similar for both indices with the exception of severe jump that BELEX15 had as a response to European debt crisis in late 2011. Similar to BIRS, SASX-10 also displayed several smaller jumps during 2014-2015 period that were not observable for BELEX15.

Both indices displayed relatively short memory as seen from short-lived peaks and sharp jumps. This is not surprising since GARCH term is comparable in size for two series as seen from the estimation results.

We now proceed to the comparison of conditional standard deviations of CROBEX with BIRS (Figure **33**) as well as SASX-10 (Figure **34**), respectively.
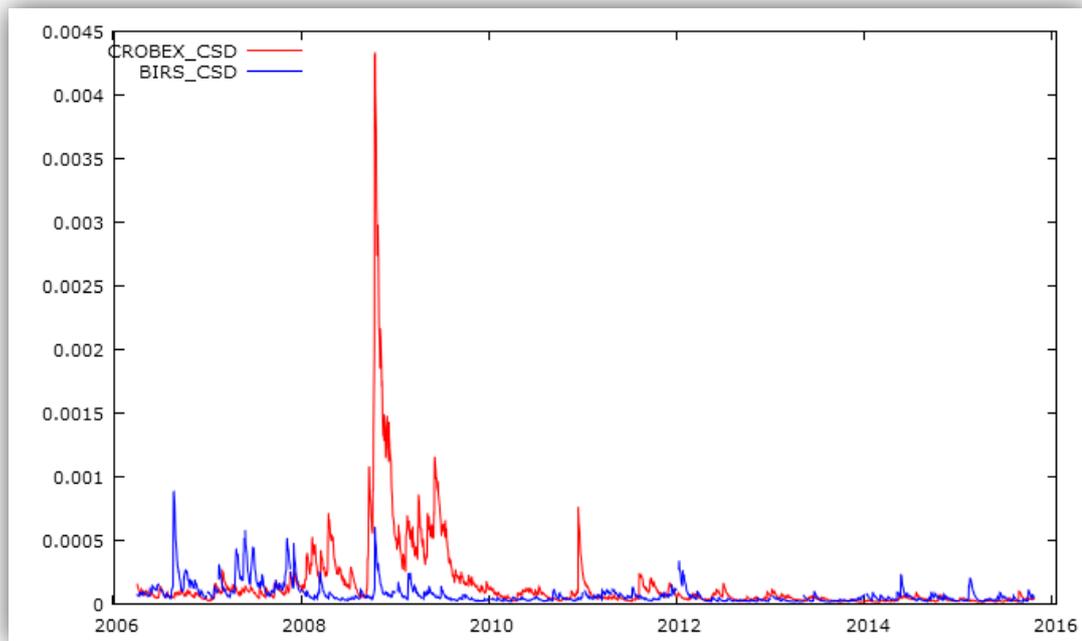
Figure 33: Comparison of CSD for CROBEX and BIRS

Before the crisis offset, BIRS and CROBEX volatility patterns were asynchronized as seen from different stages of high and low conditional standard deviations for two series. At this time, BIRS showcased higher conditional standard deviation implying higher volatility. On the other hand, CROBEX clearly had more aggressive response to the crisis itself.

Huge conditional standard deviation jump during 2009 signals that global financial crisis disrupted Croatian market more severely. CROBEX consistently maintained higher conditional standard deviation throughout crisis until 2010. Thus, graph reflects what we already knew from the estimation results: CROBEX had the higher GARCH term indicating long memory and persistence.

During 2011 CROBEX suffered from another shock (most probably due to European debt crisis) not detected for BIRS. Since 2012, conditional standard deviations are low for both indices suggesting that period of relative serenity has taken place.
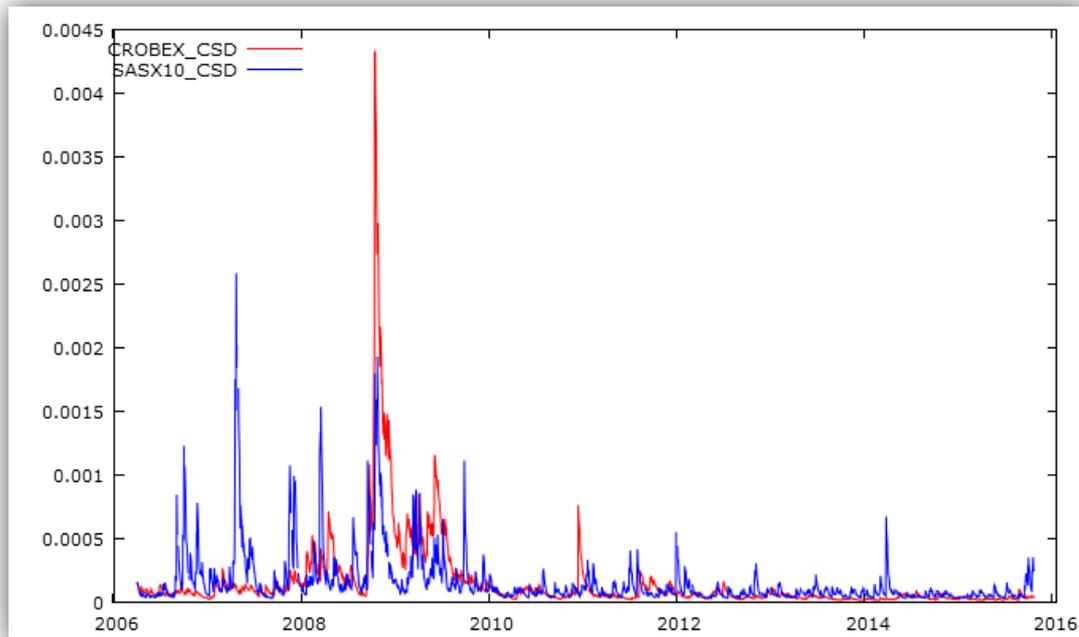
Figure 34: Comparison of CSD for CROBEX and SASX-10

Similar volatility evolution is seen after plotting SASX-10 and CROBEX conditional standard deviations. SASX-10 was substantially more volatile (and asynchronized with CROBEX) in the pre-crisis period but this shifted during the crisis.

CROBEX showcased higher conditional standard deviation jumps during this period, once again proving that Croatian market was more disrupted than the Bosnia and Herzegovina market was by the global financial crisis. In the post-crisis period it seems that SASX-10 exhibited slightly higher conditional standard deviation with several sharp peaks during 2012 and 2014.

The impact of lower GARCH term for SASX-10 was manifested as shorter volatility memory. Thus, we witnessed more sharp jumps for SASX-10 on the graph than we did for CROBEX.

Next, we proceed to comparison of SBI TOP conditional standard variations with BIRS (Figure **35**) and SASX-10 (Figure **36**), respectively.
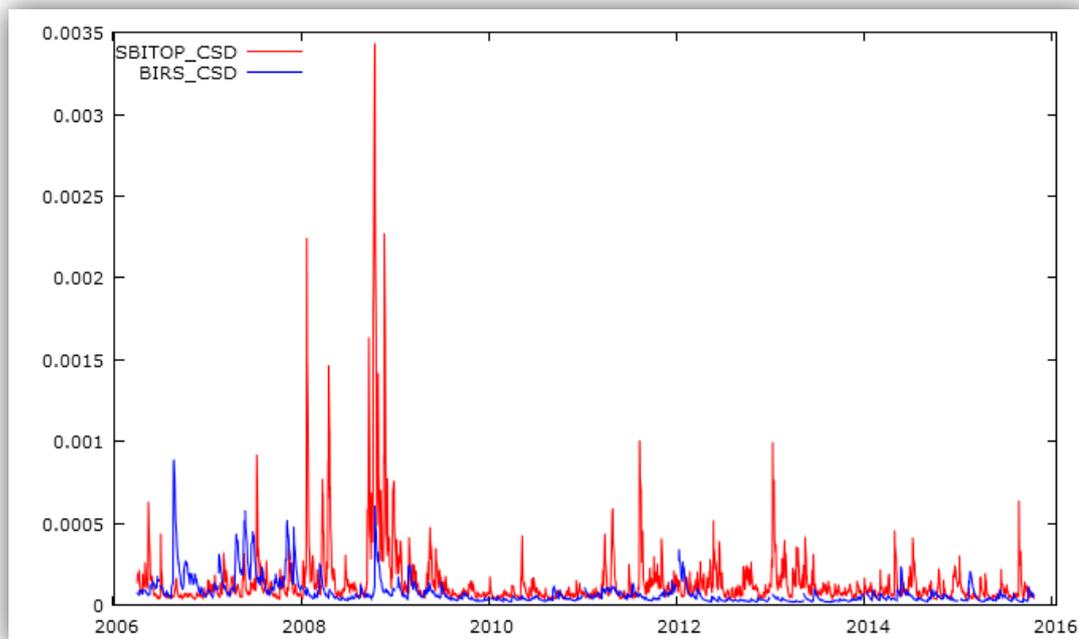
Figure 35: Comparison of CSD for SBI TOP and BIRS

Slovenian SBI TOP series have consistently maintained higher conditional standard deviation compared to Bosnian BIRS. The only exceptions were mid-2006 and early 2012. The level of synchronization between two indices was rather low: phases of high and low volatility did not match very often. Similar to CROBEX, reaction of SBI TOP to global financial crisis has been much more aggressive compared to BIRS. Among all analyzed series, SBI TOP had the sharpest individual volatility jumps. This comes as no surprise – estimation results have shown that GARCH term was the lowest among all analyzed series for SBI TOP implying short memory and low persistence.

SBI TOP series have been very aggressive in response to European debt crisis too, with sharp increases and decreases of conditional standard deviation. This is understandable since at the time Slovenian market was the only regional market fully integrated with European markets.
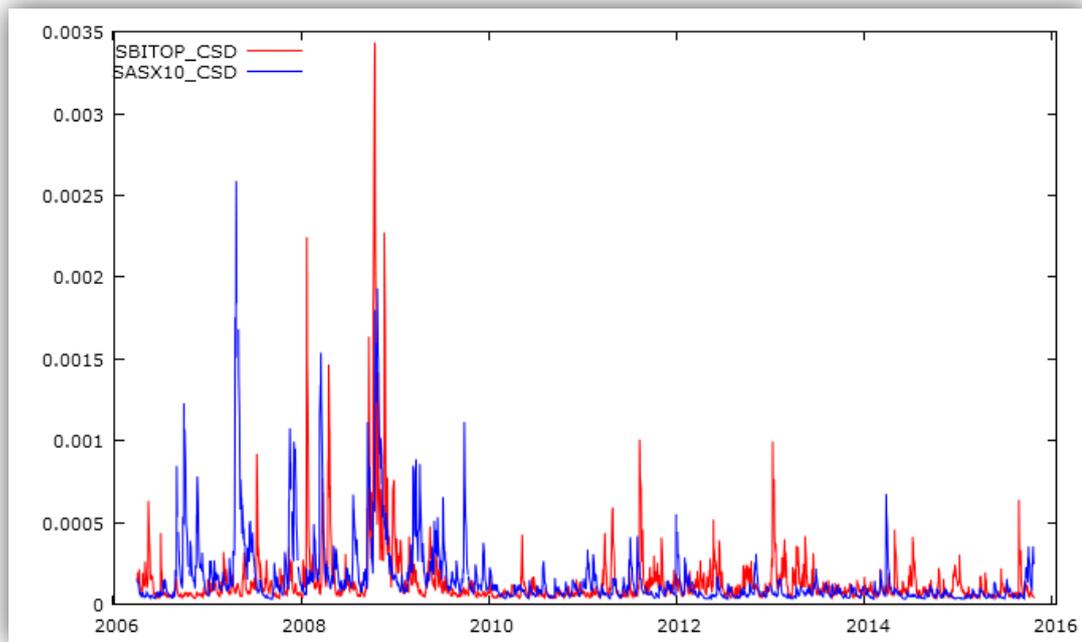
Figure 36: Comparison of CSD for SBI TOP and SASX-10

Both SASX-10 and SBI TOP had relatively small GARCH term (compared to other analyzed time series). Conditional standard deviation graph reflected this as large number of short-lived jumps for both series. In other words, both indices had short memory and persistence: volatility sharply increased and decreased during observed period.

SASX-10 has had higher conditional standard deviation just before the crisis as well as during initial stages of global financial meltdown. During the crisis and by the end of the observed period there was no clear "winner" in terms of conditional standard deviation size. Both indices were shifting from periods of high volatility to periods of low volatility and vice versa.

Next we will compare European markets (EURO STOXX50) conditional standard deviation to Bosnia and Herzegovina markets (see Figure **37** and Figure **38**).
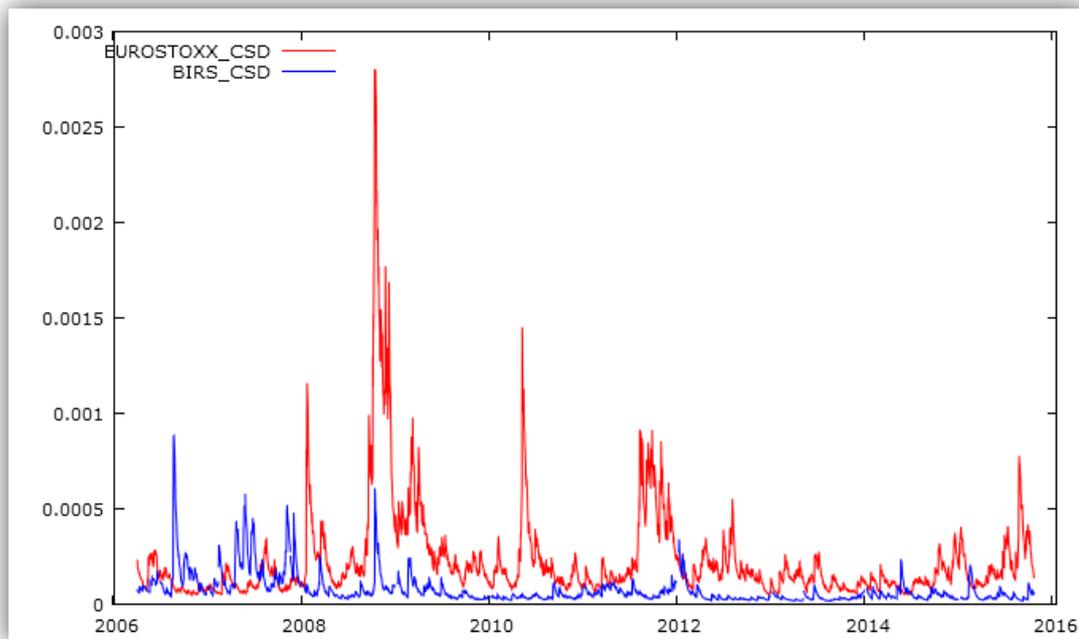
Figure 37: Comparison of CSD for EURO STOXX50 and BIRS

Looking at the conditional standard deviations graph, it is noticeable that BIRS had higher conditional standard deviation than EURO STOXX50 did just before crisis hit the markets. However, after the offset of crisis EURO STOXX50's conditional standard deviation "erupted" indicating stronger disruption in volatility. EURO STOXX50 maintained higher levels of conditional standard deviation until the end of the observed period. The only exception was early 2014 when BIRS vigorously reacted due to social unrest and political instability in Bosnia and Herzegovina.

GARCH modeling produced similar estimation results for both series - relatively high GARCH term translated to long volatility memory and persistence, as seen on the graph. This lead to the absence of prolonged periods of tranquility - it took time for the volatility peaks to die off for both series. In addition, series were quite synchronized during the crisis and post-crisis period, following similar patterns of volatility, although BIRS exhibited smaller intensity shocks compared to EURO STOXX50.

To conclude, BIRS was less volatile than European markets both during the crisis and afterwards.
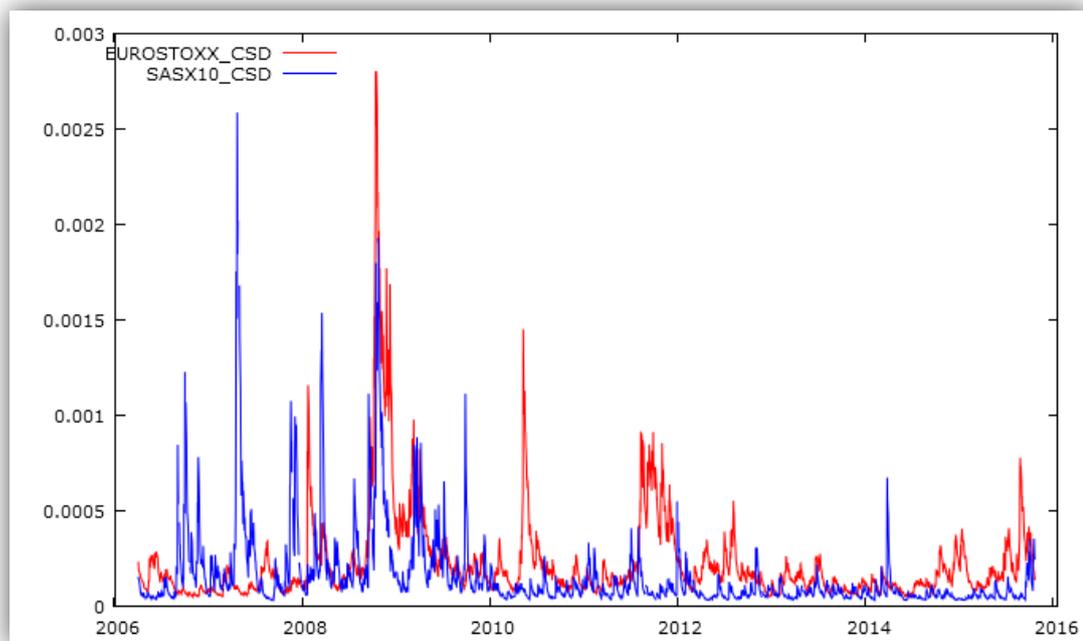
Figure 38: Comparison of CSD for EURO STOXX50 and SASX-10

Similar to BIRS, SASX-10 had much higher conditional standard deviation in late 2007 than EURO STOXX50 did. During the crisis, volatility response was quite synchronized for both series in terms of direction and intensity. Post-crisis period was characterized with lower levels of synchronization - both indices suffered shocks but increases and decreases of conditional standard deviations were not simultaneous too often.

SASX-10 had higher number of sharp short-lived peaks coming from lower GARCH term while EURO STOXX50 displayed longer memory and prolonged persistence manifesting as slow decaying peaks.

EURO STOXX50's conditional standard deviation responded sharply to early signs of sovereign debt crisis in May 2010, as investors feared this could lead to another global turmoil. Surprisingly, neither BIRS nor SASX-10 showed substantial surges of conditional standard deviation at this time. In August 2011, EURO STOXX50 reacted once more as debt crisis worsened in Portugal, Spain and Italy too. This time, SASX-10 and BIRS followed, albeit with some delay. Fear that Greece could leave Eurozone led to widespread turbulences across European markets in 2015, as seen

from EUROST0XX50's conditional standard deviation. SASX-10 response was quite synchronized although lesser in magnitude.

To summarize, SASX-10 had comparable volatility to European markets during most of the observed period. As expected, reaction was less aggressive to the shocks initiated by the debt crisis.
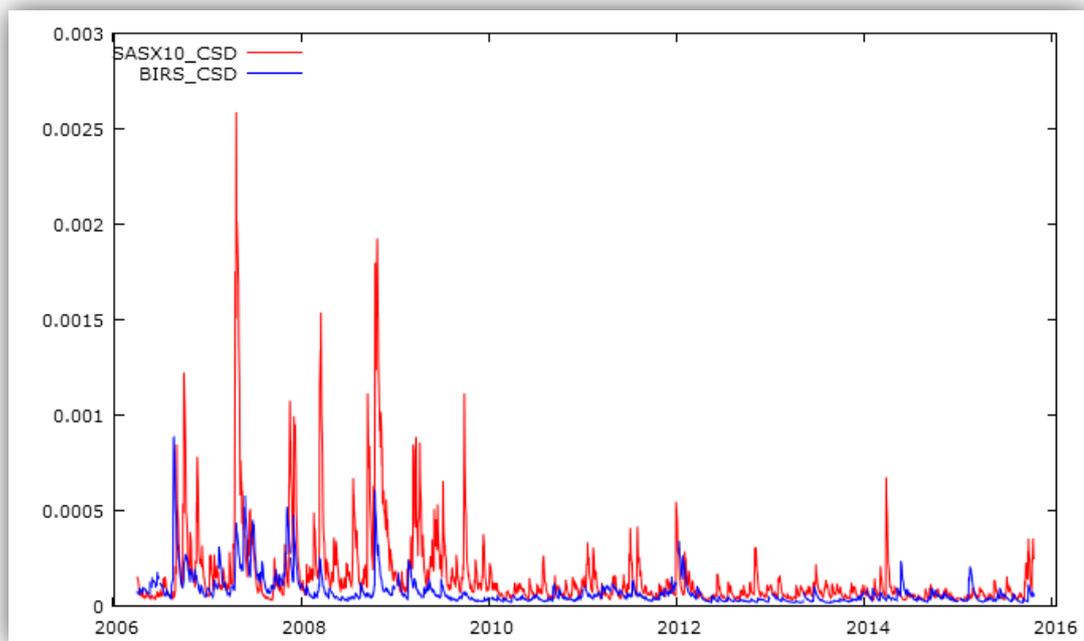
Figure 39: Comparison of CSD for SASX-10 and BIRS

Finally, we come to the comparison of BIRS and SASX-10 conditional standard deviations – the central comparison of this paper. It is evident right away from the graph (Figure **39**) that series are quite synchronized: indices followed similar volatility pattern both during the crisis and later. This comes as a no surprise since we are talking about different indices of the same country (Bosnia and Herzegovina). Hence, it is expected that market movements would be similar.

However, conditional standard deviation of the SASX-10 was consistently higher than conditional standard deviation of BIRS throughout observed period. Thus, SASX-10 has clearly proven to be more volatile. In part, this is because SASX-10

had higher long-run variance (constant) than BIRS did (as observable from the GARCH models estimation results).

Both indices displayed highest conditional standard deviation during the global financial crisis; SASX-10 being substantially more aggressive. Between 2010 and 2012 conditional standard deviations were much lower with periodic surges in volatility due to European debt crisis.

In early 2014 social unrest and political instability brought another disruption to the Bosnia and Herzegovina markets. While SASX-10 reacted with biggest post-crisis jump of conditional standard deviation, reaction of BIRS conditional standard deviation was less emphasized and somewhat delayed (compared to SASX-10).

Late 2015 was marked by another Greek debt crisis. Investors concerns over the future of Eurozone increased conditional standard deviation (primarily for SASX-10).

Estimation of GARCH under normal distribution has yielded higher GARCH term for BIRS (~ 0.88) than for SASX-10 (~ 0.78). Thus, BIRS was more influenced by past conditional volatility than SASX-10 was. As a result, BIRS was characterized by longer memory, prolonged persistence and slow decaying peaks. On the contrary, SASX-10 was represented by short-lived jumps of conditional standard deviation indicating sharp increases and decreases of volatility (i.e. short memory).

Not only BIRS was less volatile than SASX-10, it was surprisingly the least volatile among all analyzed time series. Thus, it was investors "safe haven" during the crisis and among safest former Yugoslavia markets.

However, as noted in Chapter 4, these results may be somewhat biased since both SASX-10 and BIRS had large number of identical index values for a number of consequent days. This phenomenon is most likely due to low or non-existing trading volume on specific days. Hence, to a certain extent volatility may have appeared lower than it really would be if trading actually took place.

Next, we present results of a dynamic forecast for SASX-10 and BIRS indices.

5.5. Volatility Forecast

We used Eviews 9.0 to evaluate 30 one-step-ahead out of sample dynamic forecasts for SASX-10 and BIRS time series.

Andersen and Bollerslev (1998) noted that squared daily returns are not appropriate measure for comparison of forecasting performance for different GARCH models. Hence, in order to evaluate and compare our models we used three standard measures:

a) Root mean square error (RMSE)

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\overline{\sigma_t^2} - \sigma_t^2)^2}$$

where T is number of forecasts made, σ_t^2 is actual volatility and $\overline{\sigma_t^2}$ is one-ste-ahead volatility forecast.

b) Mean absolute error (MAE)

$$MAE = \frac{1}{T} \sum_{t=1}^{T} \left| \overline{\sigma_t^2} - \sigma_t^2 \right|$$

Again, T is number of forecasts made, σ_t^2 is actual volatility and $\overline{\sigma_t^2}$ is one-ste-ahead volatility forecast.

c) Theil inequality coefficient (TIC)

$$TIC = \frac{\sqrt{\frac{1}{T} \sum_{t=1}^{T} (\overline{\sigma_t^2} - \sigma_t^2)^2}}{\sqrt{\frac{1}{T} \sum_{t=1}^{T} (\overline{\sigma_t^2})^2} \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\sigma_t^2)^2}}$$

The value of Theil inequality coefficient is always between zero and one. Zero indicates a perfect fit.

Table **11** and Table **12** present evaluation of out of sample volatility forecasts for BIRS and SASX-10 under different models and distributional assumptions.

Table 11: Forecasting analysis for BIRS index

	GARCH		GJR-GARCH	
	Normal	Student's t	Normal	Student's t
RSME	0.009103	**0.009099**	**0.009099**	**0.009099**
MEA	0.005730	0.005705	0.005712	**0.005703**
TIC	**0.940556**	0.972951	0.960586	0.977733

Source: Author's calculations

Table 12: Forecasting analysis for SASX-10 index

	GARCH		GJR-GARCH	
	Normal	Student's t	Normal	Student's t
RSME	**0.013085**	**0.013085**	0.013177	**0.013085**
MEA	0.008050	**0.008048**	0.008123	**0.008048**
TIC	0.972636	0.977535	**0.968769**	0.977535

Source: Author's calculations

Looking at the results, we see that forecasting performance of different models under different distributional assumptions is quite similar once with respect to three measures mentioned above.

For the BIRS index (**Table 11**), results are somewhat conflicting. Root square mean error produced the same values for GARCH model under Student's t-distribution and

GJR-GARCH model under both distributions. Hence, no model is clearly favored by RSME. On the other hand, mean absolute error favored GJR-GARCH model under Student's t-distribution while Theil inequality coefficient gave slight advantage to GARCH model under Normal distribution. All in all, considering all three measures combined, it seems that asymmetric GJR-GARCH model under Student's t-distribution offers best forecasting performance since it is favored both by RSME and MAE while GARCH under normal distribution offered the poorest forecasts.

For the SASX-10 index (**Table 12**), root square mean error produces ambiguous results once more: GARCH under both distributions and GJR-GARCH under Student's t-distribution have exact same value. Mean absolute error produced best results for GARCH and GJR-GARCH, both under Student's t. Theil inequality coefficient favors GJR-GARCH with normal distribution. Taking all three measures in consideration, we can conclude that GARCH under Student's distribution and GJR-GARCH under Student's t offer best forecasting performance since these models outperformed alternative models considering two out of three analyzed measures. On the other hand, GJR-GARCH under normal distribution produced poorest forecasting results.

To conclude, asymmetric GJR-GARCH model proved to be superior forecasting choice with regards to most analyzed measures. Student's t-distribution was more accurate in forecasting the conditional variance than normal distribution was.

6   Conclusion

The focus of this thesis was the analysis and comparison of stock market volatility in Bosnia and Herzegovina with three regional stock markets (Serbian BELEX15, Croatian CROBEX and Slovenian SBI TOP) as well as with European market (EURO STOXX50).

For this purpose, we utilized generalized autoregressive conditional heteroskedasticity (GARCH) type models. In particular, symmetric GARCH (1,1) and asymmetric GJR-GARCH (1,1) were developed in an attempt to model conditional volatility of the above-mentioned indices. Furthermore, conditional standard deviations were fitted from estimated models allowing us to compare evolution of volatility throughout observed period. Finally, we were able to dynamically forecast volatility for each Bosnia and Herzegovina market (SASX-10 and BIRS).

The results prove that all analyzed time series can be properly modeled with GARCH process, thus leading us to conclusion that our first hypothesis must not be rejected. Upon closer inspection of estimation results, we found that all observed time series exhibit relatively high persistence and long memory in volatility. Therefore, second hypothesis should not be rejected as well. The most persistent volatility was seen with Serbian and Croatian markets. Between two Bosnia and Herzegovina markets, Banja Luka Stock Exchange (BIRS) exhibited slightly higher persistence and longer memory in volatility. Significant leverage affect was observed for all countries except for Bosnia and Herzegovina where leverage effect was found to be negative but insignificant for both markets. As mentioned before in literature review section, Okičić (2014) also found surprising negative leverage effect on Bosnia and Herzegovina markets, so our results correspond in this aspect. However, contrary to our findings, Okičić found leverage effect to be significant. With regards to Serbian market, our results differ from Njegić et al. (2013) who found no leverage effect but are in line with Miletić and Miletić (2013) conclusions.

In terms of volatility evolution, Bosnia and Herzegovina markets display moderate to low level of synchronization with regional and European markets. Similar volatility pattern is observed for other time series: the conditional standard deviation is highest during the global financial crisis followed by period of relative serenity periodically interrupted with sharp jumps. In general, Sarajevo Stock Exchange (SASX-10) demonstrated higher level of synchronization with regional and European markets than Banja Luka Stock Exchange (BIRS) did.

While Serbian market displayed low levels of synchronization with European markets, Slovenian and Croatian markets were the most synchronized among all former Yugoslavia countries. These results further strengthen conclusions made by Horvath and Petrovski (2012) and Chocholata (2013) who also found low levels of integration and synchronization of Serbian market with European markets and moderate to high levels of integration for Slovenian and Croatian markets.

Sarajevo Stock Exchange (SASX-10) consistently had higher levels of the conditional standard deviation compared to Banja Luka Stock Exchange (BIRS), implying higher volatility of the former. Additionally, fitting of conditional standard deviation for Banja Luka Stock Exchange led us to conclusion that BIRS is the least volatile among all analyzed time series for the observed period. Hence, our third hypothesis is rejected.

Our findings provide several insights investor can benefit from. First, both Sarajevo Stock Exchange (SASX-10) and especially Banja Luka Stock Exchange (BIRS) exhibited lower levels of conditional standard deviation compared to regional and European markets during the global financial and sovereign debt crisis likewise. Thus, these markets may serve as "safe haven" at the time of a crisis for the investors. Second, it is well documented in the literature that less synchronized markets offer substantial diversification possibilities. We expect that the level of synchronization will increase as Bosnia and Herzegovina paves its way towards European Union thus decreasing diversification opportunities. However, at the time being level of synchronization is relatively low. Third, as we have seen, Bosnia and Herzegovina markets are characterized with high persistence and long memory in volatility. Hence, Sarajevo Stock Exchange and especially Banja Luka Stock Exchange may not

be appropriate for speculative and short-term trading since mean reversion is slow and shocks retain long lasting impact.

Our study has certain limitations that should be mentioned too. Developing and emerging markets are often characterized with market inefficiencies that can lead to distortion of prices and returns. Such inefficiencies have already been documented for Bosnia and Herzegovina markets by Okičić (2014). Furthermore, it is not very unusual for these time series to have same value for several consecutive days as a result of low (or non-existing) trading volume leading to somewhat biased inferences regarding volatility.

Finally, we offer some suggestions for further research. It would be interesting to see if modeling can be improved by expending the selection of (G)ARCH family models and/or by using different distributional assumption (such as generalized normal distribution). Moreover, comparison with other developing markets might provide additional insights. Furthermore, it would be intriguing to analyze if and how volatility would change as Bosnia and Herzegovina moves closer to European Union accession. At last, with high-frequency data at hands it would be possible to forecast realized volatility by using heterogeneous autoregressive (HAR) model and potentially extend inferences of this study.

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Appendix

Figure A.40: ACF and PACF for Stock Indicies Levels
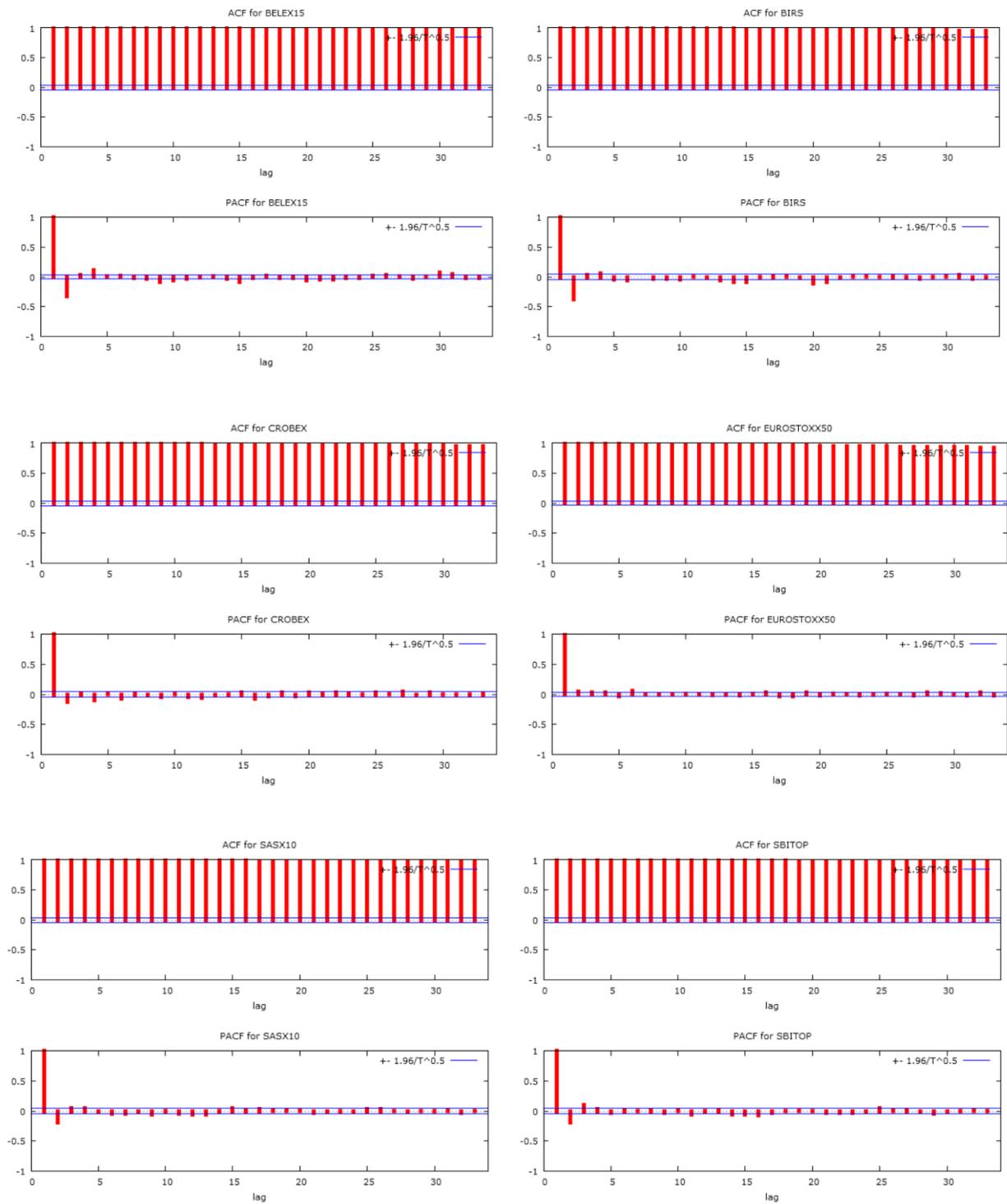
Figure A.41: ACF and PACF for Stock Indicies Returns
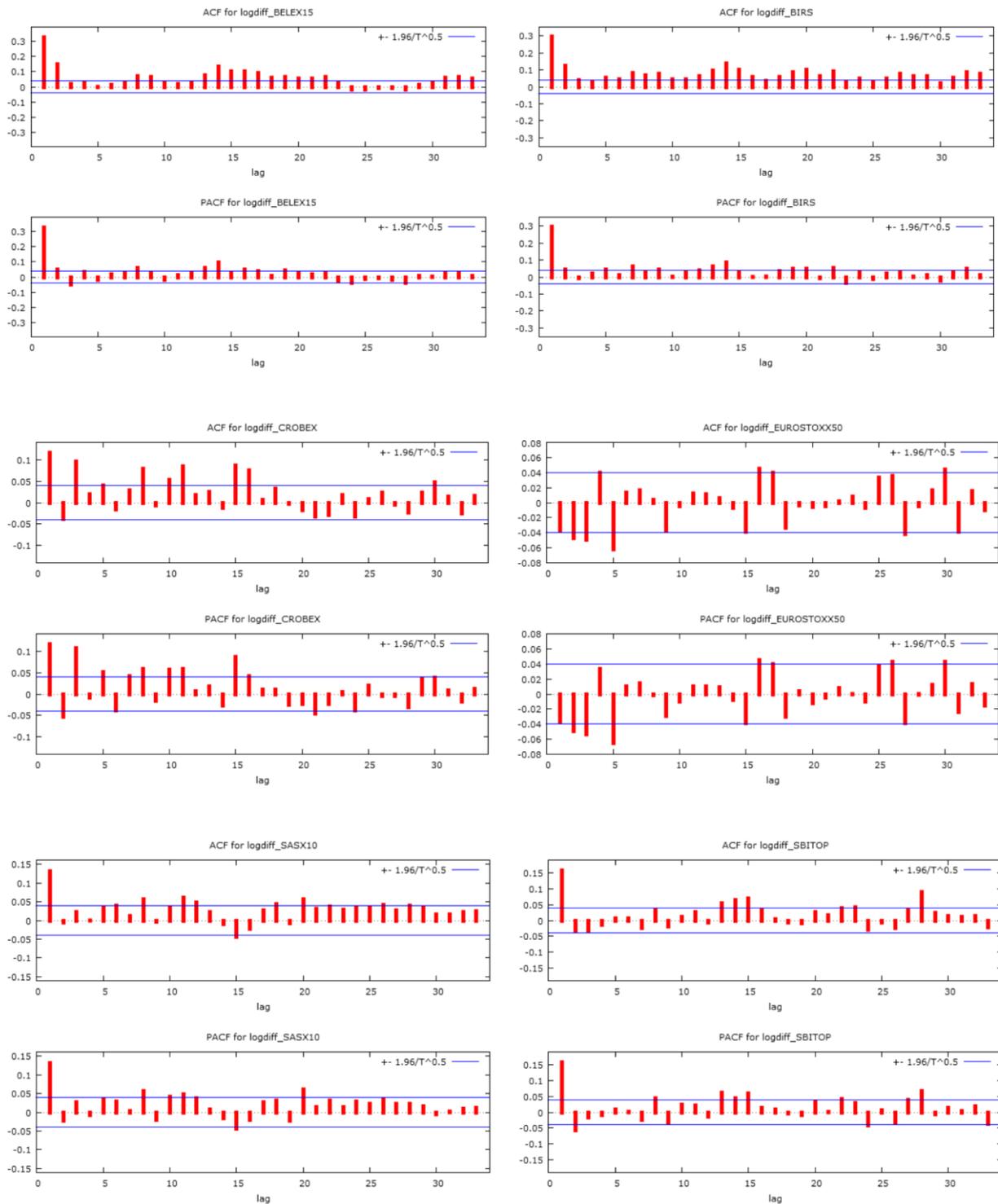
Figure A.42: ACF and PACF of ARMA residuals
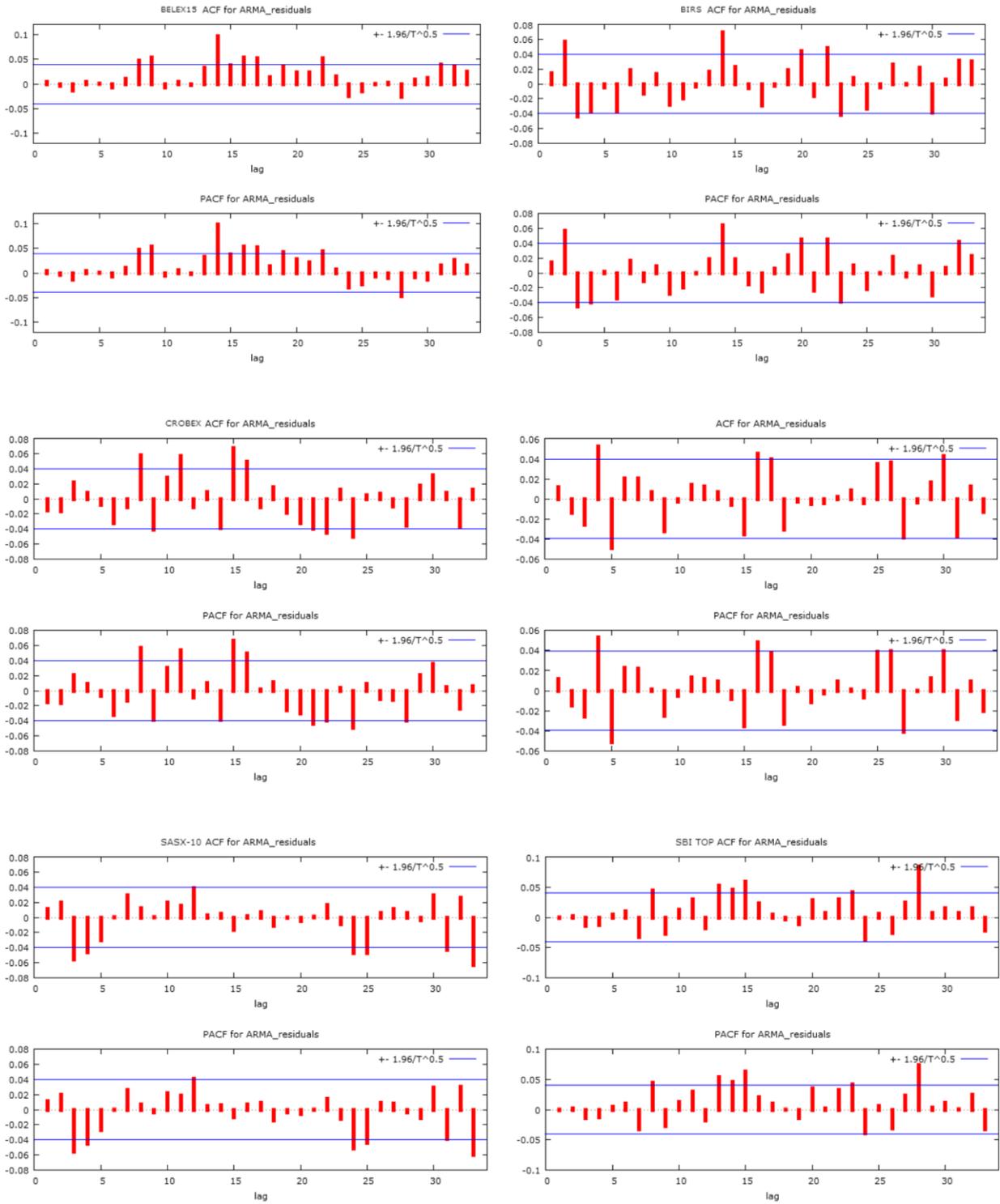
Figure A.43: ACF and PACF of ARMA squared residuals
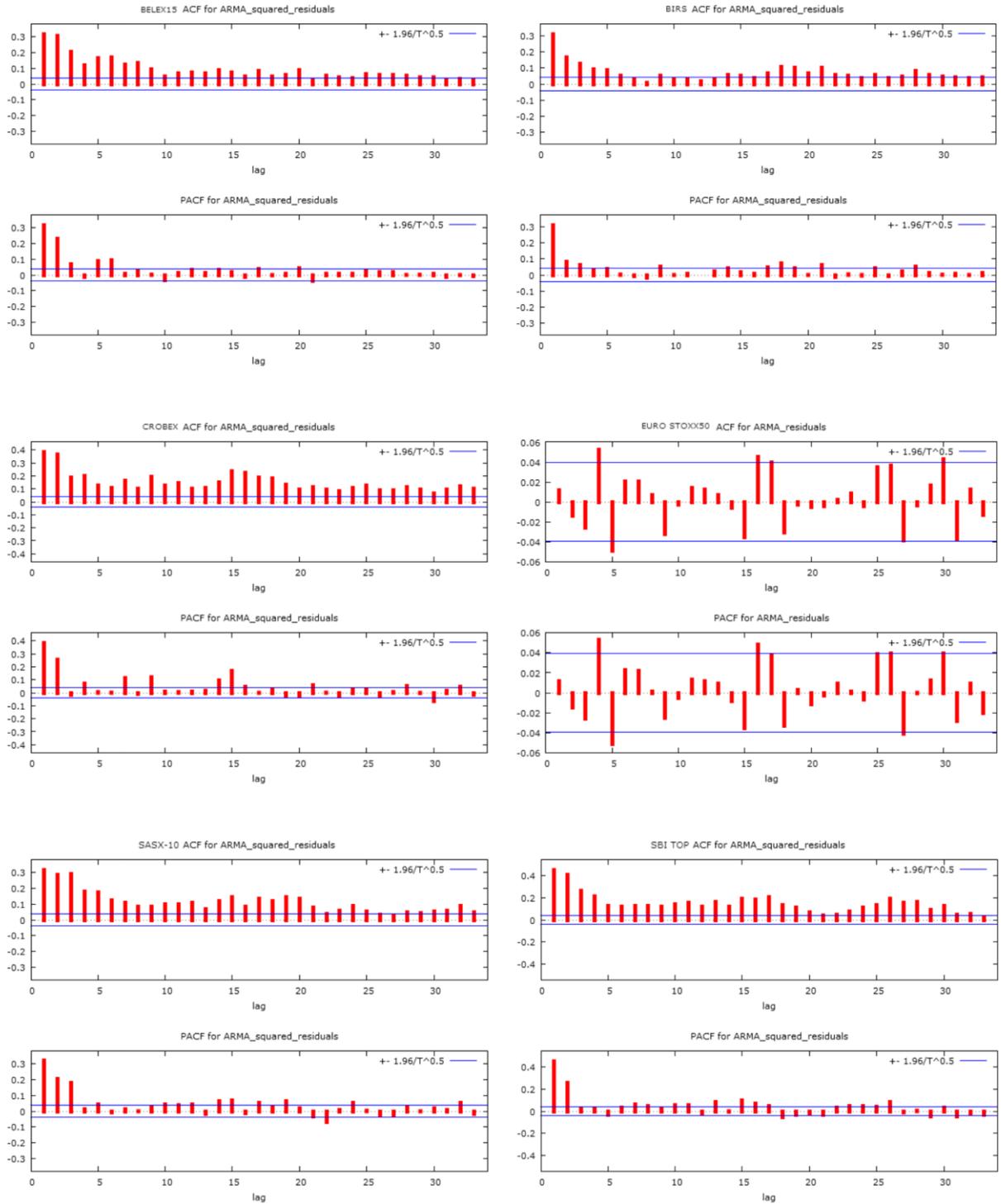
Table A.13: ARCH-LM Test Results

ARCH-LM TEST		
Index	**Test statistic: LM**	**p-value**
BELEX15	376.227	3.93364e-079
BIRS	249.582	6.75724e-052
CROBEX	486.738	5.81626e-103
EURO STOXX50	399.533	3.73984e-084
SASX-10	403.433	5.39934e-085
SBI TOP	605.944	1.0508e-128

Source: Author's calculations

Table A.14: Ljung-Box Test Results

Ljung- Box Test		
Index	**Ljung-Box Q**	**p-value**
BELEX15	0.573966	0.7505
BIRS	16.1695	0.0003082
CROBEX	2.58698	0.1077
EURO STOXX50	15.2596	0.001608
SASX-10	15.9977	0.0003358
SBI TOP	0.937302	0.8164

Source: Author's calculations

Table A.15: GARCH (Normal) estimation of post-crisis subsample

Index/Varible	BELEX15	CROBEX	EURO STOXX 50	SBI TOP	BIRS	SASX-10
Mean Equation						
C	0.000103 (0.404369)	-0.000102 (-0.864760)	0.000540 * (1.814096)	-0.000219 (-0.859217)	1.98E-06 *** (36.86388)	1.94E-06 *** (29.51362)
AR (1)	0.731521 *** (8.749563)	0.222947 *** (4.269889)	0.399940 (0.859166)	0.103457 *** (3.661729)	-0.057002 (-1.188095)	NA
MA (1)	-0.624203*** (-6.263272)	-31.62018 * (-1.908974)	178.8780 *** (38.15027)	NA	NA	-0.004595 (-0.133755)
AR (2)	NA	-0.872919 *** (-25.40629)	-0.026320 (-0.682980)	NA	NA	NA
MA (2)	NA	386.8671 *** (32.69993)	-78.25221 (-0.938527)	NA	NA	-0.004307 (-0.129725)
Variance Equation						
Constant	3.47E-06 *** (5.549358)	3.10E-11 *** (5.136688)	1.72E-10 *** (4.225924)	1.13E-05 *** (6.160024)	7.65E-13 (0.514591)	6.83E-13 *** (3.349588)
ARCH term	0.151719 *** (8.878305)	0.165164 *** (8.400102)	0.090806 *** (7.805722)	0.194720 *** (9.378533)	-0.005310 (-1.181077)	-0.036952 *** (-7.196083)
GARCH term	0.801612 *** (42.36167)	0.786067 *** (30.89584)	0.880996 *** (56.01985)	0.695462 *** (20.96828)	0.579873 (0.705922)	0.580119 *** (4.260863)
Leverage effect	NA	NA	NA	NA	NA	NA

Source: Author's calculation

Table A.16: GARCH (Student's t) estimation of post-crisis subsample

Index/Varible	BELEX15	CROBEX	EURO STOXX 50	SBI TOP	BIRS	SASX-10
Mean Equation						
C	1.55E-05 (0.067991)	-3.37E-05 (-0.203582)	0.000537 * (1.812796)	-0.000250 (-1.106904)	1.55E-06 *** (92.35020)	1.82E-06 *** (7.741914)
AR (1)	0.730243 *** (8.132924)	0.021088 *** (0.000118)	0.363008 (0.719158)	0.083221 *** (2.982327)	-0.012864 (-1.517363)	NA
MA (1)	-0.623811 *** (-5.950690)	0.014300 (8.01E-05)	876.6594 *** (38.17109)	NA	NA	-0.003490 (-0.088075)
AR (2)	NA	0.001159 (0.000110)	-0.029552 (-0.799109)	NA	NA	NA
MA (2)	NA	-0.003990 (-0.000237)	-345.0188 (-0.777829)	NA	NA	-0.016544 (-0.430853)
Variance Equation						
Constant	3.64E-06 *** (3.606598)	2.74E-05 (1.020011)	6.80E-12 *** (3.961686)	9.42E-06 *** (3.789813)	6.73E-13 *** (4.442260)	6.02E-13 (1.642099)
ARCH term	0.145110 *** (5.505223)	-0.008791*** (-4.048037)	0.089675 *** 7.569720)	0.191182 *** (5.263062)	-0.008308 (-1.200052)	-0.030574 *** (-3.856152)
GARCH term	0.804323 *** (26.64855)	0.579780 (1.369618)	0.883020 (55.68725)	0.726609 *** (15.98502)	0.551868 (1.434964)	0.570329 *** (2.494161)
Leverage effect	NA	NA	NA	NA	NA	NA

Source: Author's calculation

Table A.17: GJR-GARCH (Normal) estimation of post-crisis subsample

Index/Varible	BELEX15	CROBEX	EURO STOXX 50	SBI TOP	BIRS	SASX-10
Mean Equation						
C	0.000233 (1.155691)	-4.31E-05 (-0.190819)	-0.001004 ** (-2.026711)	-0.000445 * (-1.744931)	1.75E-06 *** (44.71500)	1.47E-06 *** (152.0938)
AR (1)	0.122795 *** (4.697023)	0.096534 (0.012138)	1.968502 *** (171.6967)	0.101300 *** (3.720571)	-0.167125 *** (-8.687557)	NA
MA (1)	3.55E+09 *** (38.20990)	-0.080343 (-0.010086)	-1.945269 *** (-148.0491)	NA	NA	-0.015059 *** (-11.07549)
AR (2)	NA	0.805286 (0.106702)	-0.972220 *** (-87.03650)	NA	NA	NA
MA (2)	NA	-0.790623 (-0.106434)	0.950778 *** (74.86472)	NA	NA	-0.024209 *** (-10.77489)
Variance Equation						
Constant	-3.660093 *** (-6.175778)	-0.287101 *** (-4.102847)	-0.246157 *** (-5.445123)	-1.341854 *** (-6.187067)	-40.51586 *** (-42.04879)	-11.53433 *** (-42.78748)
ARCH term	0.323050 *** (13.65130)	0.145718 *** (7.998104)	0.040007 ** (2.515689)	0.327535 *** (11.05858)	1.632454 *** (8.539955	1.178658 *** (7.548683)
GARCH term	0.936465 *** (85.82946)	0.982187 *** (167.6032)	0.975018 *** (205.8902)	0.882970 *** (40.64634)	-0.462832 *** (-12.63265)	0.574227 *** (55.84808)
Leverage effect	-0.000597 (-0.045473)	-0.017578 *** (-2.583833)	-0.231222 *** (-11.88469)	-0.022589 (-1.286412)	-1.537259 *** (-7.629769)	-2.832632 *** (-17.79507)

Source: Author's calculation

Table A.18: GJR-GARCH (Student's t) estimation of post-crisis subsample

Index/Varible	BELEX15	CROBEX	EURO STOXX 50	SBI TOP	BIRS	SASX-10
Mean Equation						
C	7.40E-05 (0.329016)	-0.000166 (-1.104220)	-0.000417 (-1.406569)	-0.000346 (-1.539854)	1.36E-06 *** (14090.36)	1.36E-06 *** (11851.62)
AR (1)	0.727721*** (7.998470)	-0.284680 (-1.018857)	-0.432569 (-0.980362)	0.076654 *** (2.809740)	-6.59E-06 (-0.331980)	NA
MA (1)	-0.622200 *** (-5.871604)	0.341712 (1.221744)	-214.6494 *** (-36.17379)	NA	NA	-4.90E-05 * (-1.787824)
AR (2)	NA	-0.649655 *** (-3.503400)	-0.019302 (-0.584674)	NA	NA	NA
MA (2)	NA	0.651173 *** (3.478925)	-97.36108 (-1.025672)	NA	NA	3.21E-06 (0.070792)
Variance Equation						
Constant	-0.834084 *** (-4.721000)	-0.226172 *** (-2.861733)	-10.87969 *** (-12.16398)	-1.262941*** (-4.475218)	-27.51824 *** (-11.61797)	-23.26459 *** (-15.25375)
ARCH term	0.301802 *** (8.068527)	0.108538 *** (4.345578)	0.870493 *** (11.84097)	0.335384 *** (6.636823)	-0.011745 (-0.000957)	0.016695 (0.005037)
GARCH term	0.938005 *** (55.81610)	0.985850 *** (146.4425)	0.465455 *** (10.15248)	0.891484 *** (31.72303)	0.028927 (0.345847)	0.250733 *** (5.050497)
Leverage effect	0.004516 (0.203749)	-0.030672 ** (-2.376811)	-0.257260 *** (-5.306860)	-0.010683 (-0.365982)	-0.235984 (-0.019220)	-0.146680 (-0.044250)

Source: Author's calculation